Developing the Ability to Respond to the Unexpected

Fay Turner

Faculty of Education, University of Cambridge

In this paper I present some findings from a four-year study into the development of content knowledge in beginning teachers using the Knowledge Quartet as a framework for reflection and discussion on the mathematical content of teaching. Findings which relate to the participants' ability to react to pupils' unexpected responses are discussed. Data from three case studies suggest that the framework helped participants to consider their unplanned actions when teaching mathematics. There was also evidence that over the course of the study the participants become more able to act contingently in relation to the mathematical content of their teaching.

Introduction

The basis of this study was the use by beginning teachers of a framework for the identification and development of mathematical content knowledge. This Knowledge Quartet (KQ) framework was developed by a group at the University of Cambridge (Rowland, 2008). The work of the Cambridge group focused on the classification of situations in which mathematical knowledge surfaces in teaching. The KQ offers this classification of the situations through which mathematical content knowledge of teachers is 'made visible' as a framework for the analysis of teaching. The KQ framework was developed from observation and videotaping of mathematics teaching. Analysis of this teaching produced 18 'emergent' codes (Glaser and Strauss, 1967) of situations in which mathematical content knowledge of teachers was 'made visible' e.g. concentration on procedures, making connections between concepts. These were later classified into four 'superordinate' categories based on associations between the original codes. These categories make up the four dimensions of the Knowledge Quartet; foundation, transformation, connection and contingency.

In this paper I focus on the development of teachers' content knowledge as 'made visible' through the lens of one of these categories or dimensions, that of situations in which teachers act contingently. The contingency dimension of the quartet may be considered to be about the ability to react to unplanned situations or to 'think on one's feet'. There are three codes which emerged from the empirical research subsumed under this category; deviation from agenda; responding to children's ideas and use of unplanned opportunities. Most mathematics lessons are planned before the act of teaching takes place and teachers bring their curriculum knowledge, subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Shulman, 1986) to the planning of a text for the lesson (Shulman, 1987). Teachers predict how pupils will respond to their planned teaching based on knowledge of content and students (Ball, Thames & Phelps, 2008) as well as on their previous experience of teaching, and amend their text accordingly. However, not all pupil responses can be predicted.

A teacher's ability to react appropriately to unplanned-for responses depends, at least in part, on their bank of SMK and PCK. Bishop (2001, pp. 95-96) offered an example of a teaching incident which illustrates the role of teacher's content knowledge in reacting to pupils' responses. In this example 9- 10-year-olds were asked to give a fraction between ½ and ¾. A response given was 2/3, "because 2 is between the 1 and the 3, and on the bottom the 3 lies between the 2 and the 4". The way in which a teacher might react to this response would depend on their SMK – were they aware of Farey sequences and mediants, and on their PCK – did they know how to disprove the generalisation inherent in the pupil's justification. The contingency dimension of the Knowledge Quartet therefore offers a lens through which to identify teachers' mathematical content knowledge.

The study

The aim of this study was to investigate the way in which beginning teachers' mathematical content knowledge for teaching might be developed through focused reflection using the KQ framework. Throughout the study this framework was used as a tool for analysis, evaluation and development of the teachers' mathematics content knowledge. The study began with 12 student teachers from the 2004/5 cohort of primary (5-11 years) postgraduate pre-service teacher education course at the University of Cambridge reducing, as anticipated, to 4 in the fourth and last year of the study. Data came from observation and analysis of teaching using the KQ as well as from post-lesson reflective interviews, group and individual interviews and participant written accounts. Transcripts of interviews and written reflective accounts were all systematically coded using the qualitative data analysis software NVivo. A grounded theory approach (Glaser and Strauss, 1967) was used which led to the emergence of a hierarchical organisation of codes into a number of themes. For a more detailed account of the research methodology see Turner (2008).

Case studies were built from the KQ analysis of observed teaching as well as from analysis of the data coded using NVivo. The analysis of observed teaching, using the 18 codes and four dimensions of the KQ, provided a 'spine' for presenting findings in relation to the development of participants' mathematical content knowledge. Data from the NVivo coding of interview and written data support, supplement and enrich these findings. Six themes in the development of the participants' mathematics teaching emerged from the NVivo coding. These were, beliefs, confidence, subject knowledge, experience, reflection and working with others. Discussion of findings about participants' mathematical content knowledge, including that revealed through their ability to act contingently, drew mainly on data from the NVivo themes of subject knowledge and confidence

Findings in relation to the ability to act contingently

Knowledge of errors and what they suggest about children's understanding of mathematical ideas is part of a teacher's PCK. The way teachers respond contingently to mathematical errors therefore gives some insight into their PCK. At the beginning of their teaching careers the participants did not always make good use of opportunities for teaching offered by children's errors. For example, during Amy's lesson observed during her training year in 2004/5, the class of 4-5 year old children were asked to write some 'teen' numbers. Amy focused on correcting children's reversals of digits but did not address their errors which involved writing the numerals in the wrong order e.g. '01' for ten and '21' for twelve. However, when reflecting on this lesson using the KQ framework Amy acknowledged that the ordering of the numerals was a more significant error and suggested that she should have used this to discuss place value rather than focusing on digit reversals.

The contingency dimension of the KQ framework helped Amy to think about a more useful way of responding to the children's errors. Such reflection may be described as reflection on action (Shön, 1983). The following year, Amy demonstrated the ability to respond helpfully to children's errors in action. At the conclusion of a lesson on counting some children were having difficulty counting the number of times Amy hit a chime bar, and continued counting after the last chime. Amy responded to this difficulty by asking the children to close their eyes, count the number of chimes in their heads and only give the answer once she had finished. Amy's knowledge of the cardinal principle enabled her to 'think on her feet' and suggested this effective strategy.

In a lesson observed later in 2005/6, Kate made good use of a child's error. Kate displayed a measuring cylinder on the interactive whiteboard and asked a volunteer to indicate the level to which 100ml of liquid would come. A child pointed to the interval marked '1000 ml'. Kate asked the class how they knew this did not show 100 millilitres and the children responded that it had an extra zero and was a thousand. Kate was clearly aware that children's errors can be used to advantage when teaching.

> I took advantage of Lily confusing 100 with 1000 on the interactive measuring cylinder to discuss place value. (Kate, reflective account of observed lesson, 2005/6)

Kate's use of the contingency dimension of the KQ helped her to think about how she responded to children's errors. She became increasingly confident in using children's errors to inform her teaching and appeared to relish such opportunities.

> When estimating how many cubes long a book was Harriet-Mae said "eighty" and then corrected herself to say "eighteen". I used this as an example to question the children about which of these was a sensible estimate and we discussed why 80 was not. (Kate, reflective account, 2006/7)

The use of appropriate resources in order to address unexpected difficulties is another aspect of acting contingently which was found to have developed over the course of the study, particularly in the case of Kate. An instance in which Kate made use of a resource, i.e.100 grids that she had not planned for was recorded in a reflective account of her mathematics teaching and demonstrates her concern with the flexible and appropriate use of resources.

> When we were suggesting different ways to count, the children wanted to count in 100s. Someone said 0, 100, 1000. I used 100 grids to represent units of 100 and then counted in 100s to 900. I asked the children if they knew another word for 10 hundred and someone said '110' so I had to demonstrate the difference using 10 hundred grids compared with 1 hundred grid and 10 cubes. (Kate, reflective account, 2006/7)

Jess was also aware of the need to be able to use resources contingently. Under the heading of 'Contingency', Jess recounted an incident in which she had been unable to use a particular resource to demonstrate why a child's response was incorrect.

> We were looking at lines of symmetry on the interactive whiteboard. The children were shown a variety of shapes and had to identify where the lines were and how many lines. When a child identified a line which didn't exist, I found it

> hard to prove they were wrong without actually folding paper. (Jess, reflective account, 2006/7)

Jess felt able to demonstrate that this answer was incorrect by using a different resource. However, more secure pedagogical content knowledge might have enabled her to make use of the interactive whiteboard.

Kate demonstrated secure PCK through her contingent use of a resource during a lesson observed in 2007/8. The lesson had been planned by another teacher as a Powerpoint presentation and Kate had not had the opportunity to make any amendments before teaching. Kate acted contingently when a slide was displayed showing 23 + 12 as (20 +3) + (3 + 2). This modeled a 10-10 strategy which she thought inappropriate. Kate 'deviated from the agenda' and made a new slide to model the N10 strategy i.e. 23 + 10 + 2. For a discussion of the 10-10 and N10 strategies see Beishuizen (2001). Kate's knowledge of the two methods enabled her to make an 'informed' choice about which to use. Kate further demonstrated her ability to use unplanned resources by producing a 100 grid and modeling the procedure for addition of ten by moving down one row.

Discussion of children's unexpected methods for solving problems was another form of contingent action observed during the study. Participants became more likely to carry out such discussion in their teaching over the course of the study. In the lesson I observed in her training year, Jess revised how to interpret pie charts with her class of 9-11year olds. Jess displayed a pie chart showing preferred flavours of ice cream and asked a question which involved calculating ¼ of 32. The class had had already found that 3/8 of 32 = 12 and 1/8 of 32 = 4. One child explained that he had calculated ¼ of 32 by adding 12 and 4 and dividing by two. There were rich opportunities for discussion of equivalent fractions in this but Jess simply responded "that works well". Our discussion in the post-lesson interview suggested that Jess did not see an opportunity for discussing equivalent fractions because she did not understand the child's method.

In the lesson observed early in 2006/7, Jess appeared more willing to explore a child's calculation method. Jess asked the children to record their methods for solving '20 ÷ 2' on individual whiteboards. Most children drew pictorial representations modelling the partative method that Jess had previously demonstrated. One child however, had simply written '20 ÷ 2 = 10'. When Jess asked how he had arrived at the answer he explained that he knew "ten add ten is twenty" so had "put ten on one side and ten on the other". Jess said "knowing ten add ten is twenty to find twenty divided by two is like using the opposite". Jess understood and related his method to division as the inverse of multiplication.

Over the course of the study, the participants became more likely to discuss children's methods of calculation. They were also more likely to ask for and accept children's ideas as starting points for their teaching. Amy recognised that she had missed an opportunity to work from children's ideas during the lesson observed in early 2005/6.

> The children in my 'treasure counting' group had some good ideas for how we could count all the coins more quickly. It would have been good to try out the children's ideas, despite asking for their suggestions I went ahead with what I had planned to teach them (Amy, reflective account, 2005/6).

Amy wrote this comment under the heading of 'Contingency', indicating that this dimension of the KQ had helped her to focus on how her teaching might encompass children's ideas. Comments in Jess' reflective accounts under the heading of 'Contingency' suggest that the KQ framework encouraged her to think about exploring children's thinking in order to make her teaching more meaningful.

> They often catch me out when discussing subtractions – "Why can't you do 4 – 8?" "You can, it's a minus number!" I have started to get these children to explain in more detail what they have said so I understand where they are coming from and also so some of the other children start to realise some of these things too. (Jess, reflective account, 2006/7)

During the study the participants became more willing to discuss children's methods and to explore their ideas. This was underpinned by a growing confidence that their mathematical content knowledge would enable them to understand the children's thinking and develop this in their teaching. An extension of this willingness to explore children's methods and ideas was a growing confidence to allow children to investigate mathematical ideas for themselves. In a lesson on measurement that I observed early in 2006/7, Amy gave the children a selection of objects and containers which they could use in order to investigate ideas about capacity. Amy observed what the children were doing and asked questions, made suggestions or gave them further resources to support their learning.

> Callum and Joshua filled bigger boxes with small toys and found they couldn't count that number. I don't think it mattered too much though. They were enjoying the practical experience of filling a container, they were practicing judging when a container is 'full' and they saw how they could fit more small toys in a box and less big toys.(Amy, reflective account of lesson, 2006/7)

Kate also become increasingly confident about letting children take greater ownership of their mathematics.

> I was really pleased – my upper group have finally started to work through a problem systematically on their own initiative. I didn't mention it this week as I had not really thought of that approach and they did it anyway! So we shared their systematic approach as a class. (Kate, reflective account, 2006/7)

In this instance, Kate had not intended that the children should investigate the problem in their own way. However, she was clearly happy that they did so and felt confident to discuss their strategies with the class.

Observations of the participants' teaching, and their reflective accounts showed that they became more able to respond contingently during their mathematics teaching. There was also convincing evidence from reflective accounts that the participants saw the ability to act contingently as a factor in effective teaching and that they believed they had developed in this respect over the course of the project.

> I am more experienced, so I am aware of children's common misconceptions, and can therefore adapt in response contingently, or plan for these. Generally I think there is more contingent teaching going on and I am more confident to be flexible. I can respond quickly to a child by setting up an activity I know will extend from what they are doing. (Amy, group interview,2006/7)

Kate also thought that she had become more responsive to how children reacted to her teaching.

> Quite a lot of the things that I remember talking about arose out of what the children did. One of the children who came up to write something on the board got something wrong and if he hadn't I possibly wouldn't have made that a focus. (Kate, interview, 2007/8)

By the end of her second year of teaching Jess felt that responding contingently in her mathematics teaching was something she was able to do automatically.

> I just think about contingency as a question that I hadn't thought of, that's just what you automatically do in anything, like thinking on your feet. (Jess, reflective account, 2006/7)

Conclusion

The participants' use of the contingency dimension of the KQ framework focused their thinking on the way in which they responded to unplanned-for events in their mathematics teaching. There was convincing evidence that the participants recognised the ability to act contingently during mathematics lessons as a factor in effective teaching. Over the four years of the study the participants became more able to respond helpfully to children's errors and make better 'unplanned-for' use of resources. They became more proficient at understanding, discussing and basing their teaching on children's methods and ideas. Participants' also began to adopt a more enquiry-based approach to their mathematics teaching which was more likely to require them to act contingently. These developments in participants' ability to act contingently were underpinned by development in their mathematical content knowledge. In directing participants' reflection towards their contingent actions, the KQ played a role in the development of this aspect of mathematical content knowledge.

References

Ball, D. L., Thames, M. H. and Phelps, G. 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5): 389-407

Beishuizen, M. 2001. 'Different approaches to mastering mental calculation strategies'. In *Principles and practices in arithmetic teaching* ed. J. Anghileri, 119-130. Buckingham, Philadelphia: Open University Press.

Bishop, A. J. 2001. Educating student teachers about values in mathematics education. In *Making Sense of Mathematics Teacher Education* eds F. L. Lin and T. J. Cooney, 233-246. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Glaser, B. G. and Strauss, A. L. 1967. *The discovery of grounded theory: Strategies for qualitative research*, New York: Aldine de Gruyter.

Ma, L. 1999. *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. London: Lawrence Erlbaum.

Rowland, T. 2008. Researching teachers' mathematics disciplinary knowledge. In *International handbook of mathematics teacher education: Vol.1. Knowledge and beliefs in mathematics teaching and teaching development* Eds. P. Sullivan and T. Wood, 273-298. Rotterdam, the Netherlands: Sense Publishers.

Schön, D.A. 1983. *The Reflective Practitioner*, New York, Basic Books.

Shulman, L. S. 1986. Those who understand, knowledge growth in teaching. *Educational Researcher* 15 (2): 4-14.

Shulman, L. S. 1987. *Knowledge and teaching: Foundations of the new reform*. Harvard Educational Review, 57(1), pp. 1-22.