What constitutes a 'hard' question in GCSE Mathematics: A bit of thought is required

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This paper is part of a larger project exploring the take-up of AS and A2 mathematics in a selective grammar school but comprehensive sixth form in the West Midlands. The sub-data presented here is from a survey of GSCE students asked for their reasons for doing or not doing mathematics in sixth form. Surprisingly for the selective intake most of those who were not doing AS level offered the reason that it is too hard. This then prompted an analysis (using Sierpinska, 1996) of the GCSE papers that these students were working towards and a questionnaire to AS students and teachers. The results show that any question requiring more than a memory prompt is considered hard by students and teachers.

Key words: GCSE, A-level; perception of difficulty;

Introduction

The research for this paper is a part of a larger PhD research project exploring take-up of AS and A2 Mathematics. The school is a selective grammar school in the West Midlands with a comprehensive sixth form intake. The attainment at GCSE at the school is consistently 100% of a year group of 90 achieve five A*-C grades. All the students take the Higher Tier Mathematics examination and of the cohort who completed the questionnaire, 88% gained a grade A* or A. Virtually all the students stay on to study four AS levels and are joined by approximately 160 students from other local high schools. Sixth form entry is currently 4 GCSE C grades with a Grade B from the Higher Tier to study Mathematics. At present there are 110 students studying AS Mathematics, 10 doing Further Mathematics and 5 doing the AS over two years.

The range of literature explored for this on-going research currently focuses on literature in the area of A-level take-up, in aspects of motivation and in models for understanding. Brown et al (2008) investigated the take up of AS level and found that the main reasons why students did not choose to continue their study of mathematics were perceived difficulty, a lack of enjoyment in the subject along with a lack of usefulness. My results are very similar to those of Brown despite the selective, high achieving cohort.

Background

In trying to understand perceptions of difficulty about the subject and consider how to determine the nature of 'hard' and 'challenge', the literature related to aspects of motivation was helpful. Dweck (2000) describes a *helpless* or *mastery-oriented* response to challenge and found that the nature of the goal held by a student strongly affected their response when faced with difficulty. The choice of goals consisted of *performance* and *learning goals*. The former involves a desire to obtain positive feedback on attainment, whereas the second consists of aiming to increase competence. These goals can be interpreted in the context of this project as the performance goal of wanting to achieve a certain grade at AS-Level or a learning goal of improving knowledge and understanding of mathematics.

The literature on models of understanding is large, including Skemp (1976), relational and instrumental understanding, Piaget (1971) knowledge is active incorporation of 'reality' into personal structures, Bruner (1966), active nature of learning and three modes of thought, and Pirie & Kieren (1994) understanding through a dynamic process and 'don't need' boundaries. The model I am currently using is from Sierpinska (1996) and in particular I used (and redefined) it to classify the nature of the demand level of questions in the GCSE papers taken by the students involved in the research.

Sierpinska (1996) describes a model for understanding that involves four basic mental operations *identification, discrimination, generalisation* and *synthesis*. These four acts she considers to be progressive and yet dynamic, not fixed, and personal to the individual trying to understand. The acts she also considers to not have a rigid hierarchy as they develop interactively. *Identification* she claims is made when an item is singled out and recognised as an object to be understood. I used this classification to apply to examination questions that explicitly stated which part of mathematics to apply; there would be no decision making required of the student. I also applied this classification to questions of a standard nature; the mathematics required was easily recognised (the difficulty of the mathematics in the questions is not considered; it is the process of identifying the mathematics required to answer the questions that is the focus). Figure 1 is an example of an *identification* question.

10. (a) Simplify $4p \times 5q$

.... **(1)**

(b) Simplify $d \times d \times d \times d$

.... **(1)**

(c) Expand $4(3a - 7)$

.... **(2)**

Figure 1: An example of identification. Question 10 requires no decision making by the student

Discrimination is described by Sierpinska as identifying differences between an object and others already understood. Questions in this category (e.g. figure 2) required the application of some mathematics not specifically mentioned, requiring students to make a decision about what mathematics to apply.

22. Katy drove for 238 miles, correct to the nearest mile.
She used 27.3 litres of petrol, to the nearest tenth of a litre.

$$\text{Petrol consumption} = \frac{\text{Number of miles travelled}}{\text{Number of litres of petrol used}}$$

Work out the upper bound for the petrol consumption for Katy's journey.
Give your answer correct to 2 decimal places.

Figure 2: An example of generalisation. Question 22 requires students to decide how to calculate the upper bound of a division calculation.

Generalisation is made when the object is seen as a particular case of another situation. Questions of this type require a further level of decision-making; what mathematics is needed and how is it to be applied. The example below (figure 3) requires substitution of two sets of points into an equation and the recognition that the result is a particular case of simultaneous equations.

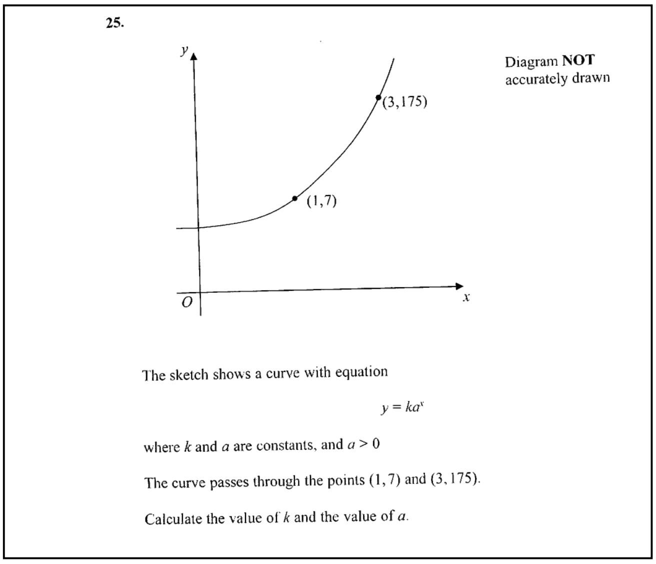

Figure 3: An example of generalisation.

Finally, Sierpinska describes a fourth category, *synthesis*, as the search for a common link, finding similarities in generalisations and forming a complete picture from previously separate concepts. I could not find any questions of this type on either of the June 2008 GCSE papers. It may be deemed appropriate by those who set the paper that questions requiring synthesis would be too hard at GCSE level. Interestingly however, there were also no questions of this type on any of the core mathematics papers (C1-C4) from the OCR examination board in June 2007.

Methods

The approach undertaken was one of mixed methods, including document analysis, questionnaire and interviews. I issued a questionnaire to 88 Year 11 students prior to them going on study leave to explore their AS choices and the reasons why they had decided to either continue their study of Mathematics or not take the subject further. I then did follow up interviews with 8 students who had decided not to continue with Mathematics. I classified the June 2008 Edexcel GCSE Mathematics higher tier papers using the Sierpinska model, and compared the results to the classifications given by 8 students who had decided to continue with AS Mathematics (4 male and 4 female, with 5 A* and 3A grades for the GCSE) and 6 Mathematics teachers. The comments from the Examiners Report were also compared.

Results and Discussion

The table below show my classification of the two GCSE papers. Some questions had different classifications for different parts, so number of marks rather than number of questions was used to compile results. The breakdown of marks for each type of understanding is shown.

Paper	Identification	Discrimination	Generalisation	Synthesis
Non-Calculator	78/100	20/100	2/100	0/100
Calculator	78/100	19/100	3/100	0/100

The grade boundaries for these papers are as follows:

Grade	A*	A	B	C	D	E
Non-Calculator	85	72	54	36	18	9
Calculator	84	68	48	299	16	9

It is significant that a student can obtain a GCSE grade A without having to make any decisions about what mathematics to apply. The predominance of *identification* questions may lead to students mainly working on questions where the mathematics required to solve them is obvious. Consequently this may lead to difficulties when students face atypical or non-standard problems.

A considerable number of high achieving students revealed in the questionnaire that they were not doing AS mathematics because they found GCSE *hard.* Of the 41/88 Year 11 students who had chosen not to study mathematics in the sixth form, 26 of these students were female and 15 were male. (All but four of these 'non-continuing' students had target grades of B or above.) The perception that the subject was hard was particularly surprising given that at the time the questionnaire was issued, (March) when the students had been revising for three months with the mathematics syllabus completed in January. In addition, this was the first cohort to sit the two-tier GCSE, where the higher paper contains significantly less A* material and more D grade questions.

So which questions and why did the students and teachers actually find 'hard'? Those interviewed were not asked to apply the model adapted from Sierpinska; instead I asked them to rate questions from the same examination paper as either 'easy' or 'hard' to prompt a decision and offered no further categories; most complied with some teachers giving split answers for different parts of questions.

There was a strong correlation between the questions classified as *identification* and those that were described as *easy* by teachers and students. Overall, students rated more questions as *easy* than their teachers. There may be several reasons for these results. Firstly teachers were asked to consider a typical student whereas the students who responded to this survey had all gained at least an A grade. Three of the students did not offer any detailed reasons for their choices, so that it may have been that they did not fully consider what was required of them in each question. This activity was given to the students when they were four weeks into the AS mathematics course and this may have altered their perception of difficulty of GCSE level questions. There are also issues surrounding whether it is easier to describe something as hard from someone else's perspective (as the teachers did) rather than admitting you find something hard (as was required of the students). Reasons given for questions being rated as *easy* by teachers were:

> Straightforward; simple; routine; common sense; easy to drill.

The emphasis on drilling and remembering suggests that students can be successful with a rote learning approach given the predictable nature of most of the examination questions. This approach to learning was also evident in the students' responses.

> Basic; learnt the rules; know how to do it; learnt it, practiced a lot

However, given that most of the examination questions are of a type that both teachers and students consistently describe as *easy*, why is it that students still perceive mathematics as *hard*? The reasons for questions being *hard* were again similar for both teachers and students and came into four main categories:

- multi-stage,
- general - involving algebra,
- unfamiliar context,
- lack of information given.

Sometimes a specific mathematical topic was given as a reason why a question was hard *"Students find fractions hard"*. Occasionally understanding was given as a reason "*Algebraic manipulation requires deeper understanding*." However, the most common reasons given from teachers and overwhelmingly from students were that memorisation was required. Typical comments from teachers were:

> Depends if they remember how to factorise quadratics;

> They usually forget where to put the boundaries;

Similarly for students:

> Can't remember the rules; forgotten technique; there are a lot of circle theorems to remember; graphs are difficult to remember. Lots of variation

> I find it difficult remembering how to get the signs the right way round. (for simultaneous equations)

These comments illustrate the difficulty with a rote learning approach to mathematics. They suggest that the perceived difficulty of mathematics may stem from the belief that students need to remember lots of separate rules requiring effort to memorise.

Memory again is typical in the comments from the students for the two *generalisation* questions:

> I have no idea what to do; can't remember ever being taught it.

These quotes highlight the problem with the rote learning approach to mathematics, as students are unsure how to approach problems that are different to those they have seen before. They find new questions hard to approach if they do not meet the conditions of methods they have learned to solve standard problems. It is particularly poignant that the latter is a quote from a student who achieved an A* grade on these papers and who has chosen to continue to study mathematics at AS-level. She is relying on her memory of previous examples rather than being able to apply her mathematical knowledge to new and different contexts.

The Examiners Report (2008) for the calculator paper indicates that the examiners are aware of the link between the standard nature of an examination question and the marks scored by students. Thus there is a significant reward for rote learning as most of the questions are of a standard type and can be successfully tackled without a deep understanding of the topic being examined. Comments include:

> … a percentage change question made a little more challenging by the relevant numbers being in a table. Many candidates had little idea how to proceed.

> … for a standard volume question this was poorly answered.

> Part (b) proved to be more of a challenge as the candidates were faced with a demand that was unusual.
>
> Candidates who had put in some preparation were rewarded on this question by a task which involved a straight substitution and it was very telling that this approach yielded much more success than that of using the given formula at the front of the paper and then manipulating to isolate cosA.

Particularly striking is the final comment which appears to suggest that examiners favour the method of rote learning the version of the cosine rule with cosine A as the subject, rather than using the version given in the formula sheet and applying mathematical skills of substitution and rearranging.

Conclusion

The title of this paper came from a teacher in response to why a GCSE question was 'hard'. He said: "A bit of thought is required". Similarly, the students claimed that the questions were hard when faced with the unfamiliar. I am left with many questions that I intend to explore through the next stages of my research.

- In a high attaining cohort, why do students find any of the GCSE questions hard?
- Why do teachers working with such students define any non-standard question as hard?
- How will students who continue with mathematics adapt to the demands of AS and A2-Level?
- How does the current performance goal environment impact on perceptions of both teachers and students on the nature of acceptable challenge?

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