Diagrams as interaction: The interpersonal (meta)function of geometrical diagrams

Jehad Alshwaikh, Institute of Education, University of London

Diagrams are part an parcel of mathematics. However, the main stream among mathematician is prejudiced against the use of diagrams in public. In my PhD study, I consider diagrams as a semiotic mode of representation and communication which enable us to construct mathematical meaning. I suggest a descriptive 'trifunctional' framework that can be used as a tool to analyse the kinds of meanings afforded by diagrams in mathematical discourse. In this paper, only the interpersonal function of the diagrammatic mode is considered with illustrations. In specific, I consider labels, neat-rough diagrams and modality as realisations of that function. Concluding remarks with challenges are presented at the end of the paper.

Keywords: Diagrams, mathematical discourse, multimodality social semiotics, representation and communication, interpersonal function.

Introduction:

Communication is inevitably multimodal where different modes, such as visual representations, gestures and actions, are used to convey meaning (e.g. Lemke 1999; Kress and van Leeuwen 2006; O'Halloran 1999; Morgan 1996). In that sense, mathematics is a multimodal discourse where different modes of representation and communication are used such as verbal language, algebraic notations, visual forms and gestures. These different modes have different meaning potentials, they contribute to the construction of meaning and the deployment of them carries the ‘unified’ meanings (Lemke 1999; Kress and van Leeuwen 2006). For example, the verbal language in mathematical texts has limited ability ‘to represent spatial relations such as the angles of a triangle (..) or irrational ratios’ (Lemke 1999). Thus we need diagrams or algebraic notations to represent these qualities or quantities.

The aim of my study is to develop a descriptive framework that can be used as a tool to analyse the role of diagrams in mathematical discourse adopting multimodal social semiotics. Halliday (1985) argues that any text fulfils three functions: ideational, the representation of our experiences in the world (e.g. the mathematical activity); interpersonal, the social relation constructed with the reader of the text; and textual, presenting the ideational and the interpersonal into a coherent text. Kress and van Leeuwen (2006) extended Halliday’s account and suggested a framework to read images and presented the notion of multimodality to express the different modes of representation and communication.

In mathematics education, a number of studies have adopted Halliday’s Social Semiotics approach, to look at different modes of communication and representation such as verbal language (Morgan 2006 has proposed a linguistic framework to describe the verbal mode of mathematical texts; 1996) and graphs and symbolism (O'Halloran 1999). Both Morgan and O'Halloran agree that, still, there is a room to investigate other modes of representation and communication of mathematical discourse.

The status of diagrams in mathematical discourse:

Mathematical diagrams are part and parcel of mathematics. They were used in ancient civilisations such as Old Babylon four thousand years ago (Robson 2008) and were an essential part of Greek mathematics (Netz 1999). Moreover, there is nearly consensus that diagrams are important in doing, learning and teaching mathematics mainly in visualisation, mathematical thinking and problem solving. However, the current mainstream among mathematicians is prejudiced against the use of diagrams or, more precisely, mathematicians 'deny' and hide the use of diagrams in their work (Dreyfus 1991). Mann (2007, 137) also states:

> When a mathematician explores new ideas or explains concepts to others, diagrams are useful, even essential. When she instead wishes to formally prove a theorem, diagrams must be swept to the side.

The main argument against the use of diagrams is that diagrams (or visual representations in general) are a) limited in representing knowledge with possible misuse of diagrams (Shin 1994); b) of an 'informal and personal nature' (Misfeldt 2007) and c) unreliable and lack rigour (Kulpa 2008). One main reason for this view is that the main stream thinking among mathematicians conceives mathematics as abstract, formal, impersonal and symbolic (Morgan 1996).

In my study, however, I consider diagrams as available resources for meaning-making and as a means for representation and communication for students to communicate with each other or with themselves in order to convey specific meanings. I suggest an analytic framework that can be used as a tool to analyse the kinds of meanings afforded by diagrams in mathematical discourse focusing on geometry. This trifunctional framework offers three interrelated different ways to look at diagrams as a semiotic resource: ideational, interpersonal and textual. In this paper I consider only the interpersonal function of diagrams because of the space available (for the ideational function see Alshwaikh 2008).

Diagrams as representation and communication: the Interpersonal Function

In the act of representation and communication the author produces an image, for example, to convey a meaning. While doing so, s/he creates a type of imaginary social relation with the viewer. Following Kress and van Leeuwen (2006), this relation is realised by contact, (social) distance, and modality.

Contact:

In his social semiotic account, Halliday (1985) (Kress and van Leeuwen 2006 follow him) distinguishes between two types of contact between the author of a text and the reader/viewer; demand and offer. Either the author demands 'something' from the viewer, for example to answer a question. Or the author offers 'something' to the viewer and in scientific texts the offer is, mostly, information. One main feature in geometry context I consider to contribute to this kind of relation between the author and the viewer in geometrical diagrams is labelling.

Labelling

In geometry, labels are given to the components of shapes or diagrams: the vertices, the sides, the angles and parts of the diagram. Labels are either of offer-labels type or demand-labels type. [i] (Indeed there diagrams where the two types are combined.)

Offer-labels: This type offers information about geometrical diagrams and does not ask any action to be taken. It expresses either a) geometrical relationships such as equality, parallelism (Figure 1) or b) specific quantities (Figure 2).

In Figure 1, all labels are presented to show properties and geometrical relationships in diagrams. The general-type of these labels suggests that they are used to introduce definitions or qualities of these diagrams. This practice often occurs in school textbooks. In other words, presenting labels in a general form suggests an authority (a mathematical one) who says what the definition of, for example, a parallelogram is.

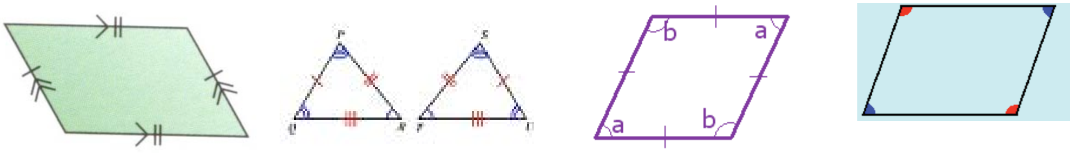


Figure 1: Offer-labels express geometrical relationships: same label (and/or colour) means equality or parallelism

On the other hand, labels in Figure 2 show specific examples with specific quantities. In mathematical discourse, this type suggests 'less' authority for the author than the previous type (general-type labels) because of the current mathematical mainstream understanding which values the general, abstract and formal prepositions, such as general properties and definitions (e.g. Davis and Hersh 1981), higher than the specific examples. Moreover, solutions to specific problems indicate that they were produced by someone with lower authority in response to a problem posed by higher authority. Thus one possible interpretation for diagrams in Figure 2 is that these are examples to illustrate the general case, 'a rectangular trapezium with bases 16 and 24 meters long and a height 10 meters' or 'this is an example of a scalene triangle'. Another possibility is that a student drew these diagrams in order to solve specific problems and s/he is showing them to the teacher/assessor.

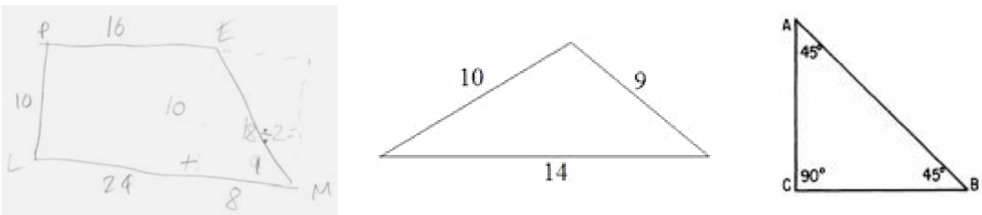
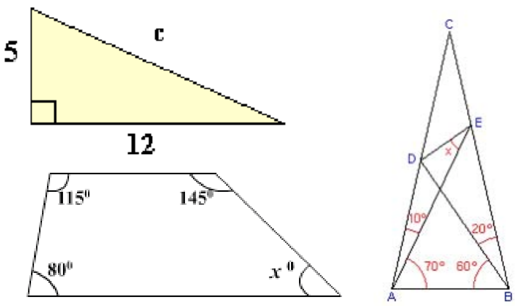


Figure 2: Offer-Labels express specific quantities



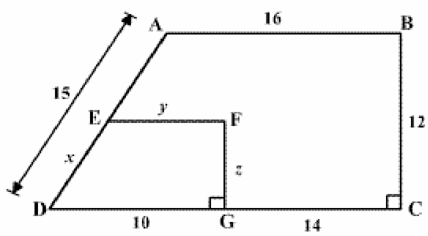


Figure 3: Demand-labels: unknown quantities

Figure 4: Demand-Labels: variable names

Demand-labels: This type of labels is realised by the presence of either a) unknown quantities (Figure 3) or b) variable names (Figure 4). It asks for a mathematical action to be done by the viewer which is to find the value of the quantity (finding the value of 'c' and 'x' in Figure 3, or the variable (x, y and z in Figure 4). In the context of school mathematics all these diagrams suggest an authority which asks a student to do something.[ii] Again, as in the offer-labels, the more general and abstract propositions the higher authority involved.

(Social) Distance

Kress and van Leeuwen (2006) consider the choices made between close-up, medium shot or long shot contribute to the meaning of image. In other words, the distance between the represented 'participants' in the image and the viewer of the image plays role in establishing a relation between them. Such physical distance is not realised in geometric diagrams. However, and following Morgan (1996), I consider that distance is expressed by the degree of 'neatness' of the diagram. In producing diagrams, the authors (mathematicians, teachers, students, etc.) draw accurate or rough diagram depending on the interest of the author, the context and the audience. A neat diagram 'indicates that the text is formal and that there is some distance in the relationship between the author and the reader' (Morgan 1996). On the other hand, a rough diagram suggests an intimate relation with the viewer or appears to be 'private' drawn while the producer works alone or for a personal use. Figures 5 and 6 show diagram drawn by students participated in my study to the same problems. As shown, they chose to present their diagrams differently.

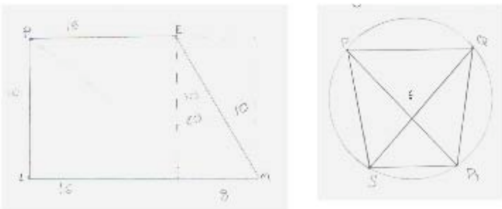


Figure 5: Neat diagrams

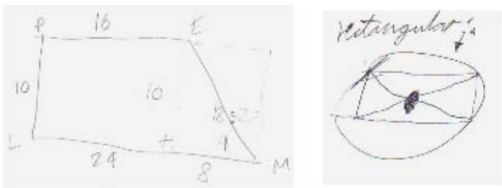


Figure 6: Rough diagrams

Modality

Modality in language refers to the degree of certainty and truth of statements about the world that is realised by the auxiliary verbs such as may, will and must and their adjectives such as possible, probable and certain (Kress and van Leeuwen 2006; Morgan 1996). In *Reading Images*, Kress and van Leeuwen distinguish between naturalistic modality and scientific modality depends on the social group which produces the image. The former refers to how the representation is 'close to' or 'true' in representing the reality and the photographs taken by camera are good examples. Scientific or abstract modality represents the reality in abstract mode such as geometric shapes or diagrams.

"Reality is in the eye of the beholder; or rather, what is regarded as real depends on how reality is defined by a particular social group" (Kress and van Leeuwen 2006). In mathematical discourse, 'more abstract approach is likely to be judged by teacher/assessor to demonstrate a higher level of mathematical thinking' (Morgan 1996). In general, it's not common to use naturalistic modality in (modern) mathematical texts, i.e. one rarely uses photograph or draw pictures to solve mathematical problem.[iii] Actually the dominant values and beliefs among mathematicians are that mathematics is abstract formal, impersonal and symbolic and school mathematics is not an exception. Hence, schematic or abstract diagrams are considered 'more' mathematical within the discourse of mathematics.

Clearly this issue is a social one which brings with it the issue of power as well. All the participant students in my study drew this type of diagrams. Drawing naturalistic or figurative diagrams to solve problems may consider by teachers/assessors as 'low level' of achievement or performance (Morgan 1996). One potential meaning arises when students draw these abstract diagrams is to announce their membership in the mathematical community or, at least, to say we know what mathematics is about. Other possible interpretation, however, is to challenge the teaching process and textbooks: to what extent do teachers and textbook present choices for students to present their own way of problem solving?

Concluding remarks and challenges:

Considering mathematics as a social and cultural practice, I argue that the use of diagrams is just as much an essential part of mathematical discourse as other modes, e.g. the linguistic and the symbolic. It is the practice of mathematicians that at some point turned to prejudice against the use of diagrams despite the fact that that use is and was essential. The suggested framework contributes to the analysis of mathematical discourse and practice in school mathematics (the way textbooks and teachers (re)present diagrams identifying the meaning potentials they carry) and how students make use of diagrams in their solutions.

However, studying the diagrammatic mode of representation and communication in mathematical discourse is not straightforward and does not lack challenges. I want to raise the challenges I face in interpreting the kind of social relations in offer- and demand-labels. Although I presented a general identification for the kind of social relation in these labels, there are different questions to think about:

- What kinds of social relations may labels offer/suggest? My point here is to ask whether students, for instance, would differently label their diagrams in solving problems to their teachers from their peers.
- Do mathematicians use labels in different ways from students? In other words, do mathematicians label their diagrams in a different way if they work on their own, with their colleagues or with their students?

These questions raise the issue of context in which diagrams are produced and used. I have to say that my study concerns about school geometry practice and that all diagrams I am using are drawn from within that context. In other words, any potential interpretations of the social function are dependent on that particular social practice. Furthermore, not only may individuals use labels (or other specific features) differently when engaged in different social practices but the ‘same’ feature may be interpreted differently in different contexts.

Notes:

[i] There is another type of labelling, *naming*, in which names are given to vertices, sides and parts of the diagram such as A, B, X, Y, etc. This type is different from other labels since neither information is offered nor actions demanded. It may be, however, considered as a reference to the viewer to refer to while 'reading' or solving a problem, and in that sense I consider it in the textual meaning.

[ii] This is also the case in an academic research article where the author may be presenting an as yet unsolved problem as an admission that their research is incomplete. This raises the issue of the context of production of diagrams.

[iii] There are some exceptions to this, especially when modelling is involved, for example, a photo of a ball being thrown used in the process of mathematising projectiles. Also there are 'incidental' photos/pictures in school textbooks– see Dowling's (1996) discussion of how different kinds of pictures may construct different readers. However, these illustrations are not considered geometrical diagrams and hence fall outside the scope of the suggested framework.

Acknowledgment: I would like to thank Candia Morgan for her vital comments on earlier version of this paper.

References:

Alshwaikh, J. 2008. 'Reading' geometrical diagrams: A suggested framework. *Proceedings of the British Society for Research into Learning Mathematics* 28 (1): 1-6.

Davis, P. J., and R. Hersh. 1981. *The mathematical experience*. London: Penguin Books.

Dowling, P. 1996. A sociological analysis of school mathematics texts. *Educational Studies in mathematics* 31 (4):389-415.

Dreyfus, T. 1991. *On the status of visual reasoning in mathematics and mathematics education*. Paper presented at 15th PME Conference, June 29-July 4, Assisi (Italy).

Halliday, M. A. K. 1985. *An introduction to functional grammar*. London: Edward Arnold.

Kress, G., and T. van Leeuwen. 2006. *Reading images: The grammar of visual images*. 2nd ed. Oxon: Routledge.

Kulpa, Z. 2009. *What is diagrammatics*? 2008 [cited 5/1/2009 2009]. Available from *http://www.ippt.gov.pl/~zkulpa/diagrams/fpptd/WID.pdf*.

Lemke, J. L. 1999. Typological and topological meaning in diagnostic discourse. *Discourse Processes* 27 (2):173-185.

Mann, K. 2007. 'Elements of reasoning with diagrams'- Review. *Metascience* 17:137-141.

Misfeldt, M. 2007. *Idea generation during mathematical writing: hard work or a process of discovery?* In CERME 5, Larnaca- Cyprus. Larnaca, Cyprus.

Morgan, C. 1996. *Writing mathematically: The discourse of investigation*. London: Falmer Press.

———. 2006. What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics* 61:219-245.

Netz, R. 1999. *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

O'Halloran, K. L. 1999. Towards a systemic functional analysis of multisemiotic mathematics texts. *Semiotica* 124 (1/2):1-29.

Robson, E. 2008. *Mathematics in ancient Iraq: A social history*. NJ: Princeton University Press.

Shin, S.-J. 1994. *The logical status of diagrams*. Cambridge: Cambridge University Press.