The effect of real world contextual framing in A-level sequence questions

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This paper provides a preliminary analysis of data from a study into the effect of real-world contextual framing in A-level sequence questions. Alternative versions of the same questions were presented in explicit, algebraic, word and pattern contexts, and set to a sample of 594 Year 13 students (aged 17-18) in a one-hour test. Facility levels of the questions were then compared. In addition, the paper presents results of a student questionnaire on real-world context which accompanied the test.

Keywords: Assessment, post-16 mathematics, real world context, modelling.

Introduction

This paper presents interim findings from a study of the effect of real world contextual framing in questions on arithmetic and geometric sequences. This study forms part of a PhD thesis whose aim is to investigate the following research questions:

1) What has led to the introduction of real-world context and mathematical modelling in A-level mathematics?
2) To what degree are 'pure' mathematics questions in A/AS level examinations capable of being framed within real-world contexts, and what is the nature of these contexts?
3) What functions are served by real-world contextual framing (RWCF) of pure A-level mathematics questions, and what are its effects?
4) What qualities determine effective use of real-world contextual framing in A-level mathematics questions?

This research has been the subject of a number of papers which have been presented at recent BSRLM conferences (Little, 2007, Little, 2008a, Little, 2008b, Little and Jones, 2007).

Design of the sequences study

The aims of the sequences study were

- to investigate the effect of real-world contextual framing on the facility of A-level questions;
- to survey student attitudes towards real-world context and its function and effect on problems.

The study comprised a one-hour test and a short student questionnaire. The test consisted of four questions on arithmetic and four on geometric sequences, given to a sample of 594 year 13 students (aged 17-18). Students were randomly allocated to one of four tests (A, B, C or D), each of which contained one of four versions of the questions. These versions were as follows:

- An *explicit* (e) version, which contained no real-world context, and explicitly defined the sequence to be used in the question;

- An *algebraic* (a) version, which used mathematical notation, such as u_n or Σ notation;
- A *word* (w) version, which used a real-world context defined in words;
- A *pattern* (p) version, in which the sequence was defined using a pattern context.

An example of a question, in its four alternative versions, is provided in Appendix 1. The solutions to each version were in most cases the same, although there were minor differences dictated by the nature of the context. The order of the questions in each test was varied in order to control for the potential effect of position in the test on facility – it was thought possible that students might 'give up' on later questions more easily than on the first few on the paper.

The questionnaire described real-world contextual framing using an example, and invited students to agree or disagree with a number of statements about pure and applied mathematics and real-world context, with a space for additional comments (see Appendix 2).

Analysis

The mean number of marks for each question is shown in Table 1.

	AI	AII	AIII	AIV	GI	GII	GIII	GIV	totals
Max score	5	6	5	4	5	3	6	6	40
explicit	4.36	4.41	2.53	2.99	3.14	1.01	3.82	5.91	28.17
algebraic	3.33	2.28	2.43	1.68	2.83	0.84	3.26	3.65	20.31
word	3.41	4.03	2.27	2.02	2.51	1.20	3.24	3.92	22.61
pattern	4.04	3.51	2.35	2.44	2.08	0.80	3.67	4.49	23.37

Table 1: Mean marks per question.

From the table, it can be seen that the 'e' versions (explicit), as might be expected, gained higher marks overall than the 'a', 'w' and 'p' versions. However, this is not true of all questions – see GII, for example, where the 'w' version (word) scored slightly higher than the 'e' version. The causes of these variations require more detailed investigation. The lower scores for the 'a' versions seem to be caused by the use of sigma notation.

The results of the questionnaire are shown in Figure 1. Two thirds of the students believed that questions set in real-world context are harder than those without context. In terms of whether or not real-world context makes questions more interesting, 33% agreed, and 30% disagreed. 55% agreed, and 30% disagreed, with the statement that real-world context shows how mathematics is useful. Over half of the students preferred pure mathematics to applied mathematics, and felt that pure mathematics is interesting in its own right.

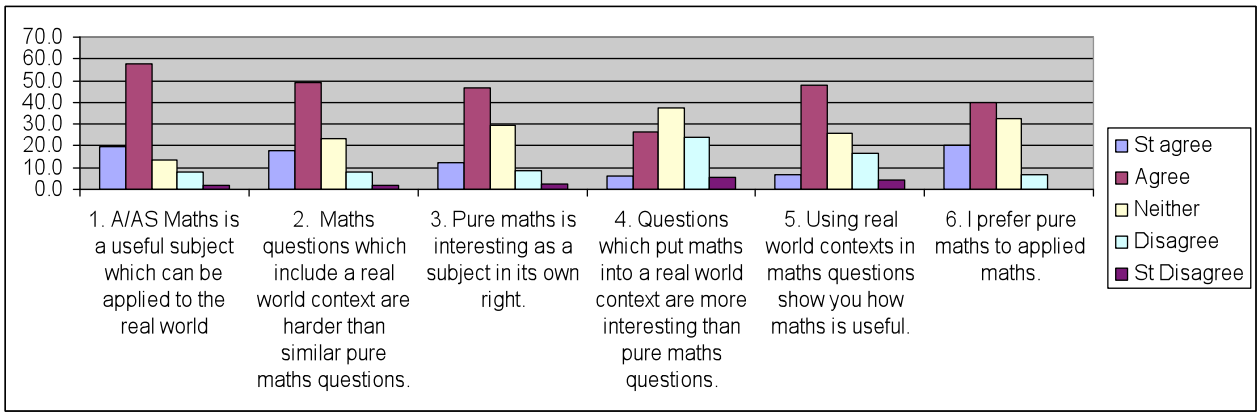

Fig. 1: questionnaire results

The variety of opinions which students hold on the use of real-world context is indicated by the following selection of comments:

> "Applying maths to real situations is certainly more difficult but (the questions) are a lot more interesting and satisfying to complete rather than straight pure maths questions."
>
> "I find maths more interesting as a pure subject but it's important to understand its relevance to the world."
>
> "Real context is harder but helps as it is what we use in the real world. Both real context and just maths should be used in exams."
>
> "Currently I enjoy learning maths without it being applied, but I imagine once in a career I will appreciate it more in a 'real world' context."
>
> "I strongly dislike real world context questions as they turn maths that I can do into something I can barely understand."
>
> "I prefer the questions which are worded (without context). I believe this is due to dyslexia, which means I find (contextualised questions) harder to understand."
>
> "Sometimes they help if you don't know terms or helps you to get an idea of what the question requires which is more comprehendable, but otherwise they are just plain patronising!"
>
> "Using questions in pure maths about how they could be used in the real world shows us how you can apply the content to real life instead of just learning pointless methods and formulas to use only in a test situation."
>
> "Real world questions don't show you how maths is useful, because questions in context, such as question 2, are not useful. It is about a beetle, not useful in normal everyday life."
>
> "Using it in 'real life' context makes it more rewarding rather than just having a number that doesn't mean anything."
>
> "I found that some of the pure maths questions are annoying and when learning about maths in a pure way you miss out on how it is useful until it comes up in later life. Though laying out questions in a pure way can help as you can get important information quicker."

Conclusions

Preliminary analysis of data from the study suggests that introducing real-world contextual framing (Little, 2008b) in general increases the demand of questions, and that students believe that this is the case. This might be because

applying mathematics demands comprehension and modelling skills from students which are not required by pure mathematical questions set in explicit forms.

Framing a question in a real-world context can provide mental scaffolding (Vappula and Clausen-May, 2006) which might assist its solution. However, these sequence questions are generally best solved by a process of transfer from the context to algebraic concepts (classification of sequence, term and sum formulae), rather than first principles thinking within the context – see Little 2008a for a discussion of this in relation to linear equation problems. When presented in explicit or algebraic form, this stage in the problem solving process is not required: it is perhaps this process of transfer to which the first student quotation is referring.

Students generally see real-world context as reinforcing the perception that mathematics is useful, although some contexts may be perceived as artificial, or even 'patronising'. These comments from students perhaps resonate with researchers (e.g. Boaler, 1994, Wiliam, 1997) who have criticised real world contexts on the grounds of artificiality.

What is therefore the function of real-world context in these sequence questions? I have argued elsewhere (Little, 2008a) that, notwithstanding that students link real-world context to applicability of mathematics, these questions have little or no practical utilitarian value. They are, however, embryonic, albeit artificial, exercises in modelling, which force the student to make connections between the world of algebra and mathematics and real-world concepts such as finance, percentages, and physical patterns and shapes.

It is perhaps unrealistic to expect short, closed examination questions to do more than present proto-modelling exercises, since a genuine modelling cycle requires more strategic thinking than is possible in an examination. Nevertheless, to reject real-world contextual framing at this level as 'McGuffinism' (Wiliam, 1997) would be to ignore an important aspect of the A/AS mathematics construct.

References

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Appendix 1: 'e', 'a', 'w' and 'p' versions of an arithmetic sequence question

A1e

An arithmetic progression has first term 7 and common difference 3.

(i) Which term of the progression equals 73? [3]

(ii) Find the sum of the first 30 terms of the progression. [2]

AIa

The *n*th term of an arithmetic progression is denoted by u_n. $u_1 = 7$, $u_2 = 10$ and $u_3 = 13$.

(i) If $u_n = 73$, find n. [3]

(ii) Find $\sum_{r=1}^{30} u_r$. [2]

AIw

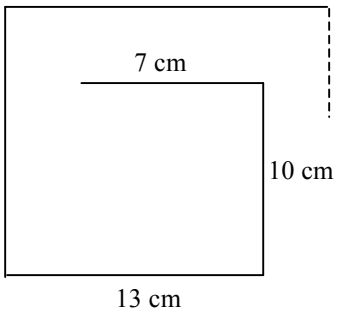
Chris saves money regularly each week. In the first week, he saves £7. Each week after that, he saves £3 more than the previous week.

(i) In which week does he save £73? [3]

(ii) Find his total savings after 30 weeks. [2]

AIp

A spiral is formed with sides of lengths 7 cm, 10 cm, 13 cm, … which are in arithmetic progression:



(i) How many sides does the spiral have if its longest side is 73 cm? [3]

(ii) Find the total length of the spiral with 30 sides. [2]

Appendix 2: Student questionnaire

UNIVERSITY OF Southampton
School of Education

Real-world Context in A/AS Mathematics Study

Student Questionnaire

Name..... *Gender Male / Female*

Is English your first language Yes / No

Which AS Maths applied unit are you doing? S1 D1 M1 Are you doing Further Maths? Yes / No

Some maths questions put the maths into a real world context, and others are just about maths.

For example, here is a question which is just about maths…

> An arithmetic progression has first term 7 and common difference 3.
> (i) Which term of the progression equals 73?
> (ii) Find the sum of the first 30 terms of the progression.

… and here is the same question using a real world context:

> Chris saves money regularly each week. In the first week, he saves £7.
> Each week after that, he saves £3 more than the previous week.
> (i) In which week does he save £73?
> (ii) Find his total savings after 30 weeks.

This questionnaire is about pure and applied maths, and using real world contexts in maths questions (in general).

Consider the following statements, and tick which option best fits your views

	strongly agree	agree	neither agree nor disagree	don't agree	strongly disagree
1. A/AS Maths is a useful subject which can be applied to the real world					
2. Maths questions which include a real world context are harder than similar pure maths questions.					
3. Pure maths is interesting as a subject in its own right.					
4. Questions which put maths into a real world context are more interesting than pure maths questions.					
5. Using real world contexts in maths questions show you how maths is useful.					
6. I prefer pure maths to applied maths.					

7. Write below any other comments you'd like to make about pure and applied maths, and using maths in real world contexts. Continue overleaf if you need more space.

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