Working group on trigonometry: meeting 1

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Introduction

For many years trigonometry has been seen as a critical hurdle for those who wish to achieve at GCSE. To understand trigonometry involves orchestrating many concepts which in themselves are known to be hard to understand: angles, rations, functions for example. This area of mathematics is therefore a rich arena for exploring how such understandings can be coordinated; how teaching might make this meaningful; as well as the nature of trigonometry. In this first meeting we developed some areas for future exploration and made a start at identifying existing work that might inform us. The complexities we uncovered made it very clear why many teachers, as well as students, take a short-term view and reach for algorithmic approaches.

We found main clusterings of initial ideas:

- Prior or concurrent understandings entailed in trigonometry
- Difficulties and needs
- Teaching approaches we knew about that addressed the meaning of trigonometric relationships
- 'Grownup' understandings that might help us understand students' difficulties

Prior or concurrent understandings entailed in trigonometry

Similarity: proportionality; multiplicative relationships: scaling of one line is experienced differently to enlargement of one line in relation to another.

Angle: measuring the amount of turn, embodied sense of turn; eventual understanding of angle as independent variable in functions.

Length: why comparing lengths multiplicatively is appropriate, rather than additively.

Enlargements as transformations; trigonometric activity as transforming enlargements.

Angle as variable; functions.

Difficulties and needs

What creates a 'need' to understand trigonometry in the students' current flow of mathematical development?

Technical language is a difficulty.

Angle is sometimes previously understood as 'vertex', usually shaped like a wedge, cheese slice, angles about 30 degrees. Vertex is seen as the pointy bits of shapes, to be counted in order to describe properties of 2-d shapes.

It is hard to find aspects of students' experience in which equivalent ratios between lengths are understood, or classes of similar triangles are understood, in ways that trigger a need for trigonometry.

SOHCAHTOA fulfils perceived immediate needs for resolving right angled triangles: but fails to show why we should be interested in by how much the hypotenuse has to be multiplied by to find the opposite or adjacent sides?

Non-right-angled triangles ‘need’ sine and cosine rules to be resolved.

Future study of physics and maths needs functions in which angle is the independent variable.

Teaching approaches

It is understandable that many avoid what is difficult, and focus on necessary algorithms only.

One approach is to describe similar triangles in terms of relationships between sides.

The unit circle approach addresses covariation of angle and position, described by height above axis.

Another approach is to work out how to define particular triangles in minimal terms: need to know relationship between sides; for fixed angles (class of similar triangles), and ask how can the sides be worked out; then specialise to right-angled triangles; defining ratios using unit circle ratios and enlargement.

‘Grown-up’ understandings that might inform teaching

We could see trigonometry as the study of classes of triangles. Specific triangles belong to classes of similar triangles (similar to the relation between particular fractions, e.g. 6/8, and rational number class to which they belong, {¾}). Do students know what they are supposed to be paying attention to? What are the relationships that hold true in each class of triangles?

Trigonometry can also be seen as study of minimal information about shapes with ratio as the main element of shapes, rather than particular lengths. Invariance of ratio of sides as angle varies: ratio is the relation between sides of triangles; ‘proportional’ describes the relationship between similar triangles: A:B as a:b, as shown in this diagram:

	Ratio relationship	
Proportional relationship ↕	side A in big triangle ↔	side B in big triangle
	side a in small triangle	side b in small triangle

The unit hypotenuse in the ‘unit circle’ model is a canonical element that can be referred to again and again (see Geometric Images (ATM publication) for an approach involving mental imagery).

‘Trig’ goes beyond properties of triangles, it is about mappings between angles and relations between lengths.

Outcomes of first meeting

As well as the notes above, a mailing list has been compiled and readings have been circulated. Other people are welcome to join this group.

Plans

Cambridge meeting: February 28th 2009

- Jeremy Burke will report on the teaching design experiment carried out by PGCE students (to be published in *Mathematics Teaching*).
- Bibliography to be compiled.
- Reports from teachers about related work.
- Further discussion of issues.
- Programme of work to be devised.

Bibliography so far:

Beeney, R., Jarvis, M., Thata, D., Warwick, J. and White, D. 1982. *Geometric Images*. Derby, Association of Teachers of Mathematics.

Thompson, P., Carlson, M. and Silverman, J. 2007. The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education*. 10: 415-432.