Symbolic addition tasks, the approximate number system and dyscalculia

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Several recent theorists have proposed that dyscalculia is the consequence of a disconnect between the so-called 'approximate number system' and formal symbolic mathematics. Such theories propose that symbolic exact mathematics is built out of approximate representations of quantity. Here we investigate this proposal by testing whether non-dyscalculic adults appear to use their approximate number systems when tackling symbolic tasks. We find a strikingly similar pattern of responses on two approximate addition tasks, one where participants saw numerosities represented as dots and one where the numerosities were represented with Arabic symbols. These findings are consistent with the view that non-dyscalculic adults do indeed use the approximate number system when dealing with symbolic mathematics.

Keywords: approximate number system, arithmetic, dyscalculia, number sense.

There is an increasing body of evidence that humans have an inbuilt 'number sense' – or approximate number system (ANS) – which supports approximate numerical operations (e.g., Cordes, Gelman, Gallistel & Whalen 2005, Dehaene 1992, 1997). The ANS, which is present in infants, children and adults, involves approximate, abstract representations of number that support both the comparison and manipulation of numerosities (Barth, Kanwisher, & Spelke 2003, Barth et al. 2006; Pica, Lemer, Izard, & Dehaene 2004). When adults and children compare or add symbolic numerals, these approximate representations seem to be activated (Dehaene 1997, Gilmore, McCarthy, & Spelke 2007, Moyer & Landauer 1967).

It is even suggested that a disconnect between the ANS and formal symbolic mathematics is the root cause for so-called mathematics disorder or dyscalculia. The *Diagnostic and Statistical Manual of Mental Disorders* (American Psychiatric Association 2000) describes mathematics disorder as a condition which causes a person to have substantially lower scores on mathematics tests than would be expected given their age, intelligence and educational background. The Department for Education and Skills (2001), using the term dyscalculia, offered a similar characterisation, emphasising how difficult it is for dyscalculic students to acquire "simple number concepts". Rouselle & Noël (2008) have suggested that dyscalculia is the result of a disconnect between approximate numerical representations held in the ANS and the symbols used in formal mathematics. They found that children classified as dyscalculic could perform well on non-symbolic comparison tasks (i.e. tasks of the form "are there more blue dots or red dots?"), but could not successfully tackle the equivalent tasks represented symbolically.

The suggestion that dyscalculia is the result of a disconnect between the ANS and symbolic mathematics is consistent with other recent claims that the ANS provides the basis of all exact mathematics (e.g. Gilmore, McCarthy & Spelke 2007). But if these accounts were correct, we would expect that non-dyscalculic adults, who are fluent in simple arithmetic tasks, would somehow harness the ANS when tackling approximate symbolic mathematics tasks. Our goal in this paper is to explore this account of dyscalculia by examining the behaviour of non-dyscalculic students during both symbolic and non-symbolic addition tasks.

Experiment 1

The primary goal of Experiment 1 was to provide a baseline for the ANS in the context of non-symbolic addition tasks. Participants saw three numerosities, n_1, n_2, n_3, represented as coloured dots. Their task was to determine which was greater out of n_1+n_2 and n_3. Previous research has found that tasks during which the ANS is used to compare numerosities tend to show a ratio-effect. That is to say that, because representations within the ANS are only approximate (internal representations of a numerosity n are assumed to be distributed normally around n), comparisons of numerosities which have a ratio close to one lead to more errors than those of numerosities with a ratio far from one (e.g., Barth et al. 2006, Gilmore et al. 2007). We would expect to find such an effect in the current task.

Method

Twelve staff and students (seven male) from the University of Nottingham (aged 23-36, M=29) received £4 for participating in the study.

Participants were tested individually. Displays were presented on a 17" Philips 170B LCD placed at eye level, and were viewed from approximately 60cm away. The stimuli consisted of three dot arrays. The two addend arrays were blue dots against a white background and the comparison set array was red dots against a white background. To prevent participants using strategies based on continuous quantities correlated with number (dot size, luminance, total enclosure area), the stimuli were created following the method of Pica et al. (2004). For each problem two sets of stimuli were created: one in which the dot size and total enclosure area decreased with numerosity, and one in which the dot size and total enclosure area increased with numerosity.

Fifty problems were used for each approximate ratio. Sum totals for number to sum ratios less than 1 were the integers from 21 to 70, sum totals for number to sum ratios greater than 1 were the integers from 11 to 60. Comparison numbers were related to sum totals by approximate ratios 8:5, 7:5, 6:5, 5:6, 5:7, 5:8. The addends were randomly chosen in such a way that neither was larger than the comparison number, or less than 5 (for those problems with a sum total less than 30) or 10 (for all other problems). Each set of dots was displayed for 300msec, in the order shown in Figure 1.

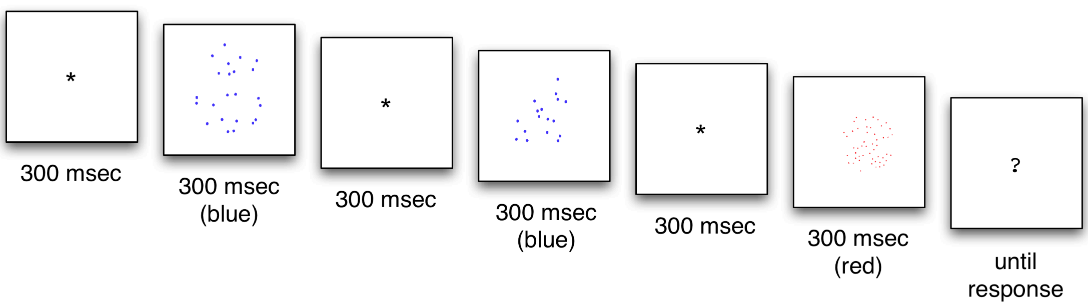

Figure 1: A non-symbolic addition task.

Each problem appeared twice: once with each stimuli set (according to Pica et al.'s method (2004)). Responses (and response times) were recorded via coloured blue (leftmost) and red (rightmost) buttons on a five-button response box.

In summary, the experiment followed a 3 (approximate ratio: 8:5, 7:5, 6:5) × 2 (ratio-direction: number-larger, sum-larger) design. This yielded a total of 600 trials for each participant. The experiment was preceded by a practice block of 10 trials.

Results

Mean accuracy rates for each ratio and ratio-direction are presented in Figure 2. These were analyzed using a 3 (approximate ratio: 5:6, 5:7, 5:8) ×2 (ratio-direction: number-larger, sum-larger) repeated-measures analysis of variance (ANOVA). There were main effects of ratio, $F(2,10)=110.72$, $p<.001$, and ratio-direction, $F(1,11)=6.09$, $p=.031$. Accuracy was higher for problems with the number larger, $M=79\%$, $SD=7.6\%$, than the sum larger, $M=64\%$, $SD=15.9\%$. As predicted, we found the characteristic ratio-effect, as there was a significant linear trend of ratio, $F(1,11) = 188.6$, $p<.001$.

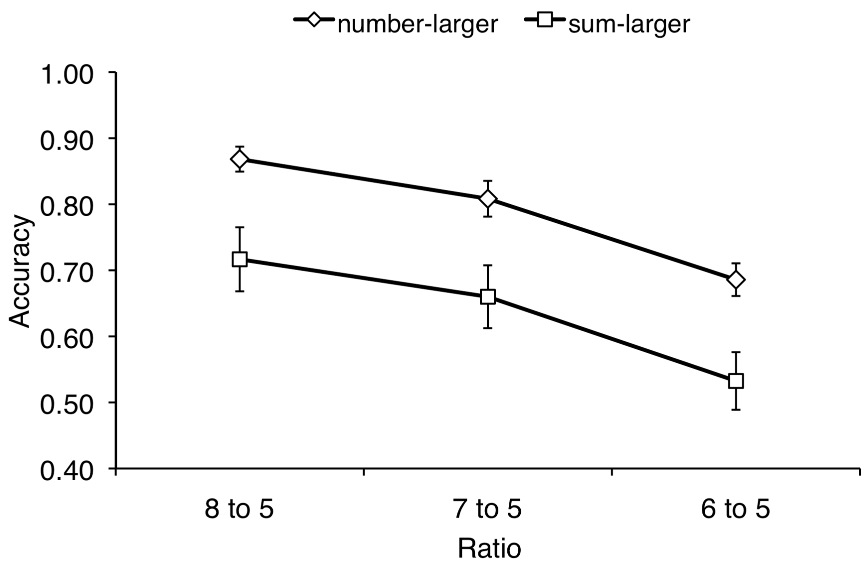

Figure 2: Accuracy rates for sum-larger and number-larger problems in Experiment 1, by ratio. Error bars show ±1 SE of the mean.

Discussion

The results from Experiment 1 clearly showed that participants were able to accurately tackle non-symbolic addition tasks at accuracies well above chance. Furthermore we found the so-called ratio-effect: accuracies on problems of ratio 5:6 were lower than those of ratio 5:8. Surprisingly, however, we also found an unpredicted main effect of ratio-direction: number-larger problems had higher accuracy rates than sum-larger problems. Note however, that like earlier researchers (Barth et al. 2006) we did not counterbalance the order in which the numerosities were presented (the sum was always presented first, followed by the comparison number). Consequently, it is possible that this effect might be a consequence of a confound in the experimental design.

Experiment 2

Our goal in Experiment 2 was to ask non-dyscalculic participants to tackle problems similar to those used in Experiment 1, but where the numerosities were represented symbolically. If, as we hypothesised, the ANS is used in such tasks we would expect to see similar patterns of results as in Experiment 1.

Method

Twelve staff and students (five male) from the University of Nottingham (aged 19-37, *M*=24) received £4 for participating in the study.

Participants were tested individually. Displays were presented on a 17" Philips 170B LCD placed at eye level, and were viewed from approximately 60cm away. Stimuli consisted of two symbolic items, a sum and a comparison number, as shown in Figure 3.

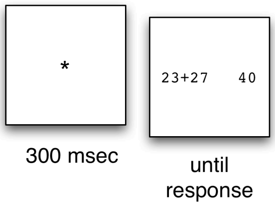

Figure 2: A symbolic addition task.

Numerosities were from the same range as those used in Experiment 1. A total of 25 problems were used for each approximate ratio, with the task being to determine as rapidly as possible which item (left or right) was numerically larger. Again six comparison ratios were used, 8:5, 7:5, 6:5, 5:6, 5:7 and 5:8. Each problem appeared four times (once as each of *a*+*b* vs. *c*, *b*+*a* vs *c*, *c* vs *a*+*b* and *c* vs *b*+*a*). Both responses and response times were again recorded via the leftmost (left-larger) and rightmost (right-larger) buttons on a five-button response box.

In summary, the experiment followed a 3 (approximate ratio: 8:5, 7:5, 6:5) × 2 (ratio-direction: number-larger, sum-larger) design. This yielded a total of 600 trials for each participant. The experiment was preceded by a practice block of 10 trials.

Results

Mean accuracy rates for each ratio and ratio-direction are presented in Figure 4. These were analyzed using a 3 (approximate ratio: 5:6, 5:7, 5:8) ×2 (ratio-direction: number-larger, sum-larger) repeated-measures ANOVA. There were main effects of ratio, $F(2,22)=22.08$, $p<.001$, and ratio-direction $F(1,11)=6.17$, $p=.030$. Accuracy was higher for problems with the number larger, *M*=97%, *SD*=2.6%, than the sum larger, *M*=93%, *SD*=5.2%. There was a significant linear trend of ratio, $F(1,11) = 6.17$, $p=.030$.

Discussion

The data from the symbolic tasks we used in Experiment 2 followed essentially the same pattern as that from the non-symbolic tasks we used in Experiment 1. We found the ratio-effect characteristic of the ANS, as well as the unexpected main effect of ratio-direction (this latter finding casts doubt upon the suggestion that it was the result of insufficient counterbalancing, as all stimuli were presented simultaneously in Experiment 2). This pattern of results is exactly what we would expect if the ANS was being used by participants in Experiment 2 to help them tackle the symbolic addition and comparison tasks we used. However, unlike with the non-symbolic tasks used in Experiment 1, participants had various different strategies available to them when tackling the symbolic tasks in Experiment 2. Might it be that adopting non-ANS strategies would result in this pattern of results? To test for this possibility we conducted several additional analyses.

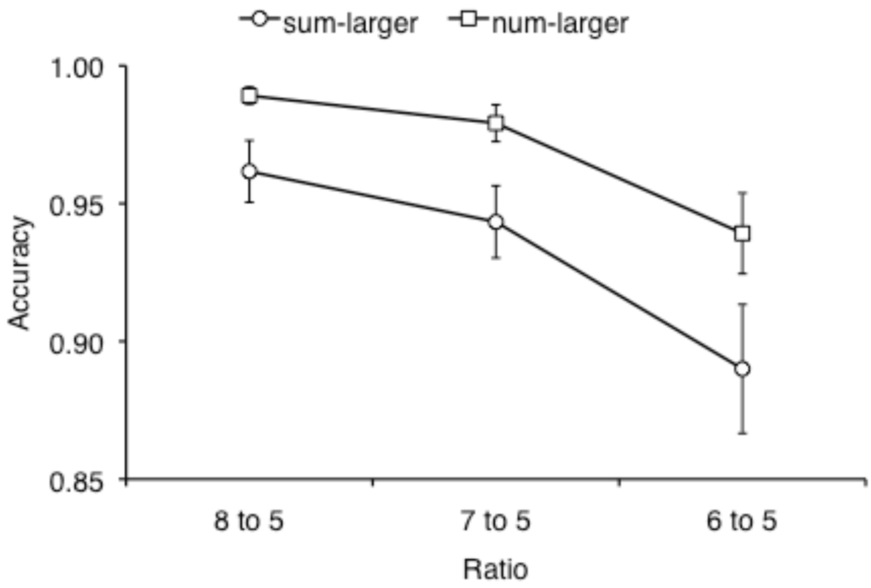

Figure 4: Accuracy rates for sum-larger and number-larger problems in Experiment 2, by ratio. Error bars show ±1 SE of the mean.

Space constraints prevent a detailed presentation of the statistical tests we used to rule out the possibility that participants were using alternative strategies. Our method in each case however, was to compare mean accuracy rates and response times on the subset of the problems where the alternative strategy would have worked with those problems where it would not have. If participants had consistently been using the given alternative strategy we would have expected to see a difference in the means associated with these two subsets of problems. In those cases where such a difference was also predicted by the ANS-based account we instead looked to see whether performance was above chance on those problems where the alternative strategy would not have worked. No evidence was found that any of the following strategies were being used by participants:

- Rounding Down. A participant using this strategy would have added the two leftmost digits of each of the addends and compared to the leftmost digit of the comparison number.
- Comparison Number Range. Here a participant would have based their answer on the size of the comparison number alone. If it was higher than the median comparison number used across the experiment it would have been selected.
- Addend Range. Alternatively, participants could have based their answer on the size of the largest addend. If it was above the median it would have been selected.
- Addend-Comparison Difference. Participants may simply have compared the size of the largest addend with the comparison number, and assumed that the sum is largest when that difference was low.

General Discussion

The ANS appears to be an inbuilt cognitive system that supports rapid – but approximate – numerical calculations. The evolutionary benefit of such a system is clear, for example one can easily imagine how a predator might find it advantageous to be able to rapidly decide which of two groups of prey is the larger. But is the ability to connect the ANS to formal symbolism a prerequisite for higher level mathematics? Here we explored the suggestion that dyscalculia is the result of a disconnect between the ANS and symbolic mathematics. If this were the case we would predict that participants tackling tasks where numerosities are represented non-symbolically would show a similar pattern of behaviour to those who tackle the equivalent tasks where the numerosities are represented with Arabic numerals. This is exactly what we found.

The ANS-symbolism disconnect proposal is a strong hypothesis, and further work is needed to investigate its potential. If it is correct, however, then it seems that an important early goal for mathematics education is to form and strengthen connections between various different representations of numerosity and the internal representations of the ANS (e.g. Wilson et al. 2008).

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