Does Articulation Matter when Learning Mathematics?

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In this paper, we set out why we feel that it is important for pupils to articulate their mathematical ideas as they come to learn mathematics, whether orally, in writing or through some other representation. We explain the connections we see between thinking, articulating, learning and building a pupil's identity as a competent user of mathematical skills, thinking and reasoning. We believe that articulating mathematical ideas contributes to building what we will call 'mathematical resilience'.

Keywords: learning mathematics; articulation; language; mathematical resilience

Introduction

There is a growing focus on the role of articulation in pupils' learning (see for example Sfard 2007 and Mercer and Littleton 2007). When we use the term 'articulation', we mean the act or process of converting a thought or idea into something that is communicable to others. There is growing evidence from Science and English curriculum areas, in particular, that learning and articulation are closely interlinked (Mercer and Littleton 2007 and Alexander 2006), and that increased articulation leads to raising attainment and giving pupils a more positive attitude to learning. 'Talk for learning' is described, by such authors as Pimm (1987), Mercer and Littleton (2007) and Alexander (2006), as increasing both pupils' knowledge and their confidence in that knowledge.

There is also evidence (Newman 2004) that articulation contributes to resilience in learning; it would seem important to explore this in the context of mathematics. Mathematics is found difficult by many pupils; Cockcroft (1982) said: "Mathematics is a difficult subject both to teach and to learn" (Para 228, p, 67). Many people understand that mathematicians struggle; they work through long streams of reasoning, apply logical thinking, and use symbols to record their thinking. Along the way they take wrong turns, misunderstand the nature of the outcome they are seeking and often reflect and re-align their thinking. We conjecture that what is missing from the current political focus on mathematical attainment at a certain age is a focus on valuing this willingness to struggle. We propose to call this willingness to struggle 'mathematical resilience' so that we are in a position to discuss what it might look like; enabling us to use the power of language … to use names and descriptions to conjure, communicate and control our images (Pimm, 1995). Translating Newman into the mathematical context, mathematical resilience is developed in part through meaningful participation in mathematical learning. Such meaningful participation requires the pupils, not only the teacher, to be actively involved in thinking and communicating in mathematics, and may be achieved in a 'conjecturing atmosphere' (Mason 1988). Mathematical resilience is the opposite of 'learned helplessness' (Newman 2004 and Dweck 2000), where pupils tend to lack strategies to cope with any barriers or difficulties. In order to develop mathematical resilience, a pupil needs a belief that they can learn and grow in mathematical knowing, thinking and understanding. Locus of control is an important aspect of developing resilience (Newman 2004). Pupils develop resilience by taking appropriate responsibility for their own learning and taking that responsibility involves pupils communicating their current understanding, needs and interests. Pimm also tells us that, by articulating, pupils are able to take control over mathematical meanings,

> "…. Children need to learn how to mean mathematically, how to use mathematical language to create, control and express their own mathematical meanings as well as to interpret the mathematical language of others" (Pimm 1995, p. 179)

However, for a variety of reasons, the potential of pupils articulating their mathematical ideas is not fully realised in English secondary mathematics classrooms in ways that best support improvements in mathematical learning (Ofsted 2006, 2008). It is clear that:

> "we have the practical knowledge needed to improve the quality of classroom talk. Yet in most classrooms, and in most educational policy, talk remains a taken for granted feature of everyday life" Mercer & Hodgkinson, 2008 p. xvii

Furthermore we are told that:

> "…to be most effective, mathematics education needs a pedagogy which explicitly encourages the active engagement of students in dialogue, and that mathematics teachers need to critically review their current practice" Solomon and Black 2008 p. 73

There has been a great deal written about the language of mathematics (Durkin and Shire 1991, Pimm 1987) and its use in the mathematics classroom, including the issues and barriers caused by the language used to express mathematical ideas (Evans and Tsatsaroni1994, Sfard 2001). Can classroom practice be developed in English mathematics classrooms to enable pupils to become more confident in articulating mathematical ideas themselves in order to increase their learning and development? Will the mathematical learning that the pupils engage with change by focusing on communicating mathematical ideas and processes (Alexander 2006, Mercer and Littleton 2007, Pimm 1995) rather than focusing on instrumental (Skemp 1976) or surface learning (Sims 2006) and completing algorithmic exercises - and if so, how?.

There are increasing concerns about the shortage of mathematicians and scientists in the UK economic system (e.g. Roberts 2002). We consider that there is a link between pupils' confidence in themselves as successful learners of mathematics, their mathematical resilience and their willingness to pursue further study in mathematics as a way to progress their career (e.g. Nardi and Steward 2003). We also see a focus on articulating mathematical ideas adding to teachers' understanding of how to respond positively to the demands for increased pupil confidence and Personal Learning and Thinking Skills (QCA 2007) within the day-to-day constraints of school, busy timetables, large classes and sometimes prescriptive schemes of work.

The theoretical basis

The role of talk or dialogue in learning has been discussed since before Plato. Both Mercer and Littleton (2007) and Alexander (2006) have expanded on the development of dialogue as a tool for increasing pupils' thinking, learning and development across the curriculum. In our experience, the methods advocated by these authors will need some transformation in order to be effectively applied to mathematics classrooms.

Both Mercer and Alexander based their thinking on the theories developed by Vygotsky. Vygotsky maintained (1981) that the act of speech creation profoundly influences learning. Speaking or otherwise communicating requires an individual to place structure and coherence on their understanding that may lead to recognition of gaps in that understanding or new connections between formerly disconnected knowledge. The interaction between speaker and listener(s) in a conversation amplifies this process as the parties attempt to reconcile differences in their perspectives, opinions, and experiences. Sfard (2007) takes this further emphasising the symbiotic relationship between thinking, learning and communicating.

Becoming able to articulate mathematical ideas, concepts and reasoning has a profound effect on the way that pupils see themselves (Lee 1998, 2006). The better a pupil can use the discourse that a mathematician may use, the more they become a mathematician, that is, someone who can solve problems using mathematics. Several authors emphasise the importance of learning to speak like a mathematician in order to take on the identity of a mathematician (Holland et al. 1998, Lave and Wenger, 1991, Wenger, 1999). Gergen (1995) considers that knowing only exists as part of a community and that in order to be said to 'know' mathematics, one must occupy a discursive position that is accorded the standpoint of authority by the social community of mathematicians. 'The ideal position is not knowing that something is the case, but knowing how to produce language that will be accorded status' (Gergen 1995, 31). That is, an individual takes on the identity of a mathematician, someone who 'knows mathematics,' by learning how to talk like a mathematician. If pupils are not given the opportunity to 'talk like a mathematician', their mathematical education is impoverished.

Anne Watson goes further to term what happens in many mathematics classrooms as 'cognitive abuse'. That is an emotive term but is a notion that many mathematical educators recognise. Abuse results in damage; cognitive abuse or neglect results in damage to the way that pupils think about mathematics and to their ability to succeed in learning mathematics. Where pupils develop resilience, through a talking and learning classroom, they can deal with the difficulties inherent in understanding mathematics.

Resilience in learning mathematics is therefore essential for pupils to succeed. Newman defines resilience as a process of 'positive adaptation in the face of severe adversities' (2004, 3). If pupils are to engage with mathematics, struggle through problems, deal with barriers and misunderstandings and work on mathematical ideas, then they will need resilience. Many pupils experience the idea of working with mathematics as facing severe adversity. We also know that many people do not develop resilience in the current system, but rather become maths anxious. (See for example, Ashcraft 2002.) There is evidence that mathematics anxiety (Ashcraft 2002) can severely compromise the ability of some individual pupils to carry out mathematical processes and that anxiety is, for many, an acquired response to school situations rather than innate. The origins of mathematics anxiety lie in part in the interactions between the learner and teacher, but it is unclear which components of teaching style might be associated with increased anxiety (Ashcraft, 1992). There is an indication that articulation of ideas improves pupils' confidence in both their learning and their competence to use mathematical concepts; that is, it increases their mathematical resilience.

The current system of teaching and testing seems to develop an entity or fixed theory of learning (Dweck 1999) that makes pupils' believe that they are either good at mathematics or they are not. Those that see themselves as good at mathematics when at school, can find mathematics straightforward and easy; every time they get stuck they ask their teacher, who ‘smoothes the path’ (Wigley 1992) for them. They do not meet problems that require ‘struggle’ and therefore do not develop ways to deal with adversity, or mathematical resilience. If they continue to study mathematics, they may then meet a barrier and this may lead them to consider they have reached the ceiling of their ability and consequently ‘drop out’. Developing mathematical resilience is important, but currently it happens by accident if it happens at all. When a teacher develops a ‘talking and learning’ classroom, the idea that people struggle to find an answer may become apparent; pupils may struggle together and see incremental learning happening. They may share ways to overcome barriers and develop a store of knowledge about what to do to struggle forward to a solution; they may develop mathematical resilience.

Approaches that could be used in the classroom

We consider that teachers could be supported in trying out a range of approaches for improving pupils’ articulation of mathematics, including, but not limited to:

- Using success criteria to guide pupil learning – the criteria are negotiated with and often suggested by the pupils and often set out the process of achieving mathematical success. The discussion in the classroom revolves around achieving the success criteria.
- Ideas devised to improve pupils’ ability to use mathematically correct vocabulary and ways of expression (The Mathematics Register, Pimm 1987) to express their own mathematical and scientific learning.
- Devising questions and activities that require pupils to talk about their mathematical ideas and reasoning using the mathematics register.
- Asking pupils to present mathematics that they have worked on with others in a small group.
- Asking pupils to work in pairs to write a solution to a maths problem, then discussing the resulting ideas as a class, asking for the reasoning behind working in certain ways.
- Assessment for learning.

Assessment for learning or formative assessment (DCSF 2007) depends on pupils’ ability to express their current level of knowledge and to discuss and understand ways to improve that knowledge. Effective use of assessment for learning therefore depends on the development of classroom practice that requires pupils to express their mathematical ideas. If pupils have problems articulating their mathematical ideas then it follows that they will be less able to benefit from the gains that have been attributed to the use of assessment for learning. The literature on assessment for learning (Black et al 2003, Lee 2006) indicates that increasing pupil involvement in the whole process of learning increases motivation and success. The more pupils articulate their ideas in the classroom the more involved they are in the learning process.

Research questions

We consider that there are two questions for research in this area:

1. What teaching approaches in the classroom most effectively enable pupils to improve articulation of their mathematical thinking and ideas?
2. What difference does improved articulation of mathematical ideas and concepts make to:

- pupils, including their self-concept and attitude to mathematics?
- the nature of the mathematics that is learned?
- standards that pupils are able to attain?

Does articulation matter when learning mathematics?

As we said at the start, we think articulation does matter when learning mathematics but we also think that we do not know everything about how and why. It seems likely that the benefits in learning seen by Mercer and Alexander based on the theories of Vygotsky (1981) can be accrued in mathematics classrooms. However, we need a focus on the translation and transformation of the methods inspired by such writers for use in secondary mathematics classrooms in order to allow pupils to benefit from an enhanced ability to use mathematical language in expressing their ideas and to learn mathematics.

We feel that increasing the articulation that pupils are required to undertake in mathematics will increase their 'mathematical resilience' and therefore their ability to engage as life-long users of mathematical skills, thinking and reasoning. However, we also know that teachers have known about the socio-cultural aspects of learning mathematics for many years (Lerman 2001) and yet Ofsted (2008) found the mathematics classroom a place that concentrates on the acquisition of skills, solution of routine exercises and preparation for tests and examinations. Teachers respond to a vision of how practice can be different, if it is developed with teachers and takes account of the constraints and affordances of real classrooms, in real schools.

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