Primary pupils in whole-class mathematical conversation

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Although plenary sessions are common to mathematics lessons, they are often characterized by traditional approaches that endorse the position of mathematics as a kind of received knowledge and the teacher as sole validator of students' input. A socio-constructivist view of mathematics calls for a more conversational style of interaction among participants. In this paper an account will be given of a lesson in which children aged 9 – 10 years calculated the sum of integers from one to one hundred. Particular attention will be paid to one pupil, Anne, and her reassessment of a conjecture that she made early in the lesson. I suggest that particular teacher 'moves' facilitated engagement of other students with her idea and that this was one factor that led to her new insight.

Mathematics as conversation

The traditional classroom interaction structure is the Initiation – Response – Follow-up (I-R-F) model in which the teacher initiates an exchange and the student then makes a contribution and the teacher then makes a follow-up move (Sinclair and Coulthard 1975). In situations where the follow-up move is 'evaluative', the pattern is describes as I-R-E (Mehan 1979). It is suggested that the I-R-E structure reinforces the asymmetry of power between teacher and pupils (Mercer and Dawes 2008; Pimm 1994) – the teacher retains the locus of control and the students do little more than infer what is in his/her mind. Mathematics because of its association with recall of procedures is particularly susceptible to the I-R-E structure. Based on her observations of mathematics lessons, Wood (1994) describes a funnel pattern of interactions as one that involves repeated cycles of I-R-E where students are provided with leading questions in an attempt to guide them to a predetermined solution procedure. It is similar to the 'elicitation' pattern described by Voigt (1995) in which students, although given the opportunity to offer different solutions, are guided by the teacher to one definite argument.

The recognition that mathematics needs to be 'co-constructed' by students and teacher has led to interest in how a participatory model of discourse might be developed. In particular, it is felt that a follow-up other than evaluation might lead to a more conversation-like genre than stems from the I-R-E model. Nystrand and Gamoran (1991) use the term 'uptake' to describe the process of incorporating student responses into subsequent questions. They make a distinction between test questions which are designed to assess if the student knows what someone else thinks or has reported and authentic questions which signal a teacher's interest in what the student thinks. When the teacher uses authentic questions s/he opens the floor to what students have to say and this leads to substantive engagement by students. Related to the use of 'authentic' questions in mathematics lessons is the 'focusing' pattern of interaction identified by Wood (1994). In exchanges of this type, the teacher draws students' attention to the critical aspects of the problem giving them the responsibility of resolving the situation. A different type of follow-up is that of 'revoicing'. It is described as "the reporting, repeating, expanding, or reformulating a student's contribution so as to articulate presupposed information, emphasize particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students" (Forman and Larreamendy-Joerns 1998, 106). A key part of the reformulation is the use of 'So you think that' or 'So Tom thinks that'. O'Connor and Michaels (1996) describe this as a layering or lamination of the teacher's phrasing onto the student's contribution. It is akin to what Rowland (2000) terms an 'attribution hedge' in which some degree or quality of knowledge is linked to a third party. This layering, according to O'Connor and Michaels, "animates the student as the originator of the intellectual content" (1996, 79) and "makes possible an expanded and more contrapuntal set of voices and participant roles in constructing an idea than does the IRE" (p.97). Coupled with the attribution 'you think', the discourse marker 'so' opens up a new slot in the conversational space, giving the student an opportunity to comment on the correctness of the revoiced utterance. The effect of this 'layering' is to bring students' ideas in contact with each other and thus to effect involvement of all children in the conversation (O'Connor and Michaels 1996; Rowland 2000).

In a study in which four secondary school mathematics teachers were observed and videotaped for two weeks, Brodie (2008) used 'follow-up' as a key category to describe a teacher move. She maintains that it is broader than 'uptake' as it can refer to a contribution made by a learner either immediately preceding or some time earlier in the discussion and also because it includes both teacher-directed and 'conversational' moves. She found several ways that the follow-up move could be used by teachers and using some of the research cited above developed the following subcategories:

- Insert: The teacher adds something in response to the learner's contribution. She can elaborate on it, correct it, answer a question, suggest something, make a link etc.
- Elicit: While following up on a contribution the teacher tries to get something from the learner. She elicits something else to work on the learner's idea. Elicit moves can sometimes narrow the contributions in the same way as funnelling.
- Press: The teacher pushes or probes the learner for more on their idea, to clarify, explain more clearly. The teacher does this by asking the learner to explain more, by asking why the learner thinks s/he is correct, or by asking a specific question that relates to the learner's idea and pushes for something more.
- Maintain: The teacher maintains the contribution in the public realm for further consideration. She can repeat the ideas or ask others for comment or merely indicate that the learner should continue talking. Revoicing fits in this category.
- Confirm: The teacher confirms that s/he has heard the learner correctly. There should be some evidence that the teacher is not sure what s/he has heard from the learner, otherwise it could be press.

Brodie sees these moves on a continuum of more to less teacher intervention; in 'insert' the teacher makes his/her own contribution while 'confirm' and 'maintain' serve to keep the learner's input in the public domain. For these reasons, 'insert' and 'elicit' are viewed as more traditional than the other subcategories. However the teachers in Brodie's study, although committed to 'inquiry' mathematics were found to use a mixture of follow-up moves in any one lesson. This is unsurprising as teacher 'telling' is sometimes necessary and desirable (Chazan and Ball 1999). In this paper, however, it will be shown how a mathematical conversation dominated by 'press' and 'maintain' (in particular, revoicing) moves by a teacher afforded space for young students to engage with each other's ideas.

Background

The aim of my research is to investigate the factors that contribute to the development of mathematical insight by primary school pupils. The methodology is that of 'teaching experiment' which was developed by Cobb (2000) in the context of the emergent perspective and in which students' mathematical development is analysed in the social context of the classroom. For a period of six months, I taught mathematics to a class of thirty-one pupils (seven girls and twenty-four boys) aged 9 - 10 years. The school is situated in Ireland in an area of middle socio-economic status. Many lessons took place over two or three consecutive days, each period lasting forty to fifty minutes. I visited the class on a total of twenty-seven occasions. All phases of the lesson were audiotaped. When children were working in pairs, audio tape recorders were distributed around the room.

Forman and Ansell (2001) contend that analysis based on isolation and coding of individual turns is too limited to bridge the individual and social. Therefore, I conducted ethnographic microanalysis, which according to Erickson (1992) is especially appropriate when the character of events unfolds moment by moment. The approach adopted was top-down starting with the molar units (lessons) and moving to progressively smaller fragments.

The lesson described here took place on a third consecutive visit to the class during a week of the Spring term. On the previous two days, the pupils had been working on a lesson entitled 'Chess', a version of the 'handshakes' problem. The object of the activity was to find the minimum number of games that could be played in a competition where each player had to compete with all other participants. At the conclusion of this lesson some pupils had found the number of games necessary in the case of one hundred participants (i.e., the sum of 1 - 99) by using a calculator while others had latched onto the discovery made by one pupil, David, that the solution could be found 'by multiplying by the number less than it and halving it' ((100 x 99) ÷2). It was my intention on the third day to begin a new lesson but first told the story of Gauss (the mathematician who, as a boy, had amazed his teacher by his rapid calculation of the sum of integers from 1 to 100) in order to see if the pupils would make any connections between it and the chess problem. I expected that talk on this problem would last no longer than five or ten minutes. However, a rich discussion followed in which I had to 'let go'. The format of the lesson was a whole-class introduction, small group work followed by whole-class discussion The focus of this paper is plenary that took place in the introductory phase. The following transcript conventions are used: T.D.: the researcher/teacher (myself); Ch: a child whose name I was unable to identify in recordings;…: a hesitation or short pause; […]: a pause longer than three seconds; ( ): inaudible speech; [   ]: lines omitted from transcript because they are extraneous to the substantive content of the lesson.

Enactment

In the plenary under discussion the following thematic units or phases were identified:

- Phase One: Summing to one hundred, e.g. fifty plus fifty or five twenties.
- Phase Two: Finding partial sum and multiplying by appropriate factor (e.g. adding one to ten and multiplying by ten).

- Phase Three: Reasoning that solution strategy suggested in phase two would yield an incorrect solution.
- Phase Four: Adding '100' pairs (e.g., 99 + 1; 98 + 2, etc.).
- Phase Five: Adding sums of decades (e.g., 91 + 92 + 93 + …+ 100 and making appropriate adjustment to find sum of other decades).
- Phase Six: Applying solutions found for 'Chess' problem to Gauss problem.

Anne made contributions to the discussion in phases two, three, four and six. In phase three she underwent a change of mind about a solution strategy that she had proposed in phase two and this seemed to be on the basis of contributions of other pupils in the class. As pupils made suggestions, I wrote them on the blackboard. The following discussion took place during phase two and concerns an input that Anne made after Barry had suggested that the solution could be obtained by multiplying fifty-five (the sum of one to ten) by nine. Teacher moves are coded using Brodie's categories (above).

	Teacher moves
50 Anne: Thirty multiplied by ten.	
51 T.D.: Thirty multiplied by ten, why would you say it's thirty? Now Barry is using fifty, and he was multiplying fifty by about ten or nine, is that it?	Revoice (Anne) Press Revoice (Barry)
52 Barry: Yes.	
53 T.D.: And you think thirty multiplied by, why do you think thirty?	Revoice (Anne) plus press
54 Anne: Because if you add from one up to ten it's thirty.	
55 T.D.: How do you know if you add one up to ten it's thirty?	Press
56 Anne: If you add one to five, that's fifteen…	
57 T.D.: Hm, hm	Confirm
58 Anne: and then fifteen and fifteen is thirty so then if you multiply that by ten.	
59 T.D.: Ok, possibly that would get it for you. Fiona?	Maintain

Prior to Anne's suggestion, Barry had proposed that the answer would be around four hundred and fifty or five-hundred by multiplying fifty-five by nine. It is most likely that he was using the answer obtained on the previous day for sum of one to ten but I had erroneously thought that he was using fifty as a 'half-way point'. In turns 51 and 53 above, I revoiced the conjectures of both Anne and Barry. I also pressed Anne for justification. These moves probably assisted Anne and other class members to see the status of her contribution (and Barry's) as a conjecture – provisional, tentative and modifiable (Rowland 2000). Once Anne had justified her solution it was left in the public domain as one *other* possibility (see turn 59). Fiona then conjectured that the solution could be found by summing to fifty and doubling or summing to twenty-five and quadrupling. Thereupon, Alan commented on Anne's idea as follows:

	Teacher moves
66 Alan: Em, well, I don't think Anne's one is right.	
67 T.D.: Why?	Press
68 Alan: Cos ninety plus ninety eight plus ninety seven plus ninety six to ninety would be around over five hundred and when…	
69 Ch: Oh	
70 T.D.: Ok, so you are thinking that, you think, you disagree with Anne because you are thinking, what Alan is doing now, Alan is thinking ninety –[ ]- you are thinking ninety plus ninety one plus ninety two plus ninety three would give you approximately how much?	Revoice (Alan) Rebroadcast
71 Alan: Em, I don't know.	

72 T.D.: But it's…	Press
73 Alan: But it would probably be over five hundred.	
74 T.D.: It would be over five hundred, so in that section, if you are thinking about all those numbers there that would give you about, even just adding ninety to a hundred so you are thinking that would give you about five hundred- [ ] Barry?	Revoice
75 Barry: Eh, well, I disagree with Anne as well because eh I counted, I counted up all the numbers up to ten and I got fifty five.	

Alan has observed an error in Anne's reasoning on the basis that the sum of numbers between ninety and one hundred would be 'over five hundred'. My revoicing in turn 70 is directed initially at Alan ('so you are thinking') to ensure that I understand him correctly and then to the rest of the class ('Alan is thinking') for the purpose of rebroadcasting his contribution. In turn 75, Barry indicates that he also disagrees with Anne – he makes reference to his earlier thinking that the sum of numbers between one and ten is fifty-five. The conversation than turned to consideration of the sum of numbers between ninety and one hundred after which Anne interjected:

	Teacher moves
91 Anne: I don't think … my answer wouldn't work.	
92 T.D.: What were you thinking your answer was?	Maintain
93 Anne: I thought it would be thirty multiplied by a hundred.	
94 T.D.: Why would it not work?	Press
95 Anne: Em, because you would have to, cos I did eh one plus two plus three plus four plus five and then em I got fifteen and then I added fifteen and fifteen equals thirty but then it would be more because you would have to add six, seven and that ()	

In turn 91, Anne is reassessing her earlier reasoning and appears to have reached a new insight. Her reasoning is based on the fact that proportional reasoning cannot used to find the sum of consecutive numbers (in this case the sum of numbers between one and ten). While my question in turn 92, could be viewed as 'press' (for recall of a procedure), it serves to rebroadcast Anne's conjecture and thus has been categorised as 'maintain'. Although Anne's line of reasoning is different to that of either Alan or Barry, it is likely that their input caused some perturbation in her thinking.

Conclusions

Most of the teacher moves in this lesson were either 'press' or 'maintain' (usually revoicing). These moves served in this instance to make ideas public so that pupils became evaluators of each other's input. There was a sense that the pupils felt free to comment on the ideas of their peers but these comments were not viewed as disrespectful by the contributor. This is evidenced by the fact that Anne seemed to have little difficulty taking part in the conversation after Alan and Barry had discussed her input. However, in another situation, a different outcome may have emerged and thus the decision to 'go with the pupils' rests on the teacher's judgement. In this regard, Alrø and Skovsmose (2002) suggest that risk can be negative as one's suggestions may be refuted but it also includes the possible excitement experienced if one's input plays a significant role in the solution process. They advise that, in an educational setting, it is important that a balance is created between the negative and positive elements of risk. In this case it seems that there was a viable balance between the two and the resulting conversation allowed for new mathematical insights to be constructed – not only by Anne but by other pupils in different parts of the lesson.

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