Engagement, abstraction and visualisation: Cognitive and emotional aspects of Year 2 mathematics undergraduates' learning experience in Abstract Algebra

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Abstract Algebra is considered by students as one of the most challenging topics of their university studies. Our study is an examination of the cognitive, social and emotional aspects of mathematics undergraduates' learning experience in Abstract Algebra. Our data consists of: observation notes and audio-recordings of lectures and group seminars of a Year 2 course; student and lecturer interviews; and, coursework and exam papers. Here we offer some observations on the students' apparently diminishing engagement over the ten weeks of the course. Particularly we exemplify from their comments on the effect that the abstract, not easily visualisable nature of Abstract Algebra has on their relationship with the topic.

BSRLM Keywords: undergraduate mathematics education, student engagement, Goldin's theory of affect, visualisation, Abstract Algebra

Introduction

Abstract Algebra is one of the mandatory courses taught usually in the second year of a Bachelor degree in Mathematics and is typically considered by the students as one of the most challenging ones. Often, after their first encounter with Abstract Algebra, students tend to avoid third-year or further courses in this area of mathematics. Previous research (e.g. Nardi, 2001) attributes student difficulty with Abstract Algebra to its multi-level abstraction and the less-than-obvious, to students, *raison d'être* of concepts such as cosets, quotient groups etc. Furthermore Abstract Algebra is amongst the first courses in which students are not able to cope with by just memorising formulas or by "just learning 'imitative behavior patterns'" (Dubinsky et al, 1994, p268). Additionally, the students' introduction to the novel ideas of groups and rings takes place in the unfamiliar academic context of large-scale lectures. This unfamiliarity is likely to exacerbate their difficulty with the topic (Mason, 2002, p52). Also, as it is often suggested by research (e.g. Millet, 2001), lecturing to large student audiences has an arguable effect on student engagement.

The ongoing study we draw on here aims to examine closely mathematics undergraduates' learning experience in Abstract Algebra. We are particularly interested in the intertwinement of cognitive and socio-affective aspects of the students' experience since, as Goldin (2000) proposed, affect is "critical to the structure of competencies accounting for success or failure" (p211). Here we offer some preliminary observations on the students' apparently diminishing engagement over the ten weeks of a Year 2 Abstract Algebra course. We exemplify from their attempts to bestow meaning to the new ideas they are being introduced to. Particularly, and given that constructing appropriate visual imagery is often described in research as providing crucial support to this meaning-bestowing process (Zazkis et al, 1996), we highlight how their relationship with Abstract Algebra is affected by the difficulty to visualise.

Aims, context and data collection of the study

The study – doctoral study of the first author – aims to examine closely as many facets as possible of Year 2 mathematics undergraduates' introduction to Abstract Algebra. At the centre of our data collection is a course currently taught in a well-regarded mathematics department in the UK. Data collection took place in the Spring Semester of a recent academic year. The course was mandatory and 78 students attended it. It was spread over 10 weeks and there were 20 hourly lectures. Additionally to the lectures, there were 3 cycles of seminars, in Weeks 3, 6 and 10. The role of the seminars was mainly to support the students with their coursework. There were 4 seminar groups, with about 20 students in each. In each seminar group there was a seminar leader, a full-time faculty member of the department, and a seminar assistant who was a PhD student. All members of the teaching team had related research interests. The students submitted the coursework at the end of the semester. This was marked and returned to them soon after.

In the lectures there was not much interaction between the lecturer and the students. The lecturer, a very experienced mathematician, was writing extensively on the chalkboard and was commenting orally alongside. In the seminars the students were expected to work on problem sheets, distributed to them earlier in the preceding weeks, and arrive having prepared questions. They had the opportunity to ask the seminar leaders and assistants anything they had difficulty with and receive help. The lecturer was also available during 'office hours' for the same purpose.

Data was collected by the first author and consists of:

1. **Lecture observation notes**. These covered: record of student attendance; instances of interaction between the students and the lecturer; verbal, body or other evidence of student (dis)engagement and emotional response to the lecture; and, general observations of lecturer and student behaviour.
2. **Lecturer notes**: notes of what the lecturer was writing on the blackboard.
3. **Audio-recordings** of the 20 **lectures**.
4. **Audio-recordings** of 24 **seminars** (2 recorders in each of the 12 seminars; one on the seminar leader and one on the seminar assistant) in which we have captured all conversations with students during which they predominantly discuss difficulties with certain items in the problem sheets.
5. **Interviews with** 13 out of the 78 **students** who made themselves available on a voluntary basis, the 4 **seminar leaders and assistants and the lecturer**. There were three cycles of interviews, at the beginning, the middle and the end of the course, in which students discussed their learning experience in Abstract Algebra. The discussions with the lecturer, who was also one of the seminar leaders, covered learning and teaching issues as well as institutional and administrative issues. Interviews with the seminar leaders and assistants were mostly about their discussions with the students during the seminars, and their general views on pedagogical issues.
6. **Student coursework**. Students were given three problem sheets in Weeks 2, 5 and 9. They had to work on these before the seminars on Weeks 3, 6 and 10. They had to work on all problems, but they had to hand in only a selection of these in Week 12. The selection of problems to be assessed was announced to them after each seminar.
7. Marker (seminar assistant) **comments on student coursework**.
8. **Student examination scripts** collected at the end of the academic year.

Preliminary data analysis: diminishing student engagement

Data analysis is currently in progress. At the time of writing a preliminary scrutiny of the lecture data (audio recordings and notes) has led to the following list of themes which we will explore further across the data from the above listed sources:

1. the apparently **diminishing student engagement** over the ten weeks of the course;
2. the variable learning effectiveness of **exemplification** in the lectures;
3. the variable learning effectiveness of **visualisation** in the lectures;
4. the limited **interactivity** in the lectures;
5. the variable effectiveness of the lecturer's **strategies for introducing new concepts**;
6. the variable effectiveness of the lecturer's tendency to introduce **non-linear, spirally structured arguments** (of the 'flash back' and 'flash forward' types);
7. the strong **link between Linear and Abstract Algebra** and the problems that emerge from the students' difficulty to establish and sustain this link;
8. the, often elusive to the students, potential **significance of the running oral commentary** that supplements the lecturer's writing on the board.

Here we focus on the first theme. For this purpose we draw on the **observation notes** taken during the lectures and on the **student and lecturer interview transcripts**.

Our claim about the diminishing student engagement over the ten weeks of the course is grounded on three sets of observations:

1. **Pathology of Absence**
   The number of students attending was gradually and significantly decreasing as the course was progressing. At the beginning of the course the attendance was around 80%. It gradually reduced to 60% and towards the end it was occasionally less than 50%. In Lecture 15, the lecturer circulated an attendance sheet and the present students were 45 out of 78. Despite this measure – even to the lecturer's surprise as expressed in one of the interviews – attendance was further reduced after this incident.
2. **Pathology of Presence**
   As the lectures progressed students' body language suggested increasing disengagement, difficulty to follow the lecturer and lack of focus. Most usually, this was evident in their talking with their peers. This often distracted the lecturer and those from the audience trying to follow him. For example, in Lecture 14, when the Euclidean Domains were introduced, audience noise was particularly high and throughout the lecture there is not even one instance of exchange between the lecturer and the audience.
3. **Explicit Student Expression of Emotion**
   Direct expressions of emotion increased gradually in frequency and power over the three cycles of interviews.

Below we sample these expressions of emotion. Before doing so however we introduce briefly an adaptation of Gerald Goldin's (2000) theory of affect, which we are currently finding useful when examining these data. Our emphasis – much like Keith Weber's (2008) – is on student affective responses to material they see in the lectures and during engagement with coursework.

Goldin's Theory of Affect and other related literature

Goldin (2000) describes affect in mathematics in terms of four elements: beliefs and belief structures; attitudes; emotional states; and, values, ethics and morals. Particularly significant to our study is his notion of "local affect", "the rapidly changing states of feeling that occur during problem solving – emotional states, with their nuances" (p210). Goldin describes eight such emotional states and several possible ways in which these affective states may lead to certain problem-solving strategies. Our data partly refers to problem-solving – as much of the student experience in Abstract Algebra revolves around their engagement with the problem sheets. However our data also involves direct evidence of students' affective responses to Abstract Algebra (for example in the lectures and the seminars) as well as accounts of these affective responses (for example in the interviews). Through our data analysis we are exploring whether, and how, Goldin's model may be expanded towards accounting for a wider spectrum of students' learning experiences, not just strictly problem-solving related ones.

According to Goldin (2000) at the first stage of problem solving the student is likely to experience feelings such as **curiosity**, **puzzlement** or **bewilderment**. Following this there are two possible *affective pathways*. [We use the term affective pathway to mean "a link between one's affective and cognitive representation systems (i.e. a consistent cognitive response to an emotional state)." (Weber, 2008, p82)]: *favourable* (i.e. emotions of **encouragement**, **pleasure**, **elation** and **satisfaction**) and *unfavourable* (i.e. emotions of **frustration**, **anxiety** and **fear/despair**). These pathways in *local* affect lead to *global* affective structures such as specific representational schemata, general self-concept structures as well as (particularly the second pathway) self/mathematics/science/technology resentment.

Weber (2008) suggests that these affective pathways may be *self-strengthening* if their duration is long. A repeated emotional experience is possible to cause stable attitudes and beliefs that may be related to particular cognitive responses (Goldin, 2000). Moreover, as suggested by Weber, mathematical understanding is organic, since, when students feel that they have achieved some understanding in one mathematical topic and consequently they find it pleasurable, they want to extend their understanding with regard to this and other mathematical topics.

The role of lectures in shaping students' affective responses is essential, since it is primarily in the lecture theatre that the students are first exposed to the new material that later on they need to work with. Claudi Alsina (2001) observes that some typical approaches to lecturing (such as prioritising deduction over induction in the lecture organisation) may have adverse effect on student affective responses to the material and therefore their engagement. For example, we agree with Mason (2002) that the lecture is there to "engage students' thinking and attention, not to show how much you can cover" (p45). There are several instances in our lecture data in which the lecturer, in his effort to cover the required material for the course, appears to give less priority to the students' ability to follow his pace. For example, especially towards the end of the course, some of his lectures were particularly dense with new definitions and theorems.

In what follows we sample from a set of 89 explicit statements of emotion, made in the 39 interviews conducted with 13 of the 78 students. A rough categorisation of these suggests statements directly related to the **lectures**, statements about other aspects of the students' learning experience (e.g. **coursework**) and statements referring to the **nature of Abstract Algebra** as a domain of mathematics.

Sample of student emotional responses in the encounter with Abstract Algebra

Despite an often promising start, students, from some point on, appear to feel lost and attend **lectures** only to copy the notes on the blackboard. Their initial excitement gives way to puzzlement and, to some extent, resignation from effort to understand.

> "At first, it was easy, like with few lec – first few lectures, and even the first problem sheet, it was a bit easy, so you sort of like – oh, yeah, this is good, and you sort of like excited about it, and then – as soon as it's getting harder, and you sort of like – go ahead and get a bit like – if you don't get any excited any more, you just – oh, I have to go to the lecture, and sort of like – so you sort of like – you don't looking forward to the lecture any more, and you sort of turn up, when the lecturer talking in the lecture, so you just sort of listen, and you copy? But you sort of don't understand what going on." [CL3, 3rd interview of Student CL]

Similar emotions were reported with regard to experiencing other aspects of the course, such as **coursework:**

> "Um, I've – can sometimes get frustrated when I'm going through the problem sheet if I can't do a question, like I might look at it like – a hundred times and still feel like I can't do it and then obviously I start to feel frustrated, and – but then I might look at it in a different way and then find, that I've managed to do it, and then obviously, I feel like quite happy that I've managed to do that, and – obviously that's quite a nice feeling to actually feel like you've achieved something? So... but obviously when – there are times when I feel like – I'm not going anywhere, I'm not getting anywhere, and then that's when I – start to feel a bit black. Yeah, so it's hard but – I think all concepts of maths are quite hard, it just takes time..." [NT2]

In the above quote what is noticeable – apart from the evident frustration – is also the substantial emotional impact of a successful problem-solving attempt.

Beyond characterising the students' response to lectures, coursework etc. the above reported emotions extend to the **students' relationship with the subject matter of Abstract Algebra** – for example, in terms of its less than evident to them 'logic', 'formulae', 'patterns' and 'methods':

> "In the other courses there is some logic behind… For example in fluids and solids there was a certain logic behind the course… there are certain formulae, there are certain methods or patterns which we have to follow in order to solve the exercises… and in the rest of courses as well… in algorithms… there is some logic that you can follow…" [MO2]

Or its increasingly, as the course progresses, abstract nature:

> "It's getting worse and I really hate, group – groups, um – I don't – yes, I don't really expect very much of it, I'll just have a look at it and it – it's just gonna be more and more... abstract, isn't it…" [KL2]

Or the fact it doesn't always lend itself easily to visualisation:

> "Yeah, and like... in the – in the proofs as well, it's like – oh, but the – that means this, and it's just trying to understand just why that means that, and because I can't see it in my head, and I can't visualise it, it just – I don't see why they're so you know – like it goes and therefore this, and I'm just like – but why?" [KL2]

> "LH: I find it hard to picture, I find it hard to... understand what's really happening… what's meant by certain things. I thought when it first started, oh this is going to be ok, but then it just was like – with ideals and stuff, it was just really confusing, I was like, oh...[…] to be able to picture it in my head." [LH3]

In LH3's statement the student's cumulative local-affect experiences (initial optimism followed by moments of increasing frustration) have resulted, in this third interview, in an overall sense of hardship and frustration. In this sense the difficulty to construct pictures-in-the-mind has an adverse emotional impact on the student's engagement with the subject (Ioannou & Nardi, 2009).

While students (e.g. as evident in the interviews) repeatedly express – often with intense emotion – a need for 'pictures' that will illuminate the nature of the novel to them Abstract Algebraic objects, they are at the same time reluctant to attempt a construction of such images (e.g. as evident in the visual scarcity of their written work) or engage with the images on offer by their lecturers (e.g. as evident in the lecture observations). Our analyses currently explore whether issues such as student lack of experience/practice with visualisation and student uncertainty about the mathematical status of visualisation lie behind this reluctance; and, how these cognitive and epistemological issues intertwine with the above recorded emotional ones.

Next steps

In Goldin's terms the emotional states recorded in the above quotations, while localised in terms of time (they are about specific moments) and in terms of context (they are about specific aspects of Abstract Algebra activity), may evolve into longer-lasting, globally 'unfavourable' emotional structures. Subsequent phases of our analysis will aim to trace these structures, and their impact on students' competence in Abstract Algebra, across all other data sources (e.g. coursework and exam papers).

References

Alsina, C. 2001. Why the professor must be a stimulating teacher. In *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, ed D. Holton, 3-12. NL: Kluwer Academic Publishers.

Dubinsky, E., J. Dautermann, J., U. Leron, and R. Zazkis. 1994. On learning fundamental concepts of Group Theory. *Educational Studied in Mathematics* 27: 267-305.

Goldin, G. 2000. Affective Pathways and Representation in Mathematical Problem Solving. *Mathematical Thinking and Learning* 2(3): 209-219.

Ioannou, M., and E. Nardi 2009. Visualisation as a meaning-bestowing process in the learning and teaching of Abstract Algebra. In *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, edited by M. Tzekaki, M. Kaldrimidou and C. Sakonidis, vol. 1, 397. Thessaloniki, Greece: PME.

Mason, J. H. 2002. Lecturing. In *Mathematics Teaching Practice*, 39-70. UK: Horwood Publishing Series in Mathematics and Applications.

Millet, K. C. 2001. Making large lectures effective: An effort to increase student success. In *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, ed D. Holton, 137-152. NL: Kluwer Academic Publishers.

Nardi, E. 2001. Mathematics undergraduates' responses to semantic abbreviations, 'geometric' images and multi-level abstractions in Group Theory. *Educational Studies in Mathematics* 43: 169-189.

Weber, K. 2008. The role of affect in learning Real Analysis: a case study. *Research in Mathematics Education* 10(1): 71-85.

Zazkis, R., E. Dubinsky, and J. Dautermann 1996. Coordinating visual and analytic strategies: A study of students' understanding of the group D4. *Journal for Research in Mathematics Education* 27(4): 435-457.