How secondary teachers structure the subject matter of mathematics

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There are no large studies that focus on how mathematical ideas are structured in those lessons which lead to the successful learning of key mathematical ideas. There are some clues in pan-cultural comparative studies that 'coherence' and 'complexity' are critical features of the way mathematical ideas are treated. In our work with about 40 lesson videos available from various studies we have come to understand that example choice, task design, variation, and certain key mathematical activities play a part in engagement and learning, whatever the teaching style, social context, lesson structure and interaction patterns.

Keywords: mathematics teaching; comparative mathematics teaching; mathematics lesson styles; mathematical understanding

Introduction

Current literature in mathematics education, world-wide, distinguishes between so-called 'traditional' teaching and so-called 'reform' teaching. The former is generally associated with worked examples, textbook-type exercises, and students working individually having first attended to the teacher's exposition. The latter is generally associated with extended tasks, groupwork and shared construction of meaning. Other perspectives on mathematics education tend to accrete around these as if they are magnetic poles: thus rote-learning, procedural understanding, and negative self-concept get attached to 'traditional', while mixed-ability grouping, realistic activity, and distributed knowledge get attached to 'reform'. This polarisation of associated understandings of the nature of knowledge, learner identity, power structures, and the processes of learning seems to me to omit something essential about mathematics – that the mathematics that is available to be learnt in all these contexts is an agreed culture consisting of conceptual, or concept-like, understandings that are manifested in agreed forms of communication. Furthermore, these relate to each other in agreed ways. Some relationships arise in the notations we use, some arise in usage, and some arise in logical implications. Thus people who engage in mathematics act as if the meanings of what they do are shared. What counts as a mathematical object, meaning or concept in a reform classroom also has to count in a traditional classroom (and *vice versa*), and also has to count in any other mathematical situation, otherwise it is not mathematics. Focusing on what is common in mathematics lessons – what makes them 'mathematics lessons'- rather than on what is different gives new ways to look at teaching and in this paper I am extending the thinking reported in two earlier papers which move in this direction (Watson 2004; 2007).

Basic meanings

The polarisation described above requires care in defining terms. When I use the word 'concept' I am behaving as if there is a body of knowledge called mathematics which consists of defined meanings that individuals have to come to understand in some private, individual, way if they are to take part in mathematical activity. I have no

doubt that the process of learning is social, and that the way mathematics has come to be constructed is through historical-cultural process, but my concern here is that individual access to these ideas should not be left to happenstance but should be structured carefully by teaching. The phrase 'structured carefully' reminds some people of the cognitivist approach of putting learners into situations in which things are very carefully done to them in a pre-determined order, such as is done by some computer-assisted programmed learning packages, or by trailing through graded exercises in textbooks. Thus 'structured carefully' joins the accretion of meanings around 'traditional' and becomes alienated from images of classrooms in which learners work together, talk, explore, generate their own understandings and so on. It also can be taken to be opposed to independent learning, and to mean the intended control of learning by the teacher. It can also be taken to mean a view of teaching and learning that runs on tramlines towards training the behaviour required to pass a test. However, if instead we take it to mean that the teacher designs the environment very carefully so that it is more likely that learners will pay attention to some aspects of mathematical objects rather than others, more likely to use certain language forms than others, and more likely to engage in some kinds of mathematical activity than others, then we can use the phrase to query all kinds of teaching, and all kinds of environment.

Individual access to mathematical concepts is structured through sequences of tasks – things learners do. The word 'task' has come to be associated with the 'reform' pole, and hence to mean something exploratory, extended, complex, open-ended. However, I use it to mean anything a learner is asked to do, or chooses to do. Listening to a teacher's explanation, doing textbook questions, constructing a decahedron, designing packaging for tennis balls are all tasks. This means that lesson design is about sequencing tasks, and embedding tasks, rather than occasionally using tasks. This view of 'task' makes it possible to think through a lesson in terms of what the learner is expected to do, and hence to think about what and how learners might learn while doing these tasks. Hypothetical learning trajectories can be imagined in terms of what it is possible to learn within the dynamic social context of doing a particular task in a particular classroom. This contrasts with predicting learning trajectories from theories of conceptual or cognitive development, and then structuring researcher-designed task sequences for teachers to follow.

The questions with which I approach lesson analysis are:
- what is available in this lesson from which students can learn mathematics?
- what are learners being asked to do?
- how could mathematical ideas develop within this lesson?

Data

The data I use for this paper come from two sources. The Changes in Mathematics Teaching Project (CMTP) involved, among other things, analysing video-lessons from three secondary schools. During this work I developed a way to look at lessons by combining ideas from several other studies (Watson 2007). We used this as a tool to prepare outline descriptions of lessons in terms of the intellectual demands of tasks, and how these were sequenced.

The second source of data was the Mathematics Knowledge in Teaching e-Research Project (MKiTeR) which we undertook with University of Auckland. We were interested to learn more about how secondary teachers' mathematical knowledge

impacted on the way they handled the mathematical content of their lessons, both at the planning stage and in the flow of teaching. For this we filmed two experienced teachers, one of whom had strong mathematics qualifications and one who had less specialised knowledge, and two new teachers with a similar range of personal mathematics. We interviewed them pre-video about their intentions for the lessons. We interviewed them post-lesson, and returned later to ask about particular features that had interested us. Each lesson was analysed by four researchers in the UK, and was also sent to Auckland for more input. The analysis was done by applying tools we had already each used elsewhere. We then compared insights and discussed how we had adapted our tools for the particular use of seeing how mathematical knowledge impacted on the lessons. In this paper I focus on the use of the tools to map the mathematical content in the lessons, and this was seen as an essential precursor to the main task of identifying impact (which is not yet done).

Analysis of lesson structures

From the CMTP project we edited particular mathematical content out of the data to construct outline structures of lessons, as shown in these examples:

Lesson A	Lesson B
• T says how the ideas in the lesson sequence are progressing and what this lesson will be about and how it relates to last lesson; • Interactive recap of definitions, facts, and other observations. • T introduces new aspect & asks what it might mean. • T offers example, gets them to identify its properties • T gives more examples with multiple features; students identify properties of them. • Students have to produce examples of objects with several features, • Three concurrent tasks for individual and small group work: • describe properties in simple cases; • describe properties in complex cases; • create own objects. • T varies variables deliberately • They then do a classification task & identify relationships within groups • T circulates asking questions about concepts and properties.	• T gives object with multiple features & asks students to "think about..." • T indicates a classification by focusing on variables and variation; • Students use prior knowledge to identify properties of diagram; • T summarises what was done before and what problems to avoid; gives format sheet has two way classification to classify shapes, • Students create objects with multiple features and classify them using the format, exemplifying different combinations of properties. • Students induce generalisation and new definitions • Students transform existing ideas in light of new experience • Predictions and conjectures are written on the public board. • Students do informal inductive reasoning. • T leads association of ideas & generalisations. • Students have to compare examples, explain and justify their comparisons and conjectures

Our aim was to describe the range of lesson-types in the project schools, but what we found was that every lesson was individual in these terms, according to the mathematical ideas being taught, the students' prior experience, and the normal practices of the classroom. All were also influenced by students' contributions, so that the raw material for tasks was often provided by students, and hence the nature of the tasks was often dependent on what this material was. For example, if contrasting examples were generated by learners, the next task might be to compare them, but if learners offered examples of all one type, the next task might be to find a contrastive example. There were also examples of lessons in which teachers merely scanned offered answers until 'the right one' appeared.

Multiple analytical perspectives on mathematical content

For the MKiTeR data, the Oxford team used the following foci:
- Lesson structure (as above)
- Operations entailed in classroom tasks (from Doyle (1983), adapted and used Andreas Stylianides). This analysis was particularly revealing with extended tasks, because it allowed us to see them as sequences of challenges
- Examples used and generated (Liz Bills). We explored what it was possible for students to infer from the examples they were given, and this gave us insight into students' unanticipated responses.
- Methods of analysis used by TIMSS to identify mathematical coherence, rationale and complexity; these methods did not give access to *what* was coherent or *how* it was achieved.
- Variation (Thabit Al-Murani). Analysis of the dimensions of variation opened up in the lesson by teachers and students, and how these were developed, gave insight into what teachers thought were the important features of a concept.

From these we found new ways to look at lessons. For example, one lesson which, on the surface, appeared to be rather disorganised included a sequence of questions which enabled students to shift from taking a discrete view of a concept to having a continuous view of it. In another there was frequent mention of mathematical habits of thinking. In another, the teacher wanted students to shift from 'own' methods to an efficient method, but the choice of examples did not make this a necessity.

Interactive teaching

Most of the lessons included features of 'traditional' and 'reform' teaching, and represents a culturally particular kind of collaborative, interactive, combination of whole-class, pair and individual work is now the norm in many English schools. This 'style' takes account of both individual and collective sense-making, and attempts to structure this towards conventional understandings. We could call it 'English interactive': the focus is on talk between students, in small groups, and between students and teacher in the whole class; the tasks which generate the talk can be of many kinds. A variety of other features, such as extended tasks, collaboration, choice of grouping, practice exercises, telling the students things, and eliciting their understandings all seem to fit into this general interactive atmosphere with teachers varying their choices according to the unfolding of mathematical ideas.

So far we have only used these analytical tools with opportunistic samples of lessons and teachers. From some of the analyses differences have emerged which could affect mathematical learning. For example, interactive teaching includes responses to students and also guidance for students, but in most lessons an inductive form of reasoning, i.e. reasoning from recent experience, was endemic rather than reasoning deductively. Where we saw deductive reasoning it was deliberately introduced and structured by teachers, such as asking for 'because…' to be added to every answer. It was worrying how few lessons included any attention to this kind of shift. The English interactive style includes aspects of transmissional, discovery and connectionist teaching (as described by Askew et al. 1997) but the nature of the connections being made varies. We found that, however 'good' the lesson by generic judgements of teaching, no non-specialist teachers made connections within mathematics, nor did they discuss mathematical implications of the work, whereas most specialist teachers did make such links. Interactive teaching also raises some equity issues about the social dynamics in classrooms outside the scope of this paper, and can give undue dominance to the mathematical ideas and social skills of certain groups of students. Another feature was a difference between teachers who posed challenging tasks to whole groups, then set about helping different students work on those tasks, and those who set less challenging tasks but targeted individual students to pose harder versions of it. Our choice of tools, applied to lessons which show high levels of interactive teaching skill, can reveal these hidden problems. However, without more knowledge about what the conceptual flow in *excellent* interactive teaching, all we can do is point to some deficiencies.

Problems with research

Because our tools focus on how mathematical content is handled, whatever the overt lesson type, they are ideally suited to producing detailed answers to questions posed loosely by the TIMSS video analysis team. Their 'mathematics experts' identified mathematical rationale, coherence and complexity as common features of high attaining countries but could not say more than that. There are studies that probe 'typicality' in mathematics classrooms, and studies that show how certain modes of teaching make a difference to students' learning, but nothing that looks at how mathematical ideas unfold in successful classrooms of all kinds. There are studies that record how much attention is given to conceptual understanding, but not how that attention is given or what conceptual understanding means. There are also problems with the definition of 'success', and how exceptionally successful teachers can be identified in order to find out what they have in common. It would be important to have a sample big enough to distinguish between those who train students well for tests beyond their predicted grades, and those who help students develop robust conceptual understanding, and combinations of these, and to encompass those who work in departments in which a team approach to good teaching is the norm, and are outstanding in that context, and those who are working in more isolated environments. In the session at the conference we discussed these matters further.

References

Askew, M; Brown, M; Rhodes, V; Wiliam, D and Johnson, D. 1997. *Effective teachers of numeracy.* London, King's College..

Doyle, W. 1983. Academic work. *Review of Educational Research* 53:159-199.

Watson, A. 2004. Affordances, constraints and attunements in mathematical activity. In *Research in mathematics education, vol. 6: Papers of the British Society for Research into Learning Mathematics* eds. O. McNamara and R. Barwell, 23-34, London, BSRLM.

Watson, A. 2007. The nature of participation afforded by tasks, questions and prompts in mathematics classrooms. In *Research in mathematics education, vol. 9: Papers of the British Society for Research into Learning Mathematics* eds. L.Bills, J. Hodgen and H. Povey, 111-126 London, BSRLM.