# ENRICHing Mathematics: Progress in Building a Problem-Solving Community

Cathy Smith and Jennifer Piggott

*Homerton College, Cambridge and NRICH, University of Cambridge*

The SHINE enriching mathematics project recruited secondary school students in two socio-economically deprived London boroughs for out-of-school workshops over the course of a year. Students worked collaboratively on open maths tasks, with discussion guided by NRICH leaders and participating school teachers. Here we outline two aspects of the project evaluation: how we analysed progress in collaborative classwork and how the students described what they had learnt. Students found Shine maths enjoyable, different and more challenging than school maths. Their teachers observed improvement in problem-solving behaviours. The model of a maths-talk learning community offered ways to categorise changing classroom behaviours, and helped to identify tensions and effective practices of classroom management.

**BSRLM Keywords: problem-solving, enrichment, assessment**

## The Shine Project and Evaluation.

This research concerns the five-year "Shine" maths enrichment project, based in two London boroughs, and initially commissioned by a national charity, Shine. Secondary school students from years 8 and 10 in local schools met at weekly out-of-school workshops over a school year, and worked collaboratively on tasks drawn from the Nrich bank of problems. One of us, Jennifer Piggott, was involved in designing and running the project, with the challenge of creating a setting and pedagogy which supported students' mathematical engagement with problems. The other, Cathy Smith, designed and implemented an evaluation of three early cohorts. As the project developed the evaluation fed formatively into project design, and we have since collaborated to report and reflect on 'Shine' as an example of collaboration between schools, LEA and enrichment providers (Smith and Piggott 2007).

The aims of the project were to raise attainment in the areas of problem solving and mathematical thinking, and to raise students' aspirations and awareness of mathematics. Two year 10 cohorts and one year 8 cohort were studied in the evaluation; each with 35 to 50 participants recruited from 5-7 local schools. The project schedule varied for each cohort, involving between 45 and 58 hours of workshops. The Shine recruitment criteria focused on problem-solving potential but in practice schools recruited willing students from their higher sets. This was not a very narrow restriction: students achieved in the top 30% nationally, expecting from A* to C at maths GCSE, and Levels 6 to 8 in KS3 SATS. Recruitment and attendance were ongoing concerns involving careful liaison with schools: recruiting 70 students from 7 schools gave 50 long-term participants whose average attendance was 67%. The participating students were representative of the major ethnic groups in the two boroughs. The high proportion of Black and Asian students (above 70%) was unusual for enrichment projects and resulted from the school-level organisation.

Shine workshops lasted 2 or 3 hours and were based on a short starter problem and one or two longer tasks, taken from a schedule which involved revisiting and making connections between related tasks. Students worked individually, in groups, and in whole-class discussion. Sessions were led by Nrich staff or schoolteachers trained in the initial phase of the project. Leaders aimed to “create an atmosphere in which they engage in dialogue and other interactions including the use of modelling and metacognition and the use of props or cues, as teaching and learning tools”(Piggott 2007, p38). Spoken maths was the main focus of public attention and development, while written working was informal and private.

The evaluation considered possible impacts on student attitudes and attainment and what features of practice contributed to them. Its design was structured by balancing two concerns: to select attributes and use instruments that were compatible with the project pedagogy, and to collect attainment data relevant for school maths and for problem solving. A range of instruments was used to focus on these different aspects:

- before-and-after student questionnaires concerning attitude, enjoyment and aspirations;
- before-and-after teacher profiling of students on 12 problem-solving descriptors;
- attainment data from SATS and GCSEs, for participants and for ‘matched’ students identified by class teachers at the outset;
- teacher and student interviews;
- videos and observations during workshops.

Changes in the student cohorts over time and missing teacher data meant that the comparison sets had to be carefully identified, and necessarily excluded the experiences of students who left, and who had certain teachers. These quantitative analyses are seen as one perspective to inform the qualitative evaluation; they do not generalise straightforwardly. For school maths, GCSE results were higher than for matched students by an average 0.3 of a grade (paired sign test, 1%; data for 66 out of 85 students) and Yr 9 SATS were higher by 0.2 of a level (paired sign test, 5%; data for 34 out of 38 students). For problem solving, teacher profiles showed significant improvements in nearly half the attributes (Wilcoxon signed rank test, 5%; data for 79 out of 123 students). The greatest improvements were in students’ ability to explain their reasoning, their interpretation and use of diagrams, and their formulation and manipulation in algebra. Details, and reasons for caution, are discussed further in Smith (2007). This paper considers two aspects of the evaluation, the analytic framework of a maths-talk learning community and students’ reflective comments, to show how features of workshop practice contributed to beliefs on maths.

## Observations

In the Nrich teaching style leaders stressed communal acts of speaking and recording, and managed transitions beween these and private episodes of individual thinking and group work. Rather than make individual performances visible, as would be the case with assessed tasks, the evaluation needed to analyse the workshops in terms of the balances and interactions between activity and passivity, public and private work, teacher and student. The framework used for this was the model for a maths–talk learning community developed by Hufferd-Ackles et al. Fuson and M. Sherin (2004) to research change during the US ‘math reform’ program. Classroom behaviour is described in terms of progress through levels 0 to 3 in each of the four inter-related areas of questioning, explaining, source of ideas and responsibility for learning.

Broadly, level 0 describes classrooms in which the teacher supplies and controls the mathematics attempted and discussed. Level 3 describes classrooms where students initiate ideas and extend each other's reasoning. At intermediate levels, teachers direct students' attention to each other's maths-talk.

Videos and field notes from 12 workshops were analysed to find examples of episodes of behaviour characteristic of each level, and then referenced to whether the observation occurred in the first half, ie 'early', or 'late' in each project. This provided a structure for producing descriptions of behaviour and change in each of the four framework areas. We discuss these below, summarising the level descriptors and exemplifying how we drew links between the supporting evidence and the features of practice that contributed to change.

### *Questioning*

At level 1, teachers use follow-up questions that probe students' methods and thinking. Their phrasing models mathematical language and values.

> But is that all the solutions? Any other possibilities to explore? What do you think? Have we covered the whole field of possibilities there – are you convinced? (Obs, Y10)

Here the leader is directive but doesn't leave time for students to respond or to ask their own questions. The public questions are intended as a model for private questions that could guide individuals' future activities and metacognitive strategies. At level 2/3, teachers ask students to comment directly on each other's contribution.

> That's brilliant! Are you hearing this? This isn't about adding up - this is about understanding. Maria, can you hear OK? Just catch on to what Melody's saying… (Obs, Y8)

These questions focus attention on the activities of understanding and listening to others rather than on task aims. They direct the social space of the classroom, stopping just short of asking Maria to comment on Melody's work.

All the workshops showed level 1 questioning; some had level 2 questions inviting students to describe others' work or compare it with their own; in a few later workshops students were asked to critique others' reasoning but this was only achieved with very directive management techniques: short questions, instructions and names. Some students felt uneasy in this atmosphere:

> Teachers can be less pushy. I sometimes found it a bit intimidating especially when I didn't know an answer (Y8 questionnaire)

No workshops matched level 3 descriptors because students did not initiate questions about others' reasoning in public. There were however examples of increased questioning of others in small group work.

### *Explaining*

At level 1, teachers elicit students' explanations but often restate and fill them out. In the following example the leader initially depersonalises the process of explaining, suggesting that 'maths' does the 'telling', but then links it back to student action when Jodi's explanation comes to a halt and she needs a prompt. Such details - the pragmatics of maths talk (Rowland 1999) - allow leaders, consciously or not, to manage student self-esteem by attributing ownership flexibly.

> T: It’s not right . Can it tell us anything?
> Jodi: the left side needs 2 more to get 11, the right side needs 2 less. So…
> T: So which can you swap? ( Obs, Y10)

At level 3 students expect that all results have to be explained. In later workshops this was clearly an understood classroom convention. For example Chaz and Iping were cautious (or lazy) when publicly contributing to a Sudoku problem:

> Chaz types in her numbers on the Product Sudoku screen.
> Iping calls out “You’ve got a lot of explaining to do”.
> Chaz: “OMG you mean I’ve got to explain it all!”, enters a few more - thoughtfully - deletes some - and returns to group (Obs notes, Y10).

This area showed the clearest change. In later sessions, leaders treated explanations as objects of mathematics available to be revised and critiqued. They might prompt students to re-state their own explanations several times in one interaction. They did not necessarily resolve students’ incomplete or contradictory reasoning at the end of each whole-class phase. These practices helped to locate the source of maths ideas.

### *Source of ideas*

In this area, there was little variation of level but some tensions in achieving it. Level 2 describes student ideas as the focus of classroom attention, with the teacher managing their interaction. The Shine workshops were designed at this level so leaders had clear guidelines that their role was to elicit student ideas and hold back from suggesting strategies. In early workshops there were episodes of unease when progress was slow and neither leaders nor students would volunteer ideas. Students who dropped out at that time described sessions as boring. Two related features that linked individual work to public work were useful in reducing these episodes. Leaders used group work time to circulate and rehearse students in ideas and explanations, allowing them to call publicly on those students later. One leader described this as a more purposeful version of his usual school practice. It was also made easier by the number of helpers present: usually 4 or 5 accompanying teachers and volunteer university students. Students found the potential attention liberating:

> Different groups have different ideas, and the teachers help with the different ideas. And if there’s one teacher then you’d only be able to help with one idea and not like everyone’s (interview Y10).

So, although there was often more help available than students used, it had a practical and symbolic role in creating space for multiple opinions, students as well as teachers.

### *Responsibility for learning*

This fourth area concerns the connections that students make between their own learning and the public practices of the maths-talk community. At level 1, students are passive listeners and reporters, co-operating with teacher instructions. At levels 2/3 they also show that they expect to understand mathematics and, if they don’t, they initiate talk about their understanding rather than seek instructions about what to do. Shine students sought help in relatively private interactions so whole-class observations showed no clear progress beyond level 1. In interviews, however, several students said that Shine maths involved more detail and persistence than school and linked this to trying repeatedly to understand:

> We look at problems and we take them apart and we try to explain every single bit carefully along the way. We try to find another way of getting the answer until

> we understand completely what the question is about, all the possible answers. (int Y8)

Even a student who was less aligned with this convention described it as part of the maths - doing the 'whole problem' – not just the requirement of an arbitrary teacher:

> You have to do the whole problem - it feels like a never-ending tunnel. I've got this bit and I can go on – now he says what about this bit! (int Y8)

In the first two areas, leaders did change the focus of their questioning and explaining, and students changed too. There were tensions when teachers directed students in unfamiliar practices but the changes were accepted, perhaps because they were recognisably linked to progress and authority. In the last two areas, the framework was used to show fixed level 2 classroom behaviour as far as this could be controlled by teachers. However, managing this level made demands for student input that conflicted with expectations about pace and social roles. Questioning and explaining were again important techniques for communicating the expectations. We observed that leaders blurred distinctions between private and public knowledge, and whether demands came from teachers or from the task, and suggest that this helped to resolve the paradox of instructing students to take responsibility.

## Students' views

The observation framework raised the question of how students might independently take on ownership of mathematics ideas and responsibility for learning. The evaluation operationalised this as students' self-assessment on progress within Shine and its transferability to their school maths, described via scored statements and open questions in questionnaires (84 paired plus 32 initial and 16 final) and interviews.

Over 90% of students felt that they had improved in all aspects of problem-solving maths. Over 80% also felt the project had helped their school maths, but only moderately and with no consensus as to how. Year 10s had the most extreme reactions either way: several who dropped out explained that it wouldn't help them at all with school maths, but some articulated benefits clearly:

> "Yes because at first in most exams, most questions I rush to do it, but this time I take time and I think of different ways to do it. When I am stuck I think of the ways I do here." (Int, Y10)

There were however a few significant changes to questions about school maths. Year 8s believed less that answers were 'right or wrong'; and year 10 students questioned whether 'you do well in maths by copying your teacher'. Both of these open the way for contributing ideas and pursuing understanding in the classroom. In addition the proportion of pupils in each cohort who said that they enjoyed school maths lessons rose after the project by 9-15%. This counters the national trend that enjoyment of maths decreases with age even as students increase in confidence and attainment (Sturman and Twist 2004).

The students drew clear distinctions between the type of maths they worked on at Shine and what they did at school. Around 80% found it 'quite a lot or a lot' different, and 60-70% 'quite a lot or a lot' harder. They commented on the increased demands to contribute and explain:

> "We are pushed more to join in, it's not book work at all, the teachers encourage you loads to answer problems" (Y8)
>
> "It is harder and focuses on why not what" (Y8)

Analysing student comments showed three main sets: two that constructed the project as an addition to school maths, and one as a challenge. One set focused on having learnt a single skill described either as *problem-solving* or *working systematically*. Another focussed on extending a problem-solving repertoire, stressing the variety of possible approaches: *"I learnt different methods in solving"* (Y10). The last set described Shine as introducing a new perception of mathematics:

> "It's like two different whole subjects […] that's very similar, not just the one whole maths being taught in different ways." (Y8)

Pleasurable excitement about this 'new' maths was more common in the Year 8 cohort than Year 10. The notion of 'two' maths was followed up in interviews. Students reconciled differences by predicting that school maths would become like Shine when they progressed to GCSE/ A-level. This related to the perceived intensity of the problems but also to their application in context:

> "Yeah in schools we look at normal maths, symmetry or anything like that … Here we look at overall, world-wide. Like - the cinema problem – we don't do this stuff in school. It's based on what we do everyday - everyday stuff." (Y10)

Problems often use contrived contexts in which to create mathematics, eg forming algebraic expressions for cinema ticket prices. Perhaps the opportunity to stay in them for an extended time while developing a strategy is as important as the superficial realism, and is missing in school maths.

Finding benefits for school maths was one motivation for students, but we were also interested in whether the project offered intrinsic motivation that could sustain involvement through the unease noted in observations. Students were asked to describe their best achievement on the project. Responses were split between a sense of achievement about being involved in difficult tasks - "*Just the fact that I'm coming every week ever since I started and got on with the problem and not gave up"* - and the social enjoyment of *"solving problems with my friends and contributing to it*".

Shine set out to create a teaching environment that approached a maths-talk learning community. The evaluation confirmed that students became more ready and able to explain their own reasoning and attend to a range of strategies, and this contributed to success in solving problems. The practices used by teachers to promote public reasoning created some social tensions for students and teachers, and strategies emerged to reduce these. Students found the project challenging but motivating, with more differences than similarities to school maths.

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