Mathematical Knowledge in Teaching: the Nuffield seminar series

Kenneth Ruthven and Tim Rowland

University of Cambridge, UK

Over the last two years, BSRLM members from several universities have contributed to a national seminar series on Mathematical Knowledge in Teaching that has met on six occasions. The final report of the series, supported by the Nuffield Foundation, is now available, and an edited book is in preparation. The seminars have examined current scholarship and research bearing on how teachers' subject-related knowledge underpins successful mathematics teaching, and on how such knowledge can be assessed and developed. As a consequence, it has been possible to identify areas where there is a need for further research in this important field.

Keywords: mathematics teaching, teacher knowledge, seminar series.

Introduction

The purpose of this paper is to report the activity and findings of a seminar series on Mathematical Knowledge in Teaching between January 2007 and June 2008. The six meetings, over seven days, were funded by the Nuffield Foundation. Further details of the six meetings, including a number of papers presented, can be accessed from http://www.mkit.maths-ed.org.uk/

The Nuffield seminar series

Intellectual aims and organisation

The overarching intellectual aim of this seminar series was to draw together current ideas and evidence about the forms and functions of the mathematically-related knowledge which enables teachers to support successful student learning of mathematics. More specific aims have been to achieve a critical conceptual synthesis, to establish significant professional implications, and to identify major research needs. Thus, the seminar series consisted of an opening 2-day conference followed by five 1-day conferences, each with a specific focus. Three of these events were concerned with taking a critical overview of existing thinking and research on the key issues of conceptualising and theorising mathematical knowledge for teaching, auditing and assessing such knowledge, and developing and deepening such knowledge. Two further events were concerned with closer analysis of a selection of research studies and teaching resources related to substantive aspects of mathematical knowledge for teaching. The specific aspects were division and fractions, and argumentation and proof, chosen because they are well-researched and contrasting aspects of mathematics addressed at multiple levels of education. The final event focused on formulating a research agenda. An Appendix to this paper provides a summary overview of contributions to the seminar programme on each of these themes; in particular, of the papers and presentations prepared by speakers.

To encourage a focus on critical synthesis and professional implications, a specification was provided for presenting speakers (in a fairly standard form, adapted to fit the focus of the particular meeting), emphasising the importance of speakers explaining how and why ideas or methods represent a significant advance on, or alternative to, earlier ones, and identifying significant implications of bringing these ideas or methods to bear on the practices of teaching, and of teacher education and development. Similarly, the specification enjoined group discussions to identify commonalities and contrasts, complementarities and conflicts between different ideas and methods; and to identify any significant limitations of the ideas and methods in illuminating important practical issues, and any significant limitations of current policy and practice in acknowledging important insights from these ideas and methods.

Professional aims and organisation

The overarching professional aim of this seminar series was to build research capacity in this area. More specific aims were to establish a working network covering teacher educators and educational researchers, but also extending to potential research user groups with a direct professional interest in the area; and to provide particular opportunities for doctoral student researchers working in this area to participate in the network. Thus the network that has been established includes mathematics education researchers and teacher educators at various career stages (including doctoral students). Recognising that future teachers develop much of their mathematical knowledge at school and university, prior to entering courses specifically devoted to teacher education, we recruited some persons active in designing and teaching undergraduate (specialist and service) mathematics courses to join the core membership. To involve practitioner and policymaker communities concerned with mathematics teacher education and training, we were fortunate to be able to recruit as core members of the seminar series participants representing the Department for Education and Skills [DfES, now DCSF/DIUS], the National Centre for Excellence in the Teaching of Mathematics [NCETM], and the Office for Standards in Education [OfStEd]. Although our attempts to recruit school-based subject mentors to core membership of the series proved unsuccessful, at each of the university-hosted meetings we were able to attract such colleagues from amongst those associated with local teacher education activities. Likewise, other doctoral students and academic staff of the host institution joined these seminars as local attendees. Finally, a number of other interested persons, primarily mathematics education researchers and teacher educators visiting from overseas, attended particular seminars.

Findings

The main findings from the conceptual synthesis which has taken place over the course of the seminar series can be related to the research needs and professional implications identified at the final meeting.

One important need is for research on mathematical knowledge in teaching to take on a more programmatic character. This has a number of aspects. First, research to date has tended to focus on particular phases, topics and processes: on primary teachers and teaching rather than secondary or tertiary; on arithmetic rather than algebra, geometry or probability and statistics; on proof and proving rather than on models and modelling. Second, a range of approaches, drawing on different sources of evidence, have been developed in order to identify what mathematical knowledge

is important for teaching, but insufficient attention has been given to triangulating – and potentially integrating – these approaches and understanding the relationships between them. A more systematic research programme would provide a stronger knowledge base for designing courses of initial teacher education and providing professional development for serving teachers.

A further need is for research which probes some of the assumptions which have pervaded the work undertaken to date, and explores the viability of alternative conceptualisations of mathematical knowledge in teaching and corresponding ways of investigating its operation. For example, much work to date has conceived mathematical knowledge in teaching as strictly individual knowledge capable of being elicited in isolation from the actual teaching situation, and as independent of the tools and resources available to support subject teaching. Equally, most research, even when it has compared educational systems, has proceeded on the assumption that mathematical knowledge in teaching is independent of particular contexts for (and cultures of) mathematics education. Finally, the field has tended to emphasise those aspects of mathematical knowledge which are specific to teaching, perhaps at the expense of more generic aspects.

A final need is for a stronger emphasis on research aimed at developing and validating tools to support the enhancement of mathematical knowledge for teaching within the initial and continuing professional education of teachers; research and tools locating such knowledge more strongly within everyday processes of teaching, and relating it more directly to providing effective support for student learning of mathematics. Examples of such tools examined during the seminar series include diagnostic ‘mathsmaps’ to help teachers reflect on, and develop, their mathematical knowledge; ‘lesson study’ as a vehicle for collaborative development of mathematical knowledge for teaching in the context of preservice teacher education; and the ‘knowledge quartet’, an analytic framework for the identification and discussion of primary teachers' mathematical knowledge as evidenced in their teaching.

The last stage of the seminar series coincided with publication of the interim and final reports of the Williams review of primary mathematics teaching (raising many issues equally applicable to mathematics teaching in other phases). During the final seminar, several papers examined the main recommendations of the Williams review, endorsing the main recommendation that professional development for teachers should focus on “the three interrelated strands of mathematical content, mathematical pedagogy and embedded practice”, but also noting some of the challenges of implementing the review’s proposals. The review’s emphasis on more informal modes of teacher development through in-school mentoring and coaching calls for investigation of how best to embed professional learning around teachers’ everyday work in school communities in ways which enhance mathematical knowledge, for example through forms of lesson-focused peer interaction. Equally, the review’s emphasis on peer interaction may underestimate the potential contribution of what have been termed ‘educative’ curriculum materials, specially designed to support the mathematical learning of teachers as well as their students.

Dissemination

Three main avenues of dissemination from the seminar series are being pursued. First, an open website was established at the start of the project (http://www.mkit.maths-ed.org.uk) to make public details of the series, and provide access to documents

prepared in the course of it. Now that the seminar series has been completed, this website provides a valuable and readily accessible archive of its work and findings.

Second, to further develop the synthesis undertaken by the seminar series, and make it more widely available, an edited book is now in preparation. We expect this to appear in 2010 in the Springer *Mathematics Education Library*. The main chapters will develop papers and presentations given during the seminar series. The editorial guidelines emphasise the same concern as the seminar specifications that authors should make clear what intellectual progress and professional implications are represented by the research that they are reviewing and presenting. This represents a valuable opportunity to refine the work undertaken during the seminar series, and to further strengthen critical synthesis, particularly through the inclusion of discussion chapters at the end of each section of the book which will draw on the seminar discussions to explicitly address that brief.

Third, we made two presentations at the February 2009 day-conference of the *British Society for Research into Learning Mathematics*. The first of these focused on conceptions of Mathematical Knowledge in Teaching, with contributions from Marilena Petrou, Maria Goulding, Jeremy Hodgen, Anne Watson, Andreas Stylianides and Jill Adler. In this session, the speakers presented snapshots of the different theoretical and practical perspectives examined in the seminar series. In the second session, Julie Ryan, Dolores Corcoran and Fay Turner reported on empirical research into the use of particular theorised tools in the development of Mathematical Knowledge in Teaching, involving pre-service and early careers teachers. Ken Ruthven concluded by outlining some areas where the need for further research was identified.

Appendix

Brief details of the six seminars are as follows.

Seminar 1: Conceptualising and theorising mathematical knowledge in teaching

January 2007, Cambridge. http://www.mkit.maths-ed.org.uk/seminar1.html

The seminar was structured around the following invited presentations:

Maria Goulding - Mathematical subject knowledge in primary teacher training: a view from England and Wales.

Jeremy Hodgen - The situated nature of mathematics teacher knowledge.

Heinz Steinbring - Changed views on mathematical knowledge in the course of didactical theory development.

Dina Tirosh & Ruhama Even - Teachers' knowledge of students' mathematical learning: an examination of a commonly held assumption.

Kenneth Ruthven – Synthesis.

Seminar 3: Auditing and assessing mathematical knowledge in teaching

September 2007, London. http://www.mkit.maths-ed.org.uk/seminar3.html

The seminar was structured around the following invited presentations:

Julian Williams - Audit and evaluation of pedagogy: towards a sociocultural perspective.

Tim Rowland - Auditing the mathematics subject matter knowledge of elementary school teachers.

Julie Ryan & Julian Williams - Mathsmaps for diagnostic assessment with pre-service teachers.

Marilena Petrou - Michigan research on developing a practice-based theory of content knowledge of teaching.

Seminar 5: Developing and deepening mathematical knowledge in teaching

March 2008, Loughborough. http://www.mkit.maths-ed.org.uk/seminar5.html

The seminar was structured around the following invited presentations:

Anne Watson - Developing and deepening mathematical knowledge in teaching: being and knowing.

Fay Turner & Tim Rowland - The Knowledge Quartet: a means of developing and deepening Mathematical Knowledge in Teaching.

Birgit Pepin & Linda Haggarty - Making connections and seeking understanding: mathematical tasks in English, French and German textbooks.

Seminars 2 and 4: Mathematical knowledge in teaching: examining the case of division and fractions (Seminar 2) and the case of argumentation and proof (Seminar 4)

April 2007, Manchester. http://www.mkit.maths-ed.org.uk/seminar2.html

January 2008, Cambridge. http://www.mkit.maths-ed.org.uk/seminar4.html

At these meetings, discussion of research studies and teaching resources was stimulated by critical reflections on published research related to the above substantive topics. These were prepared and presented by:

Dolores Corcoran, Johannes Siemons, Ray Huntley, Lara Alcock, Peter Huckstep and Sandy Pepperell (the case of division and fractions);

Maria Goulding, Marie Joubert, Andreas Stylianides, Cathy Smith and Johannes Siemons (the case of argumentation and proof).

Seminar 6: Formulating a research agenda on mathematical knowledge in teaching

June 2008, London. http://www.mkit.maths-ed.org.uk/seminar5.html

At this final meeting of the series, several core members of the seminar made brief presentations, each with a supporting paper. These presentations were organised thematically as follows.

Towards a programmatic framework

Maria Goulding & Marilena Petrou - Conceptualising teachers' mathematical knowledge for teaching.

Andreas Stylianides - Towards a research programme for identifying what mathematical knowledge is important for teaching.

Johannes Siemons - Mathematics knowledge in teaching: Formulating research.

Julian Williams - Towards a conceptualisation of a 'collective teachers' mathematical knowledge.

Towards more informed provision

Lara Alcock - The relative impact and teacher perceptions of different kinds of professional development.

Julie Ryan & Julian Williams - Teachers' stories of mathematical knowledge.

Ray Huntley & Peter Huckstep - The place of exemplification in mathematical knowledge.

Anne Watson - How can learning more maths impact on teaching?

Towards a comparative perspective

Paul Andrews - The cultural location of teachers' mathematical knowledge: another hidden variable in research on mathematical knowledge for teaching?

Birgit Pepin - What kinds of knowledge help teachers to become effective teachers of mathematics? What kinds of choices do teachers have? – a comparative perspective.

Towards a broadened agenda

Dolores Corcoran & Sandy Pepperell - Mathematical knowledge in teaching for social justice.

Marie Joubert - Using ICT in mathematics classrooms.

Into the post-Williams era

Tim Rowland & Fay Turner - Research into how deep knowledge of mathematics may be developed through ITE and PPD.

Kenneth Ruthven - The need for a programme of research on educative curriculum materials as a mechanism for the diffusion of mathematical knowledge in and for teaching.

Acknowledgements

We thank the Nuffield Foundation for providing funding to support the research seminar reported in this paper. We are grateful to all the colleagues, many of them named in this paper, who contributed to the seminar series, and to the BSRLM day-conference presentations.

Online Reference

The Nuffield Seminar Series in Mathematical Knowledge in Teaching (MKiT)
http://www.mkit.maths-ed.org.uk/