'Profound understanding of fundamental mathematics': a study into aspects of the development of a curriculum for content and pedagogical subject knowledge.

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The purpose of this paper is to review the development of a curriculum for the Mathematics Enhancement Course at a particular university, and in particular to evaluate the extent to which the course can be deemed to be successful in developing students' subject, and to a lesser extent, pedagogical knowledge.

Introduction

The Mathematics Enhancement Course (MEC) is now in its third year of national provision in England. It is a programme funded by the Training and Development Agency for Schools (TDA, formerly Teacher Training Agency TTA) which is aimed at graduates whose mathematics background is insufficient for entry to a PGCE or other route to Qualified Teacher Status in the subject, but who otherwise are suitable candidates for initial teacher education programmes. The course has a strong focus upon the development of subject knowledge (TDA 2003). The university where this study is carried out was part of the MEC pilot which ran for two years prior to expansion to national provision; thus this is its fifth year of running the course.
Within the specification for the MEC, there was a key emphasis upon the nature of mathematical subject knowledge that should be nurtured by the course:

> "Subject knowledge characterised by: connectedness as against fragmentation; multiple perspectives – a flexible and adaptable understanding; deep understanding of basic concepts" (TDA 2003, 3)
>
> The development of "profound understanding of fundamental mathematics, emphasising deep and broad understanding of concepts, as against surface procedural knowledge."(ibid, 1).

These ideas are influenced by various researchers, and in particular, the specification writers acknowledged the work of Liping Ma (1999).

Much of the literature on mathematics subject knowledge for teaching concerns student teachers in primary education. There are some important points made which are pertinent to consideration of the preparation of secondary teachers. Harries and Barrington (2001) discuss the views and knowledge that students had at the start of their course, characterising this as "a surface knowledge which lacked conceptual depth and an awareness of the interconnection of mathematical ideas" (p 21). At a more general level, Rowland et al (2003) group categories of mathematics content knowledge into four broad areas which they define as 'The Knowledge Quartet': Foundation, Transformation, Connection and Contingency. This work illuminates the interrelationship between content knowledge and pedagogical knowledge; it is difficult to separate the two. Prestage and Perks (2001, 101) distinguish between "knowledge needed to pass an exam and knowledge needed to help someone else come to know that knowledge", stating that "the first is necessary but not sufficient for the latter". They propose a model that assists in thinking about the distinctions between these types of knowledge, and they stress the importance of ongoing teacher reflection if the process of transformation of learner-knowledge into teacher-knowledge is to continue over time.

Further insight into subject knowledge for teaching can be found in the work of researchers who have studied the content and implementation of the National Curriculum for Initial Teacher Training in Primary Mathematics, which was introduced by the government in 1997 (DFEE 1997). In this curriculum, required mathematical content knowledge was specified in detail, and there was also a requirement for teacher education institutions to audit trainees' knowledge. Various authors express concerns about the audit process. Ernest (2004) analyses the ideology of this document and relates it to what he sees as a prevailing 'Industrial Trainer' approach of the government at the time, with elements of 'Technological Pragmatist' and other ideologies evident too. Goulding and Suggate (2001) suggest that "the audit process may simply reinforce an instrumental approach [to mathematics] in weak students". Murphy (2006) found that the audit process may increase student teachers' confidence in the subject, but found little evidence that it supports trainees in developing a connected view of mathematics.

Tennant (2006) studied students on a secondary mathematics PGCE. He investigated the relationship between formal academic qualifications, as measured by classification of first degree, and effectiveness in initial teacher training as measured by level of performance in the Standards for Qualified Teacher Status. Tennant found there was no correlation between his students' degree classification and their level of success in initial teacher training. This is an important result with implications for PGCE admissions criteria and selection processes. Tennant suggests that students with a degree in mathematics may actually have gained a very narrow understanding of some areas of the subject, with little sense of the overview and connections between areas, and he contends that in mathematics, degree results cannot reliably be used as indicators of subject knowledge for teaching. In this study I use methods similar to Tennant's to investigate MEC students' academic qualifications.

The study

In trying to answer the question 'is this course successful in terms of…' one has to consider which interested parties may be asking the question, and what each might deem to be 'success'. Stakeholders in the MEC include, *inter alia*, the students themselves, the TDA as the funding body for the course (acting on behalf of the government), and the university responsible for running the course. I review evaluation data and feedback from all of these sources. Initially, I outline some key aspects of the MEC curriculum; then I review evaluations from students and an external evaluator. Finally I present some new data comparing grades attained by ex-MEC course students on PGCE, comparing these with the general cohort.

This MEC is run collaboratively between Liverpool Hope and Edge Hill universities, with students spending a day a week at Edge Hill and the rest of the time at Hope. The curriculum includes a broad range of mathematics units of a level and content broadly commensurate with early degree / Further Maths. Students all take part in extended investigational work and, in the second half of the course, an individual enquiry into an area of mathematics of their own choice. Additionally there are some key features of the course that are a little different. Certain sessions are designated as 'Fundamentals' and explore lower level mathematics topics at a deep level. Also, students take part in a programme of peer teaching of topics from the school curriculum, and they assist in 'intervention' days when groups of schoolchildren come to the university to focus on particular concepts. All students take part in eight days' serial attachment in two different schools, with a brief to observe and note children's misconceptions in the process of learning mathematics. The school attachment and intervention project lead into a piece of written work for the students.

The first set of documents reviewed comprises a longitudinal study carried out by Peter Seabourne HMI, which began during the MEC pilot year of 2004 and continued to track certain students through their PGCE then NQT years (Seabourne 2004, 2005, 2006). The second set of documents comprises MEC students' end of course evaluations from 2007, in which they are asked if/how their view of mathematics has changed during the course. The tutor who designed the questionnaire used here was deliberately seeking to elicit reflective responses at a deep level. At the time this data was collected, it was not known that it would be used in a study of this kind.

Finally I gathered some data from the 2006-07 PGCE Mathematics cohort at this university. The first data set is a comparison of subject knowledge exit grades for the ex-MEC students and the general cohort. The second data set is a comparison of overall exit grades for ex-MEC and the general cohort. PGCE students are assessed regularly on-course, and given final grades on exit from the course, according to the system described below. The purpose of my enquiry into this data was to ascertain whether there was any difference between the grades attained by those students who had formerly followed the MEC, and those who had come from other backgrounds. I wished to find out if any differences existed in general, and also in particular with respect to subject knowledge.

In England, student teachers' performance across aspects of professional knowledge, teaching, planning and assessment are graded on a five point scale, consisting of three pass grades (1, 2, 3) and two fail grades (4, 5). The grading structure is used both formatively on course, and summatively to give an indication of achievement at the end. Grades awarded to student teachers by experienced teachers in school are moderated regularly by tutors from the awarding university. Accuracy of grading is encouraged, and it is not in the interest of schools or universities to grade weak students too highly. However I do not know of any formal research findings into the reliability and validity of this system, and given the variety of different persons assigning grades, within a system that requires subjective judgements, it would be very difficult to carry out such research. This is a key limitation in my inquiry, and that of Tennant (2006).

The third data set is a comparison of entry and exit grades for the PGCE mathematics cohort. Here I use ideas from Tennant (op. cit.) in seeking to determine whether any relationship can be established between formal academic qualifications prior to PGCE, and success in initial teacher training as measured by the grading system outlined above. To enable meaningful comparisons to be drawn up between scores on entry and exit to the course, it was necessary to convert degree classifications into an interval scale. I chose to do this by scoring a first class degree (or higher degree) as 1, a 2:1 as 2, a 2:2 as 3, a 3rd as 4 and a pass degree as 5. This provides data which has the advantage of being easy to analyse. However it is not true that, for example, a 2:1 degree is somehow twice as valuable as a 3rd. Degree classification data is ordinal and does not truly meet the equal-interval criterion. Furthermore, degree grade standards and the level and scope of competence, understanding and knowledge indicated by them are variable and problematic for trustworthy comparisons.

Findings

In his report on the pilot MECs, Seabourne commented on the value of the courses as specially designed for a target group. He noted improvements in subject knowledge, attitude, understanding and confidence in course participants. He commented that at the start of their PGCE year, former MEC students ‘hit the ground running’ i.e. they were well prepared for the course which was to follow. He also noted a significant incidental effect in the exposure of the MEC students to a variety of high quality teaching and new approaches to learning mathematics. Key findings from the student evaluations were students’ awareness of the importance of understanding the subject in depth and making connections; the value of collaborative working; enjoyment of engagement in mathematical activity.

Analysis of subject knowledge grades on exit from PGCE yields a mean grade of 1.6 for the MEC students as against a mean of 1.5 for the group as a whole (see Figure 1 below). On a grading scale of 1 to 3 (pass grades) where 1 is the highest, this represents a slightly lower score for MEC students, but this is not a significant result. Analysis of overall grades on exit from PGCE yields a mean grade of 1.9 for the MEC students as against a mean of 1.8 for the group as a whole (see Figure 2 below). Again this represents a slightly lower score for MEC students, but is not significant. This evidence suggests that these MEC students were almost as well prepared for PGCE as their peers from more traditional mathematics backgrounds; the difference is not statistically significant.

Sub knowledge grades	Mean	SD
All students (n=38)	1.47	0.60
MEC students (n=12)	1.58	0.64

Figure 1: A comparison of subject knowledge grades

Overall grades	Mean	SD
All students (n=38)	1.77	0.55
MEC students (n=12)	1.88	0.65

Figure 2: A comparison of overall grades

The paired data scores for entry and exit for 31 of the PGCE students showed no discernible trend and absolutely no correlation. (Full data on degree classification was unavailable to me, hence the reduced data set). From this data sample, there is no evidence to suggest that a high scoring degree classification may be a predictor of success in initial teacher education. This is in line with the findings of Tennant (2006), and runs counter to current university orthodoxies, wherein pressure is applied to course leaders to recruit PGCE students with high scoring degree classifications.

Conclusions

Attempting to measure the success of a programme such as the Mathematics Enhancement Course can never be a straightforward task. There are many variables to consider, start and end points are difficult to define, and indeed defining and characterising the knowledge that people have, and the way in which they hold it, is very complex. Perhaps one of the purposes of undertaking educational research is to be able to challenge or substantiate some of the persistent myths, or received wisdoms, that abound within the profession. This small start at analysing data connected with student teachers' perceived levels at entry to and exit from the PGCE course has yielded some results which are interesting to me because they challenge accepted positions on the value of certain academic qualifications, whilst affirming views that I hold as a result of experience.

In having the opportunity to develop courses specially for intending secondary mathematics teachers, I believe that universities offering the Mathematics Enhancement Course have a unique opportunity to respond to the challenges offered by Ma (1999). Development of "profound understanding of fundamental mathematics" as propounded by Ma takes more than a six month course. However, if universities design and teach this course well, they will have provided a solid foundation for teachers to attain and to nurture PUFM during their professional life.

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