A-level mathematics: a qualification for entry to quantitative university courses

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Meetings with concerned groups of academics with a particular interest in the mathematics knowledge of students when they arrive at university are reported. There was general agreement on two immediately tractable issues and the appropriate actions: an A-level mathematics curriculum without options, so as to maximize students' knowledge in common, and examinations that test understanding and not merely memory and manipulative skill- so as to encourage deeper learning than at present. The relatively low numbers taking A-level mathematics is a much tougher issue. The consequences include many university courses in quantitative subjects admitting students without A-level mathematics, and adapting content and teaching accordingly, so as to survive. The underlying problem is to understand the unpopularity of mathematics after GCSE and what might be done about it.

Keywords: Mathematics knowledge; Curriculum; A-level; Quantitative courses; Student numbers.

Rationale

A generation ago school maths was a route for socio-economic mobility. The author's grandfather was a railwayman, his father an inner-city secondary school maths teacher, and he himself became a university professor, and he knows others whose families made a similar journey over two or three generations. But this route no longer seems to be open- certainly not to the same extent. In fact it seems mathematics is generally unattractive to the present generation of secondary school students- despite the potential rewards.

The author has spent his career in science and technology in higher education, and is researching an alternative secondary school maths curriculum- in the hope that, by being more in tune with the needs of 21st century life, maths will become more attractive to a new generation of learners. A-level maths is an important interface- to university courses and hence to the professions. So, one aspect of the author's research involves identifying and focussing on the shortcomings of the maths curriculum at A-level, from this perspective.

Introduction

Since November the author has been visiting a range of university departments offering courses leading to various mathematics dependent professions and talking with admissions tutors and those academics with a special interest in students' mathematical knowledge. All the courses require significant mathematical knowledge- although to differing extents- and the universities occupy different prestige levels.

The sample of visited universities covers three different prestige levels:

Top-ranking- research dominated
Middle ranking
1992 (ex-Polytechnic).

The courses investigated were:

Business and Management
Economics,
Computer Science/Informatics,
Engineering,
Mathematics, and
Physical Sciences.

The aim of sampling in these two dimensions is to try to ensure that the full range of any A-level maths curriculum shortcomings and opportunities for improvement- at least as far as preparation for the professions is concerned- will be uncovered. Hopefully this sample coverage gave accurate insights across a full enough range of both subject and A-level achievement.

Discussions with academics

I had prepared a range of curriculum content questions for my meetings with the academics, including-

a. Is the maths curriculum content sufficiently forward looking, in general, for 21st century needs? For example- is there sufficient emphasis on discrete maths?

b. Is good practice in calculator usage, including emphasis on estimation, good preparation for use of the software packages used extensively in professions?

c. Relational databases are not covered at A-level and yet relational databases support most Internet applications- so they are both topical and of great practical importance- and the underlying relational algebra would surely be rewarding?

d. School mathematics uses reasoning solely for mathematical problems. Surely there is great scope to use mathematical notation, especially graphical notation, and mathematical reasoning in non-mathematical areas- for example reasoning using causal networks?

But my meetings with academics, mostly admissions tutors for HE courses with a strong mathematical content or application, turned out perhaps unsurprisingly to be dominated by their concerns, and these together with their underlying causes have since become mine too.

I encountered general dissatisfaction with the maths knowledge of students arriving at university. I wanted to know whether changes in curriculum content would improve matters significantly. The answer I received was a qualified “yes”. It was qualified because their main concern was depth of the students’ knowledge rather than specific content. I constantly heard that the maths knowledge students acquire at school is “shallow” and often is not retained after the examinations, and this is much more of an issue than the particular maths topics in the curriculum. I was curious about the causes. But an even bigger problem turned out to be the number of students studying maths to A-level, and therefore qualified to enter quantitative courses. There are fewer than there used to be, despite the expansion of universities generally, with destabilising consequences for university courses, particularly those in the middle and lower tier institutions.

So, the scope of my investigation has inevitably changed- from testing my ideas about introducing 21st Century information age content into the curriculum to aiming to understand more pressing concerns about the state of the maths knowledge interface to university courses. My initial questions are on the back burner for now.

Concerns about secondary mathematics education

The following is a distillation of what I learned from discussion with these academics together with my thinking about underlying causes. It is separated under the headings below for clarity of explanation but the topics are all interconnected.

1. Mathematics knowledge

There was agreement that Maths knowledge is for working on problems. Two kinds of knowledge, for two kinds of problem situation were recognised. *Shallow knowledge*- using particular problem solving skills in situations where knowledge is routinely matched to a class of standard problems. “Calculus for technicians” was an example quoted. *Deeper/coherent knowledge- r*ecognising what pieces of maths, that may have been taught/learned separately, are together relevant in a particular problem situation. Importantly the problem may have to be formulated before it can be solved

2. Curriculum

a. Content

The remark commonly made by colleagues in Maths education, that “the curriculum is too crowded” didn’t resonate. But there was agreement that Breadth versus Depth or Coherence is an issue- and the knowledge students emerge from exams with was described as “shallow”.

b. Deeper implies narrower

So trading breadth for depth of knowledge became a discussion issue. If the curriculum must have reduced coverage to allow room in the timetable for greater depth, which topics should go? Will the various university disciplines that use maths agree on what should be sacrificed? There was one hopeful sign- an engineer said he would “Trade calculus for better knowledge of algebra”.

c. Options

There was general unhappiness with the variety of maths coverage students had experienced at school and universal support for a Single maths A-level syllabus *without options*. Universities want this so all the students they admit have been exposed to the same topics at A-level (of course some will have absorbed more than others and some will have taken Further Maths, but the common coverage will have been maximised).

3. Examinations

The exams define the curriculum and this is inevitable with high stakes testing. The academics accepted this statement (though they, unrealistically I thought, deplored it). I am not sure they recognised all its implications. They made two distinct criticisms of the current tests: they encourage shallow knowledge- “learning for the test” (which is soon forgotten)- and they fail to identify those students with really high mathematical ability. I suggest the underlying causes are as follows.

a. Competition for business between exam boards

This is likely to encourage grade inflation- 25% A grades now, and rising. It was suggested that a single exam board would be beneficial development.

b. The cost of testing

Exam Boards are under pressure to minimise the cost of testing. The economics of large scale testing favours electronic assisted marking.

c. e-testing- the conseqeunces

Electronic assisted marking favours short questions with short answers. (The extreme case is Multiple Choice Questions which can be marked completely automatically.) And a consequence of this is fragmentation of maths knowledge into facts and short procedures which can be tested and marked cheaply. This is exactly what the academics mean by “shallow knowledge”.

d. Economics of testing

Testing deeper knowledge requires longer questions and/or project work and is significantly more costly. We can reasonably conclude that the economics of the current testing regime bears some responsibility for the shallow and fragmented maths knowledge of students and without spending more money on testing this is not going to change.

4. Numbers taking A-level maths

a. Contraction of maths numbers

Universities have expanded but maths A-level numbers have shrunk. There were 80,000 candidates in the 1980s, only 56,000 now, although numbers are rising again by about 8% per annum (STEM 2008). Adrian Smith (Smith 2004) attributes the fall to AS-level being too hard when it was introduced and deterring many students from continuing maths to A2 level. But in any case, even assuming numbers continue to increase and over time get back to the earlier level, this contrasts with an expansion in many other subjects.

b. Consequences of low numbers

The shortage of A-level maths students has greatly affected some universities and some disciplines leading to instability in the numbers meeting entry requirements at middle rank universities (see next section).

5. Effect on universities of the shortfall in A-level maths numbers

Universities have been affected differently according to their ranking.

a. Top tier universities

These have had first call on the maths-qualified students and so have been relatively unaffected and have generally expanded their intakes somewhat (but with some difficulty in the case of chemistry and physics).

b. Middle and lower tier universities

These, however, have been strongly affected by the shortfall in all quantitative courses, as follows. Departments have been required to fill student places or lose staff and ultimately close. The general response has been to lower admission criteria- ultimately reducing the maths knowledge entry requirement from A-level pass to GCSE. Even so, some courses have closed and some departments have closed. In some cases accepting GCSE maths as an entry qualification has meant doing a lot of remedial maths in the first year of the course. (The problem of these students' weak maths is compounded by their having done no maths during their A-level years. In principle a FSMQ is the answer here but, so far, there is not a sufficiently widespread take-up to allow admission tutors to insist on this. In any case they may feel they have insufficient influence over students' choices to do so.) Other courses have modified course focus- becoming more qualitative (Informatics courses, for example, have tended to do this, although some have closed).

6. What factors determine maths numbers?

a. Career choice

Maths knowledge can open so many career doors. So why don't more students take A-level maths? What career advice are students getting at school?

b. Dislike of maths

Do so many really dislike maths so much?

c. Maths is hard

This seems to be many students' perception and Adrian Smith (ibid) suggests that the GCSE exam in maths actually is harder than in other subjects.

d. Isolation of maths

Has demathematisation of the science curriculum contributed? Science and maths used to be mutually supportive.

e. Shortage of maths teachers and access to the curriculum

There is a shortage of maths teachers, particularly in some parts of the country, and some maths teachers are not very well qualified mathematically, in contrast with the subject knowledge of teachers in many other subjects. Does this effectively restrict the access of some students to maths? The maths teacher shortage seems likely to continue, or even get worse as older teachers retire. Can current maths A-level numbers be maintained/increased? If more students should want to take maths, could the schools actually accommodate them?

f. Access to Further Maths A-level

The Further Maths Network is helping mathematically ambitious A-level students to prepare for this A-level, despite the restrictions on classroom access to maths teaching at this level. Since FMN began, numbers at AS level have more than doubled to 8,000 and are up 50% at A2 (FMN 2008). This seems like a much needed ray of hope among so much gloom!

Conclusions

Modernising curriculum content is not the most urgent A-level maths reform. Two significant reforms that appear to be urgent and relatively straightforward to implement:

A single maths A-level syllabus with no options (ACME 2009) and, ideally, a single exam board, so as to maximise the maths knowledge in common of students entering university, and A-level exams that test deeper knowledge- a necessary precondition for encouraging students to acquire deeper knowledge.

It is to be hoped that both will happen soon, because there are other urgent and much less straightforward, issues as follows.

Maths A-level numbers have declined over time and by doing so have damaged universities' ability to offer courses in quantitative subjects, in contrast with their general expansion. Numbers are increasing again but have not yet returned to historic levels. For whatever reasons, maths A-level is not a popular choice. Students may not be getting good post-GCSE advice about the importance of maths for entry to so many careers. Students' choices can be dangerous for themselves and for universities!

A further concern is that teacher shortage threatens universal access to the maths curriculum, and could constrain any increase in numbers that might otherwise occur.

Future work

The author plans a further round of discussions with his university contacts, with the following objectives:

To coordinate their response in support of the ACME recommendation that there should be a Single Maths A-level Syllabus with no options, so that all students admitted to university have been exposed to the same A-level Maths curriculum;

To seek consensus on the content of the hoped-for Single Maths A-level Syllabus- what topics should covered and to what depth- and what should be left out- perhaps by ordering a priority list.

To seek a consensus on the importance of deeper assessment of knowledge at A-level- so as to encourage students' deeper understanding, while recognising that this will increase the cost of assessment.

He also intends to consider mechanisms for getting better post-GCSE advice to students about the career importance of taking A-level maths- what career doors may open for them that will otherwise stay shut.

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