Comparing Research into Mental Calculation Strategies in Mathematics Education and Psychology

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This paper argues for the importance of re-examining theoretical assumptions in research into mental calculation strategies and strategic thinking in mathematics education. By contrasting research into strategic thinking in mathematics education with that in cognitive and developmental psychology, three areas are identified where important details of the model of strategic thinking are left unexplored in education research while being dealt with more thoroughly in the psychological literature. The areas identified are: the positing of innate processes; the nature of memory; and the relation between conscious and unconscious mental processes. The status and reliability of introspective reports on mental processes are discussed as an illustration of the potential of research in psychology to further inform mathematics education research in this area.

Keywords: strategy, strategic thinking, mental arithmetic, National Numeracy Strategy

The National Numeracy Strategy (NNS), introduced in English primary schools in September 1999, gave increased prominence to informal methods or strategies and mental calculation. Doubts have been raised however about the success of attempts to teach mental calculation strategies or strategic thinking directly (Bibby, Askew and Hodgen 2003) as recommended in the NNS. In mathematics education there is a substantial body of research on strategy use and strategy selection in arithmetic in particular, although there is little consensus in the research community about strategic thinking as an aspect of mathematical thinking or about the value of the concepts 'strategic thinking' and 'strategy' for studying mental arithmetic (Threlfall 2002). Some views of arithmetic learning, although not necessarily inconsistent with strategic thinking research, focus more on number sense, mental models, or conceptual understanding. Additionally, the individualist view of mind and learning suggested by strategic thinking is challenged by socially centered views of learning such as social constructivism, social semiotics, situated cognition and hermeneutics.

The study

This research project (Craig 2008) sought to compare research into strategic thinking and mental calculation strategies in mathematics education (ME) and developmental and cognitive psychology (PSY) in terms of method and the assumptions about strategic thinking being employed. The study was motivated by a sense of ambiguity in the use of the terms strategy and strategic thinking in ME: much research into mental calculation strategies in ME seems to assume an unspecified model for strategic thinking whereas, in PSY, although research into strategic thinking shares some common assumptions, there are in fact many variations on the basic model and discussion of these differences is more common. I will provide a brief overview of

strategic thinking research in ME and PSY. I identify some overlooked assumptions about the nature of strategic thinking in research in mental arithmetic in ME: these assumptions, and their potential relevance for ME, are examined through a brief description of their treatment in PSY.

Strategic thinking in mathematics education and psychology

The term strategic thinking (ST), loosely, refers to the mental processes involved in making choices about different possible courses of action. Two common elements of most definitions of a strategy are that it is one of several options, and that it is directed towards a goal. Additional elements which are sometimes included in the definition by researchers are that the decision to apply a particular strategy and/or the operation of the processes involved in executing the strategy be conscious or at least accessible to conscious awareness. The nature of the implied difference between a mental calculation strategy and a procedure for calculation is not always made clear and there is, in general, some ambiguity in the way the terms 'strategy' and 'strategic thinking' are used in the ME research literature.

Differing research goals in PSY and ME influence the aspects of ST considered and the research methods employed. Three related research goals link ME research and PSY research: understanding cognition or the mind; understanding the development of the mind; and understanding learning. Understanding the development of the mind is the area were the two disciplines' concerns overlap most. In ME an important goal is to understand processes of teaching and learning, and understanding the development of the mind (particularly in relation to mathematical understanding, knowledge or skills) is seen by some, although not all, as an important part of that goal.

Key to the theory of ST in PSY is the observation that individuals adapt their behaviour strategically (i.e. in ways which increase their likelihood of successfully achieving goals) to regularities in their environment. Research focuses on developing and testing models of ST as a tool to understand and explain this observation, which is accepted as holding across a range of areas of behaviour. The research goal is to model strategy choice by specifying the factors which affect it and how choices are made. Laboratory-based experiments are used to explore group behaviour in problem-solving situations under various conditions. ST models (sometimes developed into computer models) aim to account for behaviour, such as the variation observed in strategy use, by replicating it, and thus to demonstrate a sufficient mechanism for increasingly adaptive strategy choice. Research in developing and testing models of ST in PSY tends to assume that the individual, and the mind of the individual, can be meaningfully studied and discussed in relative isolation from any particular situation or actions.

Research into strategy use in ME is less uniform in terms of goals than that in PSY, and consequently in the role of ST models in the research. In ME, ST has been described as "developing a repertoire of mental and written calculation strategies and informed decision making about the use of these" (Kyriacou and Goulding 2004). ST is studied as an aspect of mathematical competence – as something which it seems that successful students and adults do. Research interests are in how this competence develops, what factors affect the acquisition of the competence, and in particular how teaching can encourage or develop ST. This clearly overlaps with the concerns of PSY.

In ME, as in PSY, much research has been devoted to observing and categorizing the different types of strategy children and adults use on basic arithmetic calculations and on exploring the factors which may influence this (e.g. Baroody 1987; Torbeyns, Verschaffel and Ghesquière 2005; Selter 2001). There are some differences in the methods used, with ME research more likely to employ realistic situations and classroom observation than PSY. The difficulties experienced by some students in mastering mental calculation, and mathematical disabilities have been interpreted in terms of strategy selection models (e.g. Ostad 1998; Roberts, Taylor and Newton 2007). In some ME research adaptive strategy use has been characterized as a *goal* for direct teaching: different methods are used to encourage pupils to use a range of methods, to apply them appropriately to situations, and to respond adaptively to new situations (e.g., Selter 2001; Beishuizen 1993).

Models and assumptions about strategic thinking

The focus, in PSY, on understanding underlying mental processes leads to a ST model which problematises some aspects of ST simplified or taken for granted in ME. An examination of the theoretical literature in PSY revealed three issues which remain relatively unexplored in ST research in ME and which are relevant to the concerns of education. The treatment of these issues in PSY could provide a starting point for their discussion in ME. Although they are not the focus of this discussion, there are of course ways in which ST research in PSY would benefit from increased awareness of research in ME, for example, in terms of the way in which strategy choices and individual development play out in actual classrooms.

Memory

Models of ST in ME and PSY differ in the extent to which fact-recall and memory are problematised. In solving arithmetic calculations, particularly with smaller numbers, one option is to recall the answer from memory. In ME research the individual is implicitly characterised as having some number facts which they know and hence will state in answer to a question (although they may make mistakes) and others which they do not know. The decision to recall a number fact rather than recalculate it might be assumed to be unproblematic either on the grounds that there is no significant time delay or cognitive effort required in recalling number facts, or because individuals are assumed to have the self-knowledge to judge quickly whether or not they have a particular fact stored in memory. Rarely are these options discussed in ME accounts of ST, although measures of working memory capacity and knowledge of memorised number facts are considered important both in mental arithmetic research and in ME more generally (see, for example, the research reported in Dowker 2004).

In contrast to ME research, ST research in PSY treats mental recall of number facts as a strategy option of equivalent status to any other option, the merits of which must be evaluated and weighed up by the individual before it is settled on (see, for example, Siegler and Araya 2005). An interesting implication of this is that it assumes the ability to internally assess the accuracy of mental recall – usually considered as unconsciously held knowledge in PSY. For ME research an interesting question which would arise from a consideration of this would be whether incorrect judgments of the accuracy of memory contribute to poor strategy choices, and how these judgments are formed. Another interesting subtlety in the psychological model of ST which is lost in current research in ME is the importance of goal-related factors

in even this seemingly simple choice to recall an answer from memory rather than to calculate it afresh. The role of goals, potentially including social and affective goals, in determining strategy use has the potential to provide a different interpretation of some mathematical difficulties: in some ST models particular goal-orientations can be seen to encourage short term mathematical behaviour which does not promote long-term arithmetic development. Of course, this is not a novel insight for ME generally, but here we see the potential to build it into a cognitive account of strategy choice and mental arithmetic.

Innate vs. learned

In ME, ST is sometimes characterised as a desired end point to learning or as a teaching goal (see, for example, Heirdsfield 2000). In contrast, some psychological models characterise ST as an innate (or at least early developing) mechanism which drives later development of improved decision making, and accounts for the observed variability of strategy use in learners (e.g. Siegler and Araya 2005). As an innate mechanism ST is understood to be an equally valid description of the mental processes of those who succeed and those who struggle with mental calculation. In this model, the individual does not learn to generate appropriate strategies, nor does he/she learn to balance the relative merits of those strategies with respect to goals, rather these processes are innate, and they are said to guarantee (with some caveats) the development of increased competency in mental calculation. ME research in ST generally assumes that we can teach individuals to make 'better' choices whereas that in PSY assumes that the individual is necessarily making the 'best' choices already, given their goals, knowledge and competencies at a particular point in time, and failure must be understood not as failure to make the best choices, but as failure in the pattern of choices, over time, to contribute to development. The implications for teaching of the two perspectives are very different.

Conscious control of mental processes

In ME, it is assumed within many studies that the decision process itself can be improved by increased awareness of the different parts and that each of these is susceptible, in principle, to change and improvement. For example, discussion is considered to be important in increasing conscious awareness of the relative strengths and weaknesses of different strategies and of the problem factors which are relevant to the choice of strategy; this assumption of openness to direct conscious control and improvement underpins the teaching objectives suggested in the NNS.

Although researchers in PSY take different positions on the amount of conscious awareness and control possible over different aspects of ST, their description of the choice mechanism does not imply conscious control. This is not to say that an individual cannot consciously call to mind possible courses of action, weigh the alternatives and choose between them – but this is not the process generally labelled ST in PSY research. There is an important distinction to draw for education research: the question needs to be asked of how conscious processes of ST, such as those sometimes assumed in ME, and the largely unconscious processes studied in PSY relate to one another.

It should be said that, like ME research, much research in PSY is not explicit about which elements of ST are to be considered innate and which learned, and consequently which might be susceptible to alteration or improvement, and which elements, if any, are considered to be open to conscious awareness or control. In fact

many subtle distinctions are contained in the use of terms such as conscious, unconscious, innate, learned, and articulable which need to be explored. Differences in the way mental mechanisms, drives or structures relevant to ST are conceived in ME and PSY have important implications for the educational consequences of ST research. Currently a lack of clarity in the ME literature about the role of unconscious processes means that such issues often go unaddressed, although the importance of ideas such as tacit knowledge and pre-verbal processes in early understanding of number has been acknowledged in other areas of ME (e.g. Wynn 2000).

Introspective reports

The importance for ME of examining assumptions about the model of ST being employed, and the potential of PSY to provide a starting point in that exploration, can be seen in the use of introspective reports as data. Verbal or introspective reports from individuals on their own mental processes are the primary data source for much research into mental calculation in ME. Their validity and reliability rest on theoretical questions about the type and degree of access individuals have to their own mental processes in general and to the processes suggested in ST models in particular, and also on practical questions about how to access this knowledge. Although many researchers address the reliability of data gathered from verbal reports as a standard part of their research it is not usually explored as a theoretical issue which relates to the model of ST being assumed. In PSY, a great deal of debate and research has centred on the validity and reliability of verbal reports (see, for example, Nisbett and De Camp Wilson 1977; Gaillard et al. 2006). This debate could be a valuable resource for ME researchers in beginning to explore the implications of using introspective reports as a theoretical issue in the nature of ST.

Conclusion

Any study of the mental processes involved in strategic thinking makes assumptions about what strategic thinking is and more generally about the mind. Ontological assumptions are made about the type of objects which are appropriate to study (although the status given these objects may not be made explicit). Epistemological and methodological assumptions are made when relying on introspective reports of mental processes: assumptions about the type of introspective knowledge it is possible to have and the ways in which it can be accessed. It is important that these assumptions are acknowledged and discussed in the research in which they are made, both in order to reduce ambiguity, and because, as I have argued here, some of the assumptions have educational implications. The treatment of mental recall in cognitive and developmental psychology raises questions about the place in strategic thinking models of individuals' knowledge of their own memory processes. The distinction between innate mechanisms and learned behaviours demonstrates the need to reconsider the assumption that all elements of strategic thinking processes are open to alteration through teaching and experience. Finally the nature of and complex relationships between conscious and unconscious, attended-to and unattended-to mental processes is of real importance in understanding mental arithmetic competence from a cognitive perspective. Psychological research in strategic thinking provides a rich source of discussion and research in this area which could act as a stimulus and resource for similar discussion in mathematics education.

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