Linking Geometry and Algebra: English and Taiwanese Upper Secondary Teachers' Approaches to the use of GeoGebra

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The idea of the integration of dynamic geometry and computer algebra and the implementation of open-source software in mathematics teaching underpins new approaches to studying teachers' thinking and technological artefacts in use. This study opens by reviewing the evolving design of dynamic geometry and computer algebra; teachers' conceptions and pioneering uses of GeoGebra; and early sketches of GeoGebra mainstream use in teaching practices. This research has investigated English and Taiwanese upper-secondary teachers' attitudes and practices regarding GeoGebra. More specifically, it has sought to gain an understanding of the teachers' conceptions of technology and how their pedagogies incorporate dynamic manipulation with GeoGebra into mathematical discourse.

Keywords: Geometry; Algebra; Open Source Software; Comparative Study.

Introduction

Algebra and geometry are two core strands of mathematics curricula throughout the world and are considered the 'two formal pillars' of mathematics (Atiyah, 2001). It is therefore not surprising that they have been specifically targeted by the field of technology (Sangwin, 2007). Many researchers consider mathematics education as one of the earlier education fields to introduce technology as an assistant tool in classrooms (Papert, 1980; Noss and Hoyles, 1996).

The major application of technology in mathematics education is the integration of mathematical software in teaching practices. In respect of geometry, the most widely used computer applications, known as Dynamic Geometry Software (DGS) and include, Cabri-géomètre and Geometer's Sketchpad (GSP), etc. One important feature of DGS is the drag mode, encouraging interactions between teachers, students and mathematics (Jones, 2000). The drag mode can be used to explore and visualise geometrical properties by dragging objects and transforming figures in ways beyond the scope of traditional paper-and-pencil geometry (Laborde, 2001; Ruthven, 2005). DGS also has options to visualise the paths of objects as they move. For algebra, the most widely used applications are known as Computer Algebra Systems (CAS) and include programmes such as Mathematica, Maple and Derive. Some graphical visualisation and symbolic representations of algebraic expressions are implemented in CAS. Using the metaphor of the two 'formal pillars' of mathematics, geometry and algebra are afforded prominent positions especially at the secondary level (Hohenwarter & Jones, 2007). However, the connection between geometry and algebra, is apparently missing, as evident that in some countries geometry and algebra are entirely separate in their curricula (ibid). Although research into current technology use of computer algebra and dynamic geometry in teaching practices separate each sphere into distinct areas for study; I argue against this separation as there are areas overlapping algebra and geometry such as functions and graphs (Dubinsky and Harel, 1992). Examining both together has great educational implications and the connections between the two should not be ignored (Edwards & Jones, 2006). However, there is a gap in the literature dealing with this linkage between both fields and the use of technology. Despite an awareness of the need for a combination of DGS and CAS (Hohenwarter & Fush, 2004), software designers struggle to combine them as there are completely different constructs in software design. GeoGebra could be seen as pioneering software, although whether or not it is successful in linking DGS and CAS still needs research as the supporting evidence is limited at present.

Comparative Study

Recent research has indicated that culture influences the ways that teachers behave and inter-culture differences appears to be stronger than intra-culture differences (Schmidt et al., 1996; Givvin et al., 2005; Andrews, 2007). In particular, comparing eastern and western traditions with their respective Confucian and Socratic underpinnings can be enlightening as there are great differences in teacher beliefs and practices (Leung, 1995; Tweed and Lehman, 2002; Andrews, 2007). There is little comparative research of technology use in mathematics education, especially between Eastern Asian and Western countries (Graf. and Leung, 2001). Consequently, seeing how culture influences technology-mediated mathematics teaching is a pertinent issue.

There are large-scale quantitative studies such as TIMSS and PISA and small-scale qualitative studies. These studies highlight both similarities and differences between mathematics education in different cultural contexts in depth and in breadth. Quantitative studies such as TIMSS have also been reproached for their uncritical evaluation and for promoting globalisation over curricular and cultural diversity (Andrews, 2007). In contrast, small qualitative studies acknowledge cultural differences without attempts for generalisation. Particularly, when comparing East Asian and Western traditions with their respective Confucian and Socratic underpinnings, there is a significant difference between what are classically designed with the educational traditions (Leung, 1995; Kaiser et al., 2005; Tweed and Lehman, 2002). In particular, Kaiser et al. (2005) proposed a framework analysing East Asian and West European cultural traditions in mathematics education. The framework by Kaiser et al. (2005) is adapted partially in terms of teaching styles as I undertake a small-scale qualitative study in countries that exemplify East and West with a focus on teachers' perspective and their use of technology in mathematics teaching. The Eastern country chosen is Taiwan since it is viewed as 'the one most often cited admiringly by educators in the West for the level of its students' educational achievements (Broadfoot, et al., 2000: 13)' and a high mathematics performing country in international comparative studies such as TIMSS and PISA (Mullis, 2003; OECD, 2004; 2007). The Western country chosen for the study is England due to its contrasting educational system (Broadfoot et al., 2000). A cross-cultural study between Taiwan and England helps obtain a sense of the commonalities and discrepancies of teachers' conceptions and practices in relation to GeoGebra use. I have chosen to research at the upper-secondary level (students aged 15-18) as this level is less researched but is a crucial step for bridging students' secondary mathematics learning and higher education. Therefore, the overarching research questions are: (1)What are the upper-secondary mathematics teachers' conceptions of technology in relation to GeoGebra in England and Taiwan? (2) In what manner is GeoGebra used for the teaching of geometry and algebra by Taiwanese and English teachers? (3)How are the teachers' conceptions of technology and GeoGebra related to their teaching practices in both countries?

Methodology

Since there is little research into GeoGebra usage to date, this study is exploratory (Marshall and Rossman, 2006; Creswell, 2007). In brief, exploratory and multiple-case studies are my chosen methodology as the research focuses on this particular mathematical software, requiring specific teachers who utilise GeoGebra to teach upper-secondary level mathematics. Comparing and contrasting cases of teachers with interest in using GeoGebra from Taiwan and England provide a comprehensive understanding of how GeoGebra can be used in two very different cultural traditions, pedagogies and curricula.

I define mathematics teaching with the use of GeoGebra in Taiwan and England as the two main units of analysis. These have embedded two cases of teachers who use this software. Moreover, within the units, four cases of English and Taiwanese teachers are studied to obtain evidence of their views on GeoGebra teaching practices. To achieve the comparability between cases and units, pre-determined themes: teacher background, views on technology and GeoGebra, software comparisons and ways of using GeoGebra have been set for research design and data collection. A complete set of data was collected from four school visits. All of the interviews were audio and video-recorded, lasted for approximately an hour each and took place in classrooms using either a laptop or a computer connected to an interactive whiteboard. During the interviews the teachers demonstrated ways they utilised the software. The interview data were collated and summarised for each of the four cases. Two of the cases will be introduced in the following passage.

The cases

Li

Li has thirteen years of teaching experience at the upper-secondary level in Taiwan. Since his first degree was in applied mathematics, he gained an interest in IT during his undergraduate study. He was enthusiastic about new technologies and volunteered to translate the Traditional Chinese version of GeoGebra. Moreover, he had been creative in using different software packages, free software in particular, and trying to use a combination of different open-source software to make teaching materials. He has written some journal articles comparing new, open-source software packages detailing how they might be incorporated into mathematics teaching for Taiwanese teachers. In addition, he conducted GeoGebra training courses and workshops for teachers in senior high schools in Taipei. He had also set up his website and school website and uploaded his up-to-date GeoGebra materials and step-by-step tutorial materials for students or teachers. Li had a similar opinion to Jay on students' and teachers' attitudes towards the use of computers. However, he was positive that exploiting GeoGebra can change students' attitude towards mathematics. Some of his designed teaching materials and tutoring examples of using GeoGebra in solving examination problems were displayed on the websites. He also encouraged students to use the websites for reference and discussion. The salient categories are listed as:

Tool use	Graphing, calculations, demonstration, problem-solving, revision, investigation, and interaction
Mathematics topics	Geometrical topics and algebraic calculations
Teaching style	Curriculum-based, textbook-oriented and exam-driven, self-developed teaching materials and website with GeoGebra
Infrastructure	Home, IT room or computer and projector in classroom

Tyler

Tyler has taught mathematics to 11-16 year olds in a college for twelve years. He has also acted as an AST1 supporting schools and as a part-time school consultant, cooperated with the NCETM GeoGebra project and hosted a GeoGebra training workshop at his college. Tyler's utterances reflected a view of GeoGebra as an environment for exploring dynamic geometry rather than algebra. He viewed GeoGebra as a replacement to Cabri, which he used before GeoGebra. However, he mentioned that his experience with GeoGebra was approximately half a year, which meant that there were areas of using GeoGebra that were under-explored and underdeveloped, such as using GeoGebra in teaching algebra.

Some criticisms about current usage of technology in schools were brought up in terms of the IT rooms and school websites. He described his intention to change the way his pupils work from being passive to actively involve in learning through software. Moreover, he did not expect that students would not undertake much thinking in the IT room. In addition, some school mathematics websites have mathematics tests for pupils to log on to at home with their personal passwords which, in his view, allowed no room for discussion and interaction. He pointed out that GeoGebra is interactive and intuitive so he could set up diagrams and activities for students to interact with easily: 'This is different. This is maths by interacting; this is maths by trying things out, by conjecturing, by having a go.' He emphasised that GeoGebra could not only be used as a presentation tool by teachers but also as an investigation tool for pupils. An enthusiasm for GeoGebra was apparent in Tyler's strategies of using GeoGebra in mathematics teaching.

Overall, Tyler was reflective and explorative about different practices with GeoGebra, and eager to find out possible areas where GeoGebra could be useful in mathematics teaching. He also drew a distinction between 'knowing how' to use it and 'getting used to' using it in relation with GeoGebra. This inferred that he acknowledged the differences between using GeoGebra and teaching with the use of GeoGebra. The salient categories emerged from the data are listed as follows:

Tool use	Demonstration, interaction, investigation, exploration, testing hypothesis, creation, projection capability and the slider
Mathematics topics	Mainly geometrical topics
Teaching style	A whole-class teaching activity
Infrastructure	Home, IT room or computer and projector in classroom

Findings

Analysing the data thematically across the case studies revealed four salient dimensions in relation to GeoGebra-assisted teaching: **educational tools, teacher transition, mathematical scope and infrastructural change**. The findings are

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introduced in the following, which indicate that understanding the linkage between teachers' conceptions and practices is crucial. Firstly, the teachers' conceptions of GeoGebra seemed to be strongly rooted in their conceptions of the effectiveness and infrastructure of technology. The English teachers imbued a more positive attitude towards technology than their Taiwanese counterparts. However, teachers in both countries expressed favourable opinions regarding GeoGebra's agreeable contribution to their teaching. Secondly, GeoGebra was commonly used as a tool for visualisation, demonstration and interaction of mathematical topics, whereas for algebraic topics it was rarely utilised in England. It appeared that the English teachers associated GeoGebra primarily with geometric topics. Conversely, Taiwanese teachers worked with GeoGebra on both geometric and algebraic topics as they did not consider algebra and geometry to be necessarily separate; possibly as a result of the structure of Taiwanese curriculum and textbook-oriented culture. Thirdly, there were three different environments where teachers engaged with GeoGebra: - preparation of teaching materials at home, presentation and interaction in classrooms and activities for pupil investigation in IT rooms. Teacher transitions evolved from and were influenced by the infrastructure as they moved from preparation to presentation, incorporating interaction with pupils and finally encouraging investigation.

Conclusion

There are three aspects generated from the data that could be seen significantly different between the cultures in England and Taiwan. Firstly, teachers' attitudes towards technology in both countries varied. The participated Taiwanese teachers held negative conceptions of technology use for teaching practices, whereas the English teachers were positive about it not only because they were confident and comfortable about using ICT but also students seemed to have higher level of acceptance. Secondly, the Taiwanese teachers experienced greater difficulties pertaining to infrastructure as the classroom settings were not particularly designed for technology use in Taiwan whilst the English classroom settings implemented interactive whiteboards and projectors which offered convenience for teachers. Finally, in terms of pedagogy, the Taiwanese teachers tended to follow a curriculum based teaching strategy and mostly related GeoGebra exercises to textbooks; therefore, GeoGebra was used specifically for assistance of visualisation of textbooks examples. Again, the English teachers appeared to be more creative and flexible in choosing their teaching methods. As the Taiwanese educational system has an examination-driven culture, there are several areas being used extensively such as problem solving for university entrance examinations and proof of theorems as well as revision for examination preparation. In contrast with Taiwan, the English educational system has a focus on individual learning, therefore, there seemed to be a stress on students' individual investigation and interaction with GeoGebra.

Despite the potentiality of GeoGebra, teachers have not fully discovered its capability to link geometry and algebra but acknowledged that it offers pervading possibility in teaching practices. As Markus Hohenwarter puts it, 'GeoGebra is free software because I believe education should be free. This philosophy makes it easy to convince teachers to give this tool a try, even if they haven't used technology in their classrooms before'.

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