

## **The T-shirt task: Using a mathematical task as a means to get insights into the nature of the collaboration between in-service teachers and researchers**

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By following a mathematical task, from its design by researchers to its implementation by a teacher, it is possible to get some insights into the collaboration between researchers and teachers. Activity Theory is used as a theoretical approach in this research.

**Keywords: designing and implementing tasks, collaboration between researchers and teachers, activity theory.**

### **The research setting**

The research reported here examines the way a mathematical task is transformed and adapted, from its design among researchers to its implementation by a teacher in lower secondary school level. This research is situated within an ongoing research project at the University of Agder (UiA) called TBM (Teaching Better Mathematics). The name of the project reflects two goals: the first one concerns Teaching (Better Mathematics) and aims at developing better understanding of, and competency in mathematics for pupils in schools. The second one concerns (Teaching Better) Mathematics and aims at exploring better teaching approaches in order to achieve the first aim. The research involves collaboration between researchers from university and teachers working at different levels, from kindergarten up to upper secondary school. Within the project, we collaborate with 4 kindergarten, 6 primary schools and lower secondary schools, and 3 upper secondary schools. The collaboration with the teachers is organised around workshops which happen approximately once a month and consist of a plenary presentation of a theme within mathematics and some group work where the teachers have the opportunity to work collaboratively with colleagues at the same school level. In addition, within each school, a group of two or three teachers (called the TBM group) is responsible for the continuity between the work done at the university during the workshops and the teaching of mathematics in their respective schools. An important feature of this research is the recognition of the researchers and the teachers as working together as co-learners and getting the opportunity to develop a better understanding of each others “world and its connections to institutions and schooling” (Wagner 1997, 16).

### ***The centrality of inquiry***

The idea of inquiry plays a crucial role in the TBM project, as we consider that through the project we, as researchers, are able to address inquiry at three different levels (Jaworski 2006): at a first level, as pupils engage with a task and inquire into the mathematics, at a second level, as teachers engage with inquiring into the developmental process of planning for their teaching, and at a third level, as researchers engage with inquiring into the research process of systematically exploring the developmental process and practices as presented at the two first levels. Thereby, all participants of the TBM project engage with inquiring into how to improve mathematics learning and teaching in classrooms.

Furthermore, we consider that inquiry could form a basis for the teachers' teaching practice in mathematics.

### ***Activity Theory as a means to characterize development***

In our project we use Activity Theory as a means to describe and characterise both the researchers' and the teachers' development. Within Activity Theory, the idea of "activity" endorses a precise meaning: human activity is understood as directed by a motive and is firmly rooted in actions and goals. Furthermore, these actions are carried out through operations and conditions (Engeström 1999). Thereby, we see our motive within the TBM project as being to engage, collaboratively with teachers, in inquiry about teaching and learning of mathematics in order to improve pupils' achievement in mathematics. Our actions consist of organising workshops and designing suitable mathematical tasks, while the notions of operations and conditions refer to searching and collecting ideas and adapting these to the workshop environment and classroom situation. In order to deepen the ideas of actions and operations, I propose to introduce the theoretical constructs of "didactical aim" and "pedagogical means" (Berg 2009). Didactical aim refers to the choice of a particular area or subject-matter as for example symmetry, algebra or proportionality, while pedagogical means refers to a task which is chosen and used in order to address the chosen didactical aim. Here I consider that the construct of "didactical aim" as a useful theoretical tool enabling me to pull out, articulate, and make visible central issues in relation to both the collaboration with teachers, and more specifically, the design and preparation of workshops, and the teachers' planning and preparation of their own teaching. Thereby, "didactical aim" relates both to the researchers' actions (preparation of workshops) as it emphasises which mathematical goal the researchers plan to address during the workshops, and to the teachers' actions (preparation of teaching) as it emphasises which mathematical goal the teachers plan to address during their teaching. Likewise, "pedagogical means" serves as a theoretical tool describing the result of the processes of choosing, transforming and adapting a particular task to a specific social setting in order to address a chosen didactical aim. Thereby, "pedagogical means" refers both to the researchers' operations in the sense of assembling a set of tasks and adapting these to the group session during the workshops, and to the teachers' operations as they prepare and adapt a particular mathematical task to their own classes. Therefore I consider that "by presenting a particular task within a specific social setting, a didactician creates a mathematical environment whose characteristics depends both on the mathematical task and on the setting" (Berg 2009, 103). In this article, I present and compare two mathematical environments: the first one relates to a specific task prepared by the researchers in order to engage collaboratively with teachers during a particular workshop, the second one relates to how a teacher implement this particular task in his teaching.

### ***Focusing on a specific task***

In my current research, I follow a particular task, the T-shirt task, from its design during a meeting among researchers at UiA (TBM meeting, 26.11.08), its presentation to the teachers during a workshop (03.12.08), and to its implementation in a primary school (11.12.08) and in a lower secondary school (05.05.09). The rationale for choosing this particularly task is the following: in December 08, a teacher from primary school contacted me and invited me to follow her teaching as she wanted to implement a task from a previous workshop in her teaching. Similarly, in May 09, another teacher (Per) from lower secondary school contacted me as he planned to implement a task from a previous workshop in his teaching. The fact that these teachers were referring to the same mathematical task (the T-

shirt task) encouraged me to focus my research on the processes behind the design and implementation of this specific task. Because of space limitation, I focus on the results of the analysis of the mathematical environment created by Per.

### ***The TBM meeting***

The T-shirt task was elaborated during the TBM meeting on November 26<sup>th</sup>. During that meeting, the discussion among the group of researchers was focusing on how to address “communication in mathematics classroom”. This theme had been chosen in advance by the TBM group of the different schools. While exploring different possibilities for addressing this subject, we decided to contextualise the discussion and to address “communication in mathematics classroom” through engaging with a particular task. We chose the T-shirt task as we agreed that this specific task offered a rich approach to communication. As one of the participants emphasised:

... in communicating mathematics, questions are a far more effective way of communicating than telling. In order to make sense of mathematical knowledge, pupils need to take the responsibility for exploring which means questioning the teacher, questioning others. The fundamental aspect about communication is *questioning*. (TBM meeting, 26.11.08, translated from Norwegian by the author)

Thereby, our group decided to focus on the ability to ask “good” questions, in the sense of engaging with task by inquiring into the mathematics (Jaworski 2006). The T-shirt task was elaborated in the following way: the context is a phone call where one person has to explain to another one the motive of a logo to be reproduced on a T-shirt (see Figure 1).

#### **T-skjorte oppgaven**

*Mål: Matematisk kommunikasjon, bruk av spørsmål, terminologi og begreper*

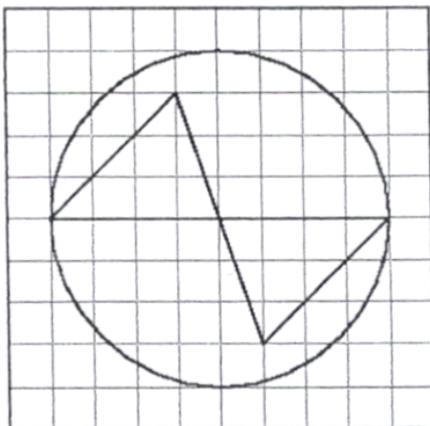


Figure 1: The T-shirt task presented within a grid system behind the logo

### ***The workshop (03.12.08)***

Usually the workshops are organised according to the following pattern: first one of the didacticians from the university gives a plenary presentation on a chosen theme. Second, a group session is organised where all participants are divided into groups according to the level at which they teach. Finally, all participants gather together in order to exchange experiences from the group sessions. The title of the workshop on December 3<sup>rd</sup> was “To ask good questions in mathematics”, and it was one of the didactician from the university who had the responsibility to introduce to the participants of the workshop (teachers from different school levels, from kindergarten to upper secondary school) ideas related to questioning and

communicating in mathematics and to link these to the idea of “inquiry”. During the group session, all teachers engaged with the T-shirt task and discussed different approaches to it with emphasis on the kind of questions which are relevant to ask in order to reproduce the logo in an accurate way. In addition, they discussed the possibility of integrating, changing and adapting this task to their respective classes.

### ***Summary of the design process***

Looking at the design process of the T-shirt task, the discussion during the TBM meeting concerned “Communication in mathematics” and we decided to emphasise questioning and its relation to inquiry. Furthermore, we chose to contextualise the discussion through presenting a mathematical task and the T-shirt task was selected as a relevant example. Using the theoretical constructs from Activity Theory, I understand the idea of “actions” as referring both to the researchers’ preparation of workshops, and to the teachers’ preparation of their own teaching. These actions are linked to the didactical aims chosen by the researchers and by the teachers. Furthermore, I consider the construct of “operations” as referring both to how the researchers assemble a set of tasks and adapt these to group sessions during the workshops, and to how the teachers prepare and adapt tasks to their own classes. Likewise, I consider that these operations offer a description both of the researchers’ and of the teachers’ pedagogical means.

### **Implementation of the T-shirt task in lower secondary school**

Before observing how the teacher, from lower secondary school, implemented the T-shirt task in his class, I had the opportunity to interview him and, thereby, to make visible his aims for the teaching period. Likewise, I conducted an interview right after the teaching period in order to summarise and evaluate it with the teacher. In this article I focus on the interview before the teaching period.

#### ***Visiting a lower secondary school: in class with Per***

On May 5<sup>th</sup>, I had the opportunity to visit Per and to observe his teaching in grade 8. During the interview he explained the rationale for implementing the T-shirt task in his class and he emphasised particularly on the following aspects:

From the curriculum, there are first of all two aspects which I would like to have as goals for my teaching, it is the use of coordinate system, and the second is the introduction of functions... And you can say, what I want to emphasise is communication, I would like the pupils to have an understanding of how one communicates in mathematics.

From his utterance, it seems that Per decided to implement the T-shirt task in his class since he could recognise the possibility to address two aspects from the curriculum through this task. The first one refers to the use of coordinate system, the second one relates to the introduction of functions. In addition, Per emphasised communication in mathematics. It was the first aspect (use of coordinate system) which was emphasised during the teaching period I observed (right after the interview). Per organised his teaching by dividing the lesson into two parts. During the first part, he asked one pupil to be responsible for explaining the logo of the T-shirt task to another pupil who was sitting behind a blackboard and could not see the logo. Here Per had prepared in advance a slightly different representation of the logo: the logo of the T-shirt was drawn *without* a grid system behind it and he presented this version to the pupil. From the classroom observation, it is possible to follow how the pupil struggled to

explain the respective positions of the circle and triangles, as drawn on the logo. During the second part of the lesson Per asked another pupil to be responsible for explaining the logo of the T-shirt task, this time *with* the grid system (see Figure 1). Here it is possible to follow how the pupil used the grid as a coordinate system and could refer to the circle and the triangles by indicating the coordinates of particular points. During both exchanges (description with and without a grid system) the rest of the pupils could follow how the communication of the logo was influenced by having the opportunity or not to use a coordinate system.

My interpretation of Per’s way of implementing the T-shirt task is the following: as the T-shirt task was introduced during the workshop of December 3<sup>rd</sup>, Per was able to identify the possibility to address two aspects from the curriculum, the use of coordinate system and the introduction of functions. I understand these as didactical aims which Per plan to address during his teaching, or using the theoretical tools from Activity Theory, these didactical aims are the “goals” for his “actions”. Thereby, during the lesson I observed, Per was addressing the use of coordinate system as one of his didactical aims. At the same time, as he implemented the T-shirt task by contrasting the presentation of the logo *with* and *without* the grid system behind it, he was in a position of emphasising communication in mathematics. Thereby, my understanding of Per’s teaching is that his didactical aim for that lesson was to address the use of coordinate system and he modified and adapted the T-shirt task to his class in order to achieve his didactical aim. Here Per’s “operations” were to produce the modified version of the T-shirt task (presentation *with* and *without* the grid system behind the logo) and therefore this new task acted as a pedagogical means which was chosen and used in order to address the chosen didactical aim. In addition, this comparison allowed Per to emphasise on communication in mathematics.

**Comparing the researchers’ and Per’s didactical aims and pedagogical means**

Looking back to the researchers’ elaboration and preparation of the workshop comparing to the interview with Per before his teaching, it is possible to observe an inversion between the didactical aims and pedagogical means:

	Researchers	Per
Didactical aim	Communication	Coordinate system
Pedagogical means	T-shirt task (coordinate system)	T-shirt task (communication)

Table 1: Comparing the researchers’ and Per’s didactical aims and pedagogical means

A possible explanation for the observed inversion consists of taking into consideration and recognising the fact that the teacher and the researchers belong to two different activity systems. Furthermore, this recognition begs the following question: how can this inversion be understood using the theoretical constructs available within Activity Theory? As explained earlier, I conceptualised our research group at UiA in terms of activity system where our activity is motivated by a desire to engage, collaboratively with teachers, in inquiry about teaching and learning of mathematics as a means to improve pupils’ achievement in mathematics. The goals of our actions, consisting of workshops and school visits, were to initiate and make possible the collaboration with teachers. In addition, according to Engeström (1999), an activity system is defined by the “rules”, the “community”, and the “division of labour” followed by the activity system. Concerning our research group, the community of researchers consists of 5-6 persons working within the TBM project, and the

division of labour is made visible as we prepare the different workshops and are responsible for school visits. Concerning the dimension called “rules”, one aspect of it is visible in our community as our group takes into consideration the wishes emerging from the teachers’ TBM group at each school in relation to the choice of didactical aims. The teachers usually suggest several themes to us and from these ideas we discuss how to organise the different workshops in a coherent way. Thereby, within our activity system, one of the rules is to recognise and elaborate on the different themes proposed by the teachers. Communication in mathematics is one example of the teachers’ suggestions. Looking at the teachers’ activity system at their respective schools, I understand the motive for their activity as creating opportunities for engagement with mathematics, and offering critical guidance for what mathematics achievement means. Concerning the goals of their actions (teaching), I see these as being the organisation of pupils’ participation into mathematics classrooms. At the teachers’ schools, the community consists of all teachers and colleagues working within the administration, and the division of labour is clearly decided by the head teacher within each school. Concerning the dimension “rules”, one of the constraints for the teachers is to follow the curriculum. It seems that it is this rule which became visible through Per’s utterance. Thereby, I consider that by following a specific mathematical task from its design by researchers to its implementation in Per’s class, the research reported here enables me to compare and to observe an inversion between the researchers’ and Per’s didactical aims and pedagogical means. A possible explanation for this inversion consists of recognising the researchers’ and the teachers’ communities as belonging to two different activity systems each of them having different rules. I argue that this recognition helps us, as researchers, to get deeper understanding of each others’ world (Wagner 1997).

## References

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