Interpretations of, and orientations to, "understanding mathematics in depth": students in MEC programmes across institutions

Jill Adler[1], Sarmin Hossain[1], Mary Stevenson[2], Barry Grantham[2], John Clarke[3], Rosa Archer[4].

[1]King's College London, [2]Liverpool Hope University, [3]University of East London, [4]St Marys College Twickenham

In this paper we present initial findings from our study of interpretations and orientations to 'understanding mathematics in depth' among students in selected Mathematics Enhancement Courses (MEC) in the UK. The MEC is a 26-week pre-Initial Teacher Education (ITE) 'mathematics subject knowledge for teaching' course designed for, and undertaken by, graduates wishing to teach mathematics at secondary level, but do not have a Mathematics degree. It is completed before commencing with a PGCE. A common theme running through the MEC documentation is the importance of 'understanding mathematics in depth'. We are interested in what and how MEC students interpret and orient themselves towards 'understanding mathematics in depth'. In designing and conducting our empirical work we have drawn upon a related project in South Africa, which is exploring 'mathematics for teaching', specifically what and how mathematics and teaching are co-constituted in mathematics teacher education programmes. The MEC is an interesting empirical context for such study, as it is a mathematics course, or set of courses, specifically designed for future teachers. We have collected data through guided, semi-structured interviews with 18 students and 4 lecturing staff at three different institutions. The interpretations and orientations of MEC students towards mathematics and the notion of 'understanding mathematics in depth', we contend, provide additional insight into the developing notion of mathematical knowledge in and for teaching.

Keywords: Understanding mathematics in depth, Mathematics Enhancement Course, Mathematics initial teacher education

Introduction

This study conducted in the UK extends from the QUANTUM project which is currently on-going in South Africa. Our focus here is on 'understanding mathematics in depth' as interpreted within the Mathematics Enhancement Course, (MEC). The MEC provides an alternative route into mathematics teacher education. It has been designed for graduates who do not have a mathematics degree but wish to teach mathematics at secondary level. It is a is a 26-week pre-Initial Teacher Education (ITE) 'mathematics subject knowledge for teaching' course which is completed before commencing with a PGCE. The programme has been running across a number of institutions in the UK for the past four years. The motivation in the UK, has been to encourage and attract more graduates into retraining as a secondary mathematics teacher. Graduates entering these programmes are required to have an A-level pass in mathematics, or some indication of post secondary study with mathematics. This seems to vary across institutions. Overall, students in these courses are moving from only some post school mathematics, to preparation for being a secondary mathematics

teacher. The MEC programmes are thus focused both on deepening and extending mathematical knowledge in ways appropriate to the profession of teaching.

The empirical field of QUANTUM in South Africa (SA) has been focused to date on upgrading programmes for teachers whose qualifications were limited by apartheid teacher education policy and practice. Orientations in South Africa similarly contain intentions to deepen teachers' subject knowledge in ways that are appropriate and useful in teaching. Despite differences between the UK and SA, programmes share the phenomenon of providing mathematical education specifically geared to the profession of teaching. The UK can be seen as another context where ideas and experience about content knowledge for teaching mathematics are being developed, particularly within the MEC. While the comparative advantage offered by looking across these two contexts will illuminate the field in interesting ways, the study in the UK will be of interest in the UK context itself, and more directly of benefit to the shared understanding that we hope develops across MEC course participants through their activity in the study. Of particular interest to us is the expressed commitment in MEC course materials to "a deep understanding of fundamental mathematics". We are interested in what and how MEC students interpret and orient themselves towards 'understanding mathematics in depth'.

Our empirical work has been conducted across three UK institutions: a MEC tutor and six students from each of the institutions have been interviewed. We report here on our findings from the student interviews only. Data has been gathered related to: MEC students' motivations for and concerns in pursuing a teaching career in mathematics and joining the MEC; the structure of the MEC and its activities; students' orientations to learning on the MEC and the meanings (interpretations) they attach to 'understanding mathematics in depth'. We are in the early stages of analyzing the data and in this paper we present and discuss emerging trends that lead us to suggest that MEC students' 'understanding of mathematics in depth' is a discourse of mathematics interwoven with discourses of teaching and learning.

Background

In official and institutionally specific documentation for the MEC, there is a common theme that mathematics teachers need to know and understand 'mathematics in depth'. This is a particular description of the specific ways in which teachers need to know and use mathematics in order to teach well. Within the field of research on mathematics teacher education (see Sullivan & Wood 2008; Even & Ball 2009) over the last 30 years there has been focus and a growing interest in content knowledge for teaching. There is increasing evidence of a positive relationship between student learning gains and what is being referred to as teacher's mathematical knowledge for teaching (Hill, Rowan & Ball 2005) and that the nature of teachers' mathematical knowledge and its use in practice matters for effective teaching. Ball, Thames and Phelps (2008) for example, have examined what teachers know and are able to use in practice and through this, elaborated and extended Shulman's (1986) categories of content knowledge for teaching that included: subject matter knowledge, pedagogical content knowledge and curriculum knowledge. They distinguish common, specialized, and horizon content knowledge, as forms of subject matter knowledge; and knowledge of mathematics and students, mathematics and teaching, and mathematics and curriculum, as forms of pedagogical content knowledge. Rowland et al. (2005) also built on Shulman (1986) and developed a grounded model of mathematical knowledge used in teaching – the Knowledge Quartet.

These and other studies related to mathematical knowledge for teaching (e.g. Neubrand 2008) do not cohere into a unified frame. However, they consistently reinforce Shulman's insight that knowing mathematics for oneself is not synonymous with enabling others to learn mathematics. An expansive knowing of mathematics is illuminated in the MEC through its intention that prospective teachers need to understand mathematics 'in depth' and hence provides an interesting context in which to investigate what and how 'understanding mathematics in depth' is interpreted by MEC students who are close to completing their MEC.

Research Method

Three institutions named A, B, C, for the purpose of anonymity, took part in this study. The sample included 6 MEC students from each institution. The selection of students was purposive, and guided by a set of criteria so that the sample included students with: mathematical and non-mathematical backgrounds; different cultural and educational backgrounds; and ranging participation and performance in the course. A total of 18 semi-structured interviews were conducted, guided by an interview agenda lasting approximately an hour. The interviews were recorded and transcribed. Each interview was conducted by a researcher from Kings College London and one of the MEC tutors in an institution which was not their own. This collaborative approach is beneficial. . Firstly, as insiders, the MEC tutors are well placed to probe students' interpretations by drawing on aspects/activities of the MEC programme. Secondly, the tutors are all 'new' researchers and are enjoying participation in a collaborative research community.

Sample Description- Within the study sample 14 students were educated in the UK and 4 educated abroad (2 students from Nigeria, 1 from Cyprus, 1 from Pakistan). Furthermore the point at which the students joined the MEC in their lives varied: 3 students joined straight after finishing their degrees at University. 4 students had had a short career and 11 students had had a long career before entry into the programme. Out of the 18 students, only 5 students had some form of teaching career or teaching experience before joining the MEC. In terms of educational qualifications 16 students had bachelor's degrees, 1 qualified in a PG Programme and 1 qualified in Access to Primary School Teaching. The subjects studied by the students at degree level ranged from: Educational Studies, Business, Computer Science, Engineering, to Sports Science. In regard to mathematical background/qualifications: 12 students had a Level 3 Mathematics qualification (i.e. A Level); 3 students had an equivalent of a Level 3 Mathematics qualification and, interestingly, 4 students had a Level 2 Mathematics qualification (i.e. GCSE or its equivalent), with some mathematics related study or experience in their educational and work histories.

The Interviews- Students were probed on: a). How and in what ways their learning of mathematics in the MEC has been different to their learning of mathematics at school and/or at University b). What and how they interpreted "understanding mathematics in depth"? c). How they rated five different statements related to "understanding mathematics in depth". These statements had been elicited from earlier interviews with MEC tutors where the MEC tutors had defined in their own terms what it was to "understand mathematics in depth". The MEC students were asked to put these five statements in the order of importance and then probed on their specific ordering. The statements included: Understanding mathematics in depth means being able to: 1). Justify your mathematical thinking. 2). Explain and/or communicate mathematical ideas and thinking to others. 3). Understand why and how these procedures work. 4). Make the connections between concepts and between procedures. 5). Identify structure and generality.

Data Analysis- We are in the initial processes of the data analysis, and will be concerned with 1). MEC students' backgrounds, concerns and motivations for pursuing the MEC. 2). Their preferences within MEC activities. 3). Their orientations to mathematics in the MEC and their learning and interpretations of understanding of mathematics in depth. As a mathematics teaching focused programme, we anticipate interpretations and hence discourses (by which we mean representations of social practice, Van Leeuwen, (2008)) of mathematics, of teaching and of learning to be present in the data. We are also interested in whether and how the students affiliate or dissociate themselves in their talk about mathematics, teaching and learning. We have developed an initial coding based on these theoretically informed assumptions, and our coding has been extended through our interaction with the data. For example, we found that students often associate mathematics and teaching elements when making reference to their Self and their Environment. For example, "I am more mathsy now" and "I am more confident to teach" in these instances for example the text would be coded as the "Self and mathematics", and the "Self and teaching" respectively. These instances often show how the students are foregrounding themselves either in mathematics, in teaching or in both mathematics and teaching as they journey through on the MEC programme. So far the data has been analysed thematically. Our next step is to refine our analysis by conducting a critical discourse analysis, attending more closely to students' positive and negative descriptions (e.g. what mathematics in depth IS and what it is NOT); and to whether and how they associate or distance themselves in what they say (e.g. I think this means vs. they say it means).

Preliminary Findings

As noted, to date we have focused our analysis on students' spontaneous responses to what "understanding mathematics in depth" means for . As anticipated, there are mathematical, teaching and learning discourses that thread through the transcripts.

Below is an example of some of the mathematical discourses that were prevalent across the data.

> Em, what it means to me is understanding, you know, the different concepts, how they originated, you know, what the idea was, therefore, a concept came about. How it can be applied is a useful lesson in everyday life, and how it's interconnected with other aspects of mathematics. So that's how I feel. We will have a deeper understanding of something. And, er, it's like an open box – we don't just look at the problem and say, 'Okay, this is the calculus,' ... No, you can have, you can try different methods, you can be flexible, you... and see what applies. So if you have a deeper understanding of it, you're not going to be scared of anything because you know there is a solution, or you can attain to solution.

Here understanding Mathematics in depth includes

- Discourses of knowing the history of mathematical concepts
- Discourse of mathematics as connected knowledge

Other mathematically related discourses in the data were:

- Discourses of proof, knowing basic proofs or first principles
- Discourses of understanding formulae and their parts; when these are used

Also reflected in the above quote is a discourse of learning, of acquiring a disposition towards mathematics as something you can do; knowing mathematics in depth means being able to be flexible. Across the data there were learning related discourses, where students talked about understanding mathematics in depth in terms of relearning mathematics, or learning new maths in particular ways. For example,

> [Understanding mathematics in depth] means that whenever it comes to understanding it ... you're not blinkered by what you're relaying to someone else. ... you've actually looked at something, tried to look at it in depth... you may have been led down some paths that come to dead ends, you may have gone down some further paths that leads you to something else. Er, I think through doing that you're more able to build analogies within your own mind, so therefore if you're trying to explain something to somebody. Because you've gone a bit beyond or possibly a bit more beyond what's expected of you, or what you think should be expected of a student, you actually look at it from different perspectives. So I think whenever it comes to actually passing that knowledge on, you actually get an enthusiasm for it, and I think if you can actually gain the enthusiasm for what you're looking at, there is a point beyond which it loses an effect, but if you get it just right, I think you can actually – I wouldn't, I'm not going to say inspired because I think that's an utterly wrong thing to say, but I think you can actually enthuse someone...

And similarly, discourses of teaching thread through discussion of learning – indicating that students interpret understanding mathematics in depth as needed for teaching. For example:

> But the fact that, you know, you should never give a... a kind of an equation and there's... there's always like a practical example or an example in layman's terms that can help somebody understand it rather than, em, put it into a lot of letters or numbers or something straightaway, because, when you... when you... when you... you often put, em... or express it in terms of, em, letters, for example. I'm... I'm... I'm one of these people that if I just see a lot of letters, it's very intimidating, especially for children, like for somebody to see that, it's very intimidating, which can even... even stop them starting the problem at all. So put that into a numerical example and then, for example, linking for patterns, for example, to start off with, and then going on to try and express that in terms of first principles or, em, or algebra..... it's a way... well, it works for me, I think, em, and I think... I think it works well with children.

Here and across the data is a representation that understanding mathematics in depth is enabling others to know mathematics. Within this are

- Discourses of dealing with 'difference' (learners are diverse)
- Discourses of making mathematics 'practical'

Discussion/Conclusion

In summary, our initial analysis of students' orientations to 'understanding mathematics in depth' in these interviews reflects an amalgam of mathematical, teaching and learning discourses. Understanding mathematics in depth meant: knowing that and knowing why (knowing basic proofs or being able to work from first principles; understanding formulae, their parts and when these are used; knowing the history of mathematical concepts; and being able to connect different aspects of mathematics and its applications. These are resonant with Shulman's elaboration of the Subject Matter Knowledge (SMK) that forms part of a teacher's professional knowledge base (Shulman, 1986). Of course, these were not uniformly described with apparent different emphases a function of different schooling histories (e.g. some students were schooled in very different educational cultures outside the UK), different mathematical histories and different experiences in the MEC course. Across the interviews, however, was a relatively strong affiliation to teaching requiring depth understanding of mathematics, appreciation of opportunities to engage with mathematics in extended ways in the MEC, and particular appreciation of activities in the MEC that closely aligned with teaching (e.g. peer teaching activities). Many of the students pointed to the social relations in the MEC and the care taken by lecturers in their learning of mathematics. Some

expressed directly that they believed the MEC would give them a 'leg-up' in the PGCE, as they had had opportunity to revisit school mathematics and relearn it 'in depth'. The next question of course, is how their appreciation for understanding mathematics in depth recontextualises into the actual teaching experience and practices and this is what we hope to be pursuing next.

References

Adler, J., D. Ball, et al. 2005. Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics* 60(2): 359-381.

Adler, J. and Davis, Z. 2006. Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education* 37(4): 270-296.

Ball, D.L., Thames, M.H., and Phelps, G. 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5): 389-407.

Even, R., & Ball, D. L. 2009. The Professional Education and Development of Teachers of Mathematics. The 15th ICMI Study Springer.

Hill, H. C., B. Rowan, and Ball, D.L. 2005. Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal* 42(2): 371-406.

Neubrand, M. 2008 Knowledge of teachers—Knowledge of students: Conceptualisations and outcomes of a Mathematics Teacher Education Study in Germany. *Proceedings of the Symposium on the 100th Anniversary of ICMI.*

Rowland, T., P. Huckstep, et al. 2005. Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education* 8(3): 255-281

Shulman, L. S. 1986. Those who understand knowledge growth in teaching. *Educational Researcher* 15(2): 4-14.

Sullivan, P., and Wood, T. (Eds.). 2008. The Handbook of Mathematics Teacher Education. Knowledge and Beliefs in Mathematics Teaching and Teaching Development. (Vol. 1). Rotterdam: Sense Publishers.

Van Leeuwen, T. (2008) Discourse and practice: New tools for critical discourse analysis. Oxford. Oxford University Press.