Working group on trigonometry: meeting 2 (Cambridge, February 2009)

Notes by Anne Watson

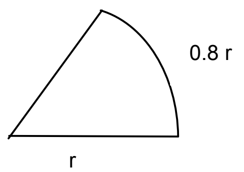
Department of Education, University of Oxford

These notes record the discussion at the second meeting of this working group. The focus was on the fundamental ideas involved in trigonometry and a programme of work was devised.

Keywords: angle, ratio, similarity, trigonometry

Angle measurement as arc length

Sarah Aldous talked about using a different approach to measuring angle with her students, after reading Thompson, Carlson and Silverman's paper (2007). This approach depends on expressing angle as an arc-length, a multiple of the radius. Thompson et al. suggest that this approach results in the angle and sine/cosine all being in the same units, so that expressions such as sin(sin x) can have meaning, while preserving the notion of angle as 'turn'. Her students used string to measure angles rather than protractors, and this had to be done on a large scale.



We discussed whether students would appreciate that the multiple would be the same for every radius, and Sue Elliott suggested that students could be 'tied' to a rope at various points and then, keeping the rope taut, act out the turn. They would have a sense of the distance they had to travel, and also that this increases with the radius. This could avoid the common misconception that angle size depends on the length of its legs.

In this measure system, the right-angle has no obvious value. This could be a disadvantage because right angles are what students are most familiar with from everyday life, and also will have already 'named'. Other than introducing π, which seems too complicated at this stage, and radian measure we could not think of a way round this problem but it does explain why we need various ways to measure angles, since 360º appears to have been adopted historically as a measure of full turn because it gives a way to express many different angles precisely (and roughly equates to a year's worth of days).

Intuitive static understanding of angle

This led us to consider circumstances in which one might *need* to understand angle so that teachers can draw on students' experience of them. We generated a rather short list which seemed to fall into two groups: those involving rotation and those involving 'angle subtended'. We decided to think about 'angle subtended', a static notion of the pointiness of angle, as an embodied notion arising from the way we see things, that is as a measure of optical scope. This appears not to relate to turn, since when we look at things we do not normally scan our heads from left to right extremities of the object, nor even our eyes.

We devised an experiment, lining up three paper rectangles vertically on a desk. The rectangles were A4, A5, and A6 in size, and hence similar. One of the group then observed these and we adjusted their position until he saw them as 'the same'. His actions and words indicated a static cone-shaped experience of 'looking' which we recognise mathematically as embodying similarity, and depends on the ratios of different aspects.

Ratio as the starting point for trigonometry

This led to the suggestion of thought experiments in which ratio is taken to be the fundamental starting point for trigonometry, rather than angle-measure. Thus one might start with triangles which are in proportion and not even mention angle while the trig functions are being established, only later recognising that they can be used to define and give values for the relevant angles.

With this approach, the key idea is ratio encapsulated as scale factor. This, of course, is a central idea in lower secondary mathematics signalling important shifts from additive to multiplicative views of number, from multiplication as repeated addition to multiplication as scaling, and from dealing with quantities to dealing with relations between quantities. A related idea would be to teach spatial enlargement separately from other transformations because it relates to closely to these key ideas, rather than ghetto-ise it with other transformations.

It occurred to us that, because the ideas in the previous paragraph are all hard to teach and hard to learn, the pedagogic reasons for reducing trig to mnemonic mechanisms for finding missing properties of triangles are all-too-obvious.

Programme of work

A programme of research work was devised to be carried out before the next BSRLM meeting in Bristol on June 20th.

Group members volunteered to:

- Read about the historical roots: Thales, Euclid, Euler etc.
- Locate and send out the old MA report on the teaching of trigonometry
- Investigate further our awareness of similarity
- Send brief questionnaire to a range of schools asking about teaching approaches to trigonometry
- Carry out small teaching experiments with a ratio approach to classifying angles, drawing on work done by Jeremy Burke and colleagues.

Plans

We shall review some of this work at the next BSRLM meeting. This is an open group and all are welcome to join. If you would like copies of previously-circulated readings please contact anne.watson@education.ox.ac.uk.