Ethics, performativity and decision mathematics

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The spirit behind the incorporation of Decision Mathematics (and Discrete Mathematics) into the A-level Mathematics Curriculum was that of encouraging the creation of algorithms and problem solving. However, some students and teachers tell us it is "boring" and not "proper" maths. In order to have an insight into the role that Decision Mathematics plays in the current programme of mathematics, we observed some Decision Mathematics lessons and interviewed teachers and departmental authorities. We found that Decision Mathematics in the classroom involves the students being programmed to carry out the algorithms, without any sense of "agency" in problem solving, creating or evaluating algorithms, and with few significant connections or applications of their pure maths. The Decision Mathematics option is said by teachers to provide an "easy" way for students to gain marks, and a "relaxing" module for them in contrast to core pure modules. Performativity and the "league tables" culture of the education system in Britain thus ensure that alternatives to Decision Mathematics may not be considered an 'equal' option especially for weaker students, whatever the needs of the students or the teachers' professional beliefs and ethics.

Introduction

This paper draws on the ESRC-TLRP project "Keeping open the door to mathematically demanding Further & Higher Education programmes"[1]. As part of the project, we observed AS level mathematics lessons at five colleges across the UK and interviewed students, teachers and authorities about their views and experiences in relation to mathematics, and how these affect the students' aspirations and decisions to continue studying (or not) mathematically demanding programmes in the future.

Here we report on the case of a particular (elective) module of the AS mathematics programme: Decision Mathematics (D1, hereafter DM). The DM syllabus focuses on the concept of algorithm, which is central to all mathematics, and the ways in which algorithms are created and used to solve decision making problems in a variety of contexts, from optimisation problems (linear programming) to behavioural competitive situations (game theory) to planning transport routes (networks). DM was incorporated into the mathematics curriculum of post-compulsory education almost 20 years ago, and it was expected that it could "provide a suitable means of adopting approaches in the teaching of mathematics different from the usual exposition and practice of routines" (Burghes and Huntley 1984, 6).

Our concern about this course originated from the fact that several students and teachers in our case studies identified it as being "different" from other modules, sometimes saying that it was "boring", too "mechanistic" and not "proper maths" and others identifying it as being "easier" than other modules like Statistics (S1), Mechanics (M1) or the Core modules (C1, C2). And yet, exam board figures show that between 2001 and 2007 student passes in D1 have doubled from about 15000 to 30000 while S1 and M1 have hardly changed (actually this overall comparison hides reductions in 2005 and increases in other years).

In order to gain a better understanding of the role that DM plays in the policies and programmes of these colleges we systematically searched our database and analysed lessons and teachers' interviews that related to DM. We hypothesised that, despite DM's initial purpose of stimulating creativity and inquiry in mathematics and making "mathematics teaching relevant to its applications" (Berry et al 1986), the pressure of performativity and the audit culture that is increasingly common in the UK might in some cases be defeating this purpose and distorting educational decision-making (Williams 2007), at the expense of the needs of the students or the teachers' professional beliefs and ethics.

Methods

For the purposes of this paper, we based our analysis in the data that came from the DM lessons of two teachers in our case study colleges, as well as in the interviews to these two teachers after their lessons and to the head of department of one of the colleges. We analysed the lessons in terms of how the teachers dealt with the mathematical concepts, i.e. if the lesson pedagogies were 'teacher-centred' (transmissionist) or 'student-centred' (connectionist or discovery) (Swan 2006, Pampaka et al 2007) and in terms of how the learners engaged with the tasks and problems presented, and how they recalled their activity in interviews. All the students' interviews were coded in Atlas.ti for all instances connected to DM, and analysed in relation to beliefs about mathematics and pedagogic and institutional practices.

Results

The lessons we observed were highly 'transmissionist', where the norm was to present the topics in a procedural way, with little interaction between the teacher and the students or amongst the students, and virtually no discussion of the mathematical concepts behind the algorithms. The lessons were full of comments from the teacher of the sort of "there are few steps to the algorithm… the first step is… find the blank…", or "I think I've armed you now with enough information to do the Chinese postman one on the paper".

For reasons of space, we will describe in more length only one of the lessons. This lesson was centred on the topic of Dijkstra's algorithm. This algorithm was developed in 1959 by Edsger Dijkstra, a Dutch computer scientist, to solve the shortest path problem, i.e. it obtains the shortest route from an initial vertex to any other vertex in a network (see Figure 1).

In this lesson, the teacher used a PowerPoint presentation to explain the algorithm. In the first part of the lesson, he introduced the terminology of the algorithm (vertex, temporary numbers, permanent label, etc.) and ran through an example (Figure 1) as a way of explaining it.

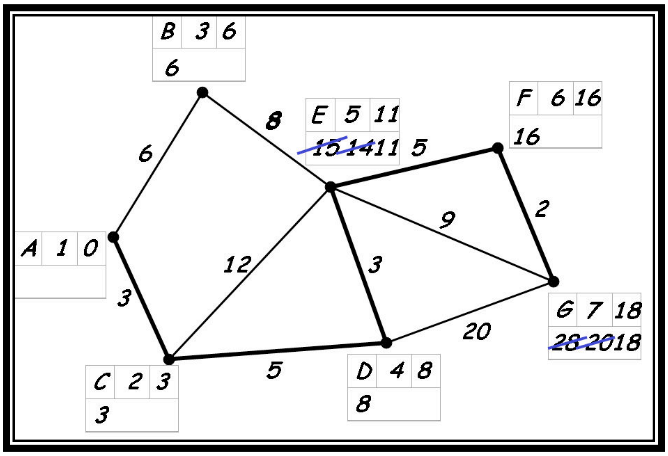

Figure 1. An example of Dijkstra's algorithm to calculate the shortest route from A to G

The algorithm was presented emphasising the procedure. This was clear in several occasions:

"What I want you to do is *watch me* run through the algorithm…"

"When you put these numbers, these temporary numbers, you *simply pick the smallest one"*.

"Once you got the idea of the algorithm, it *is the same all the time"*.

It struck us that when a student was trying to work out part of the algorithm by himself, the teacher interrupted him and said:

"Just wait a second while we run through it first, and then you'll get the chance to run through it yourself, ok?"

And even when, in the second part of the lesson, the teacher allowed students to work on their own on a "more difficult" example, he still directed the students to the solution in a procedural way:

"Second part of the question… so that now the part D-F can not be used. So *that means* our answer must be something bigger than 20. To answer this part of the question *look at all the vertices* that are connected to F that can be used…"

One of the DM teachers, Stephen, talked about DM:

S: In terms of the weaker ones, they actually… this is kind of what you do… and yeah ok, you might suggest that the answer is 2y is x is there and…

I: So for the weaker ones you don't really expect them to understand, you just do it… isn't that a bit harsh?

S: Probably harsh, maybe that's true. I expect them to know the steps but not the thinking or the reasoning behind the steps.

I: Is that just Decision Maths or everything?

S: No *the core is a bit different*. I think the core you can work it out yourself a bit more. *The Decision, just the way the... it is a bit more, the steps, yeah*. You have to plan more for decision than core, which is a bit more about understanding…

The role that DM has been playing recently in the A-levels mathematics programme at these colleges became clear during the interviews with these teachers and departmental authorities. Because DM is about algorithms which can be (and actually are in most cases) taught in a procedural way (it is more about the "steps" and less about "understanding"), even weaker students found it easy to get some marks on their exam, whether they understand what they are doing or not.

As John, the head of the mathematics department at Stephen's college, said:

J: Well, they do need (understanding). For some of the questions they do need it. *I teach them the tricks and hope that most of the time, that's enough to get them to do the exam questions* but, you know, if they set it in a slightly different way, they haven't got that understanding. Well, some, obviously the better ones pick up on it and they know how to adapt, but *the weaker ones just haven't got a (clue)…*

Hence, the policy in these colleges is to offer this module to the majority of the students entering college along side the compulsory pure units, in particular to the 'weak' students so that they can "bump their marks up" and at the same time ensure the college a general good "pass" rate, something that is harder to achieve in other modules of the programme. For example, John said:

J: *We used to think it was quite easy*, and we did that because at the end of the day the certificate didn't use to say what they've done. It wasn't like in the old days, pure and mechanics. *So we were just looking to get the results better, we were looking for a module we thought would be easier*, and I'm not entirely convinced, because well, I mean the alternatives are stats or mechanics. Now, we have reintroduced, because I stopped doing it, for those that are doing physics, they can do a M1 module for next year. Now I stopped doing that two years ago because I knew they were so weak at the pure maths, I thought well *they need an easy module here to bump their marks up*. And we used to think D1 because *it's just algorithms. You know, if they can follow the algorithm, hopefully they can pick up some marks*.

And Daniel, the teacher from the lesson on Dijkstra's algorithm, said:

D: I used to think that *lower level kids can always pass D1 easier* than pass say, S1 or M1. **(I:** Right)

D: They can pass it. *They may not do very well at it but they can pass at it. There's enough routine things for them to pass on.* (…)

D: I think enrolment is a key issue anywhere in a place as big as this and you're always going to have certain guidelines. An interesting thing, I don't know if this is relevant, but an interesting thing *in our department is that we generally advise weak students to do decision maths because we think that that will make the transition a lot easier for them.*

DM has become a module where students can perform better in their exams by following 'steps', even at the expense of their need for understanding, and hence their future prospects and their intellectual development. When asked about these ethical issues, Stephen said:

S: I must clear these exam questions, I would have to say yeah… *question - answer, what's this… key point, can you remember this…* but, then today what did we do… today we keep on… because they had been doing exams questions all last (week)… but yeah, that is true. *I feel a bit…it's difficult because ideally, it would be nice to sort of talk about understanding but you do appreciate that in the real world we are all measured by results.*

Stephen realises the ethical problem of teaching for understanding against the reality of the exams. He tries to solve this issue by differentiating himself from other teachers, incorporating in his teaching the practice of extending questions to those who are more able. In this sense, he believes he is not 'sacrificing' those who could achieve a greater understanding. He said:

S: I suppose I'd be similar (to other teachers) in the sense that we use the same sort of hand out… so all teachers use the same handout. I guess it's different in

> the emphasis, I might *place less emphasis on the actual formula - hopefully the reasoning behind to get too…* that sort of thing might not be included on the handout. I might be different as well in terms of *extending those that are quite able*. It's important that if you are able, you're not just sitting there thinking I can do this quite easily. I hope (I am) different in the sense that I'm getting *the one's that are able to sort of do a large amount of work or work that's more challenging*. That's differentiating me.

Discussion

DM was originally incorporated into the A-level mathematics curriculum as a module which could stimulate in the students creativity and inquiry, encouraging investigation and the design of algorithms, relating pure Mathematics to its applications. However, nowadays we find that in some cases DM is taught in a procedural way, forming students who can follow rules ("there's enough routine things for them") but that have little understanding of the mathematical concepts involved and the relation of these with their applications ("the weaker ones just haven't got a clue…"). DM in some colleges has become a way to achieve a good pass rate and institutional policies ensure that students, in particular the 'weak' ones, have no real alternative to it as an option.

Our analysis of the data showed that some teachers and departmental authorities are under great pressure to obtain the best exam results possible, as this could mean, for the college, getting a better position on the league tables and, for the teachers, promotions or in the worst case, keeping their job. In the "real world we are all measured by results". This effort to perform well within the audit culture poses important ethical issues: are the colleges acting in the best interest of their students, attending to their needs and future possibilities? Are the teachers acting in accordance with their professional values ("ideally, it would be nice to sort of talk about understanding") or do they have to 'betray' these and instead teach 'for the exam'? Are they also letting most of their students down, especially those who require more help, disabling rather than enabling them to become inquisitive and creative students, capable of reaching understanding? Are the 'stronger' students the only ones who have the right to choose and benefit from 'mathematical' options, or even more connectionist pedagogies?

These issues are of ethical and practical relevance and suggest the value of further research. We particularly need to enhance our understanding of how socio-cultural contexts mediate educational practices and how this relates to these ethical concerns. However, we argue that action should be taken now to ensure that institutional decisions do not hinder students' aspirations or worse, negate their possibilities of accessing education in the future. This requires diligence on behalf of examination authorities to ensure that some options are not institutionally preferred because they demand less mathematical understanding to acquire grades, and that individual students with a genuine 'interest' in or need for other options really do get access to them.

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