Interactive geometry in the classroom: old barriers and new opportunities

Chris Little (ctl@soton.ac.uk)

School of Education, University of Southampton, Southampton, UK

Although computers and calculators have had a massive effect on some areas of school mathematics, some evidence suggests that geometry teaching has been slow to utilise computer software. This paper discussed three barriers to implementing dynamic geometry in the classroom:

- curriculum scope: teachers need to be convinced that they can teach geometry more effectively
- accessibility of computers: teachers and/or students need ready and regular access to computers
- accessibility of programs: they must be easy to learn so that the emphasis is on learning the maths, not the program.

With the development of internet-based freeware, such as Geogebra, it is possible that these barriers may be being overcome. The greater accessibility to students, and the availability of digital projectors, then presents the issue of who should be in charge: the teacher leading whole-class discussion, or the student engaged in individual 'guided re-invention'.

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1. Introduction

There is no doubting the enormous impact on the school mathematics curriculum over the last 50 years of calculator and computer technology. We have come a long way from the days of mathematical tables, 'bar' numbers and logarithms and slide rules, via the first hand-held calculators, short basic programs on punched cards, the BBC 'B' and RM 380Z microcomputers, graphics calculators, to today's classrooms, many equipped with digital projectors and interactive whiteboards. It is instructive to look back at an early A level paper from the 1950s, and attempt to solve the questions armed only with the computing power of pencil-and paper and four-figure tables (if you can still find a copy). The impact on the curriculum of enhanced calculation tools has been dramatic, enabling rapid solution of problems using realistic numbers, numerical methods for solving for equations, and instant access to trigonometric, exponential and logarithmic functions.

Equally influential, in the post-16 curriculum at least, has been the development of graph plotters, enabling functions to be drawn instantly, and statistical data to be displayed graphically. Programs such as Autograph have gained a regular place in the A level classroom by combining graph plotting and statistics windows, and offering the teacher a range of bespoke tools designed with the A level mathematics curriculum in mind – for example, derivatives, areas, numerical methods, approximate integration methods, histograms, box-and-whisker diagrams, and, more recently, images of three-dimensional graphs including vectors, lines and planes.

However, the impact of new technologies on geometry has perhaps been less evident. It is still possible to cover the current GCSE shape and space curriculum without the aid of computers, despite the enhanced opportunities offered by interactive geometry packages such as Cabri and Geometer's Sketchpad. A survey conducted between 2000 and 2003 by the Fischer Trust in 373 secondary departments (Fischer Trust 2004) found teachers reporting low to moderate use of these packages, and my own personal experiences based on my knowledge of schools and colleges in Hampshire is that only a few teachers use these packages at all. This paper proposes some of the reasons why progress in using computer technology in geometry has been relatively slow, and speculates about whether this may change in the near future.

2 LOGO

Arguably, the first dynamic geometry computer package was LOGO, which facilitated the dynamic drawing of geometric shapes using a programming language which young children could understand, for example:

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To Square
Repeat 4
Forward 10 Left 90
End
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LOGO was easy to learn, and capable of producing exciting shapes. It encouraged systematic thinking skills in children, and in forcing the learner to analyse how shapes were constructed, opened up opportunities for developing genuine geometric insight in an enjoyable way. Yet, despite the vast body of research it spawned, LOGO has come and gone with little lasting impact of the mathematics curriculum, primary or secondary. LOGO was perceived by many teachers as a diversion from the serious business of teaching geometrical 'facts': it had no impact on the examined curriculum, and with the pressure of high stakes examinations, and latterly the national curriculum, the examined curriculum has become, to all intents and purposes, the taught curriculum. Despite the obvious geometrical richness of LOGO, the canonical geometry curriculum remained unchanged, and LOGO has effectively died, in secondary classrooms at least.

This illustrates the first essential requirement for new technologies to survive, and eventually influence, the curriculum: they must be seen to have relevance towards teaching and learning the existing canonical school curriculum. Given the pressures exerted by a heavily prescriptive curriculum and school league tables, teachers must be persuaded that using computers will enable them to teach the existing curriculum more effectively.

3 Interactive geometry

Following LOGO, the next major contender for impacting on the teaching of geometry, was interactive geometry, developed in the 1980s through the Visual Geometry Project, which spawned Nick Jackiw's Geometers Sketchpad, and in France through the Cabri project, developed by Jean-Marie and Colette Laborde. I personally remember well the excitement of encountering this for the first time at ICME VII in Quebec 1992, and attending a 'Universite d'été' in Grenoble in 1993. It would enable students to perform Euclidean constructions, gain a deeper understanding, through the distinction between 'figure' and 'drawing', the geometrical properties of polygons, and discover and re-invent for themselves non-intuitive geometrical theorems such as the angle properties of a circle – I remember writing a paper at the time which suggested that a new era of exciting geometry teaching might be upon us! (Little 1993)

Did it link sufficiently with the geometry curriculum? At the time Ros Sutherland and I wrote some curriculum materials using Cabri to teach angle properties, transformations and trigonometry. These sold poorly and had no impact. Cabri, initially a very pure Euclidean geometry package, was forced to adapt in its later versions to include measurement and transformation geometry tools. Despite its more obvious links to pre-16 geometry topics, these programs remained the preserve of a few interested teachers, and made their way into relatively few classrooms, even in France. Fifteen years' later, and the situation has not to my knowledge changed greatly.

Why? One possible barrier to implementation has been access to computers. Although the provision of computers in schools has advanced rapidly, most of these machines are still in computer labs, which require booking in advance, and are often unavailable for maths lessons. Digital projectors have recently improved access by allowing the teacher to display geometry dynamically to his or her students. Before this, studentsand teachers needed time to learn how to use the software effectively.
This in itself is an issue. The mastery of generic IT packages such as Word and Excel has become mandatory, given the universal availability and usage of Microsoft packages. Gradually, we have come to rely upon students' being educated in using a spreadsheet like Excel, and thus being able to utilise this as a calculation tool in mathematics, without having to teach the basics of the package. Students may be reluctant to learn how to use Cabri or Sketchpad, purely as a tool to developing geometrical skills, as the IT 'payoff' is not there.

Even with the advent of digital projectors, teachers still need to have the skills to use these programs effectively. Although the design of the programs, with the use of icons and drop-down menus, has facilitated their use, teachers still need time to develop sufficient confidence to use them in front of classes, and these skills require constant usage to be maintained and developed. The additional pressure of dealing with unreliability of hardware, the fact that not all classrooms have projectors, the unsuitability of lighting in some classrooms: all these practical difficulties have impeded all but the keenest teachers from utilising the software in their geometry teaching. Larry Cuban, in his study of under-usage of computer technology in schools and universities (Cuban 2001), listed the practical questions which teachers ask before embracing new technology in their teaching:

- Is the machine or software program simple enough for me to learn quickly?
- Is it versatile, that is, can it be used in more than one situation?
- Will the program motivate my students?
- Does the program contain skills that are connected to what I am expected to teach?
- Are the machine and software reliable?
- If the system breaks down, is there someone else who will fix it?
- Will the amount of time I have to invest in learning to use the system yield a comparable return in student learning?
- Will student use of computers weaken my classroom authority?

My own experience as a teacher resonates with these questions.

4. Geogebra

Are some of these impediments to using dynamic geometry in classrooms diminishing? I have recently been impressed by Geogebra, and new dynamic mathematics package created by Markus Hohenwarter. This package clearly builds on the developmental work of pioneering packages like Cabri and Geometer's Sketchpad but enables the user to input algebraic functions and variables via an algebraic 'window.' This enhances the usability of the package for coordinate geometry, functions and calculus, thus broadening its curriculum scope. Experience of using the package with teachers suggests that the interface with the user is sufficiently simple to require little prior training or expertise before it can be used in the classroom. NCETM has provided a small grant for a project which is looking into devising workshop materials for teachers.

Geogebra is freeware, written in Java, and readily downloadable from the internet. This greatly enhances the accessibility of the software, both for teachers in schools and for students at home. It is possible to write worksheets or investigations, save them as HTML files, for access by students at home via the internet. There may be difficult financial issues raised by offering such software as freeware – who, for example, will bear the considerable developmental costs of mathematics software in the future – but free and universal access will certainly remove one barrier to implementation. Newly qualified teachers will no longer find that their favourite package, encountered during their training, is not available in their new school.

Web-based software also enables communities of teachers to grow organically and share ideas and experiences of using the software. a recent meeting in Cambridge convened under the banner of the Geogebra Institute attracted delegates from Spain, Norway, France, Germany, Austria, Finland, Poland, Hungary, Iceland and the United States.

Cuban (2001, p …) comments on the lack of teacher involvement in the development of software:

> Although there is much talk of respecting teacher expertise, recognising exemplary teachers, and appointing occasional teachers to blue-ribbon commissions, most teachers have had little to say in designing and implementing technology plans. Even fewer teachers design professional development programs specifically targeted toward their peers. When teachers do engage in such deliberations and when they design programs for themselves, when their opinions are seriously considered, changes in classroom practice occur that even the teachers themselves had not contemplated.

The Geogebra website and wiki encourages teachers to involve themselves in proposing materials, airing faults in the program, and suggesting future developments. Perhaps the NCETM portal might be another forum which will encourage democratic teacher led development.

5. Designing classroom tasks

An issue that has been raised at Continuing Professional Development (CPD) sessions which I have been involved in has been hinted at already: who has control of the computer? If students can download and access the programs from home, this increases their access, and enhances the possibility of the discovery or 'reinvention' (Freudenthal 1991) of mathematical ideas. One of the factors which inhibits students from doing this is the nature of the instructional material requires to guide their 'reinvention'. There is a danger that worksheets can be written as a set of detailed instructions about operating the package, which have little to do with, and focus attention away from, the mathematics. This point was raised at a recent meeting to discuss Geogebra workshop materials, at which one of the teachers remarked that students are used in Information Technology (IT) classes to following tightly prescribed, detailed worksheets, and in doing so are not encouraged to think strategically.

My experience of writing curriculum materials for computer packages suggests that this is indeed a problem: give too little direction and students become bogged down with some aspect of the program; but give too much detailed instruction and students 'cannot see the wood for the trees': they become too preoccupied with following the requirements of running the program, at the expense of focusing on the mathematical ideas.

It is important that the software is readily usable, without the requirement to dot every 'i' and cross every 't'. Here, having a well-defined structure to the menus, with succinct and easily accessed on-screen 'help', may avoid the need for an over-instructional style, and enable the mathematics to come to the fore. There is also a temptation on the part of developers to add too many 'buttons' in attempting to enhance the curricular scope of the program.

In draft workshop material for Geogebra, I have tried to overcome this problem by using the words 'create', 'construct', 'move', and 'measure', in bold to refer students to the corresponding menus in the program, as the following example (Figure 1) of an introductory worksheet on calculus illustrates.

The Gradient of f(x) = x^2

1. Input f(x) = x^2.
2. Create a point on f(x).
   Re-name (right click) the point as P.
3. Construct tangent to the curve at P.
4. Measure the slope (gradient) of the tangent (m).
5. Move P, and record the slope of the tangent at points with x-coordinates −2, −1, 0, 1, 2, 3 in a table.
6. Predict the gradient at the point with x-coordinate (a) 4, (b) 0.5, (c) −2.5.
   Verify your prediction.
9. In your own words, write a rule for calculating the gradient of f(x) = x^2.

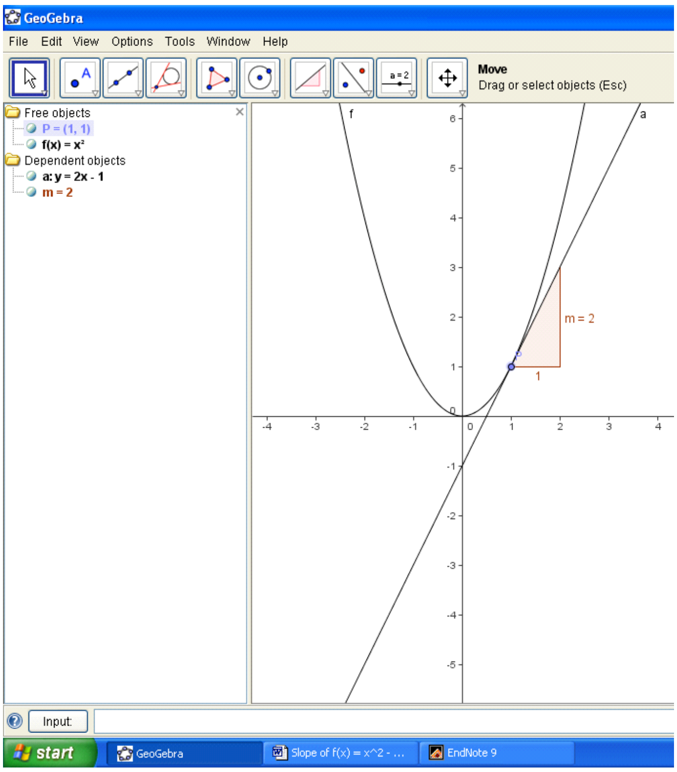

Figure 1 Example worksheet

The effectiveness of such worksheet material needs to be researched, as does the more general issue of whether there are pedagogic gains in students working individually with computers, as opposed to the use of classroom demonstration and discussion using a digitally projected image.

Of course, this question of whether learning mathematics is best orchestrated by a teacher as a collaborative activity, or as an individual working at his or her own pace and making their own discoveries and advances is not confined to computer aided instruction. In practice, my guess is that it is blend of the two, and the difficulty lies in the unfortunate fact that one blend does not suit all, and that we all differ in out style of learning.

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