The impact of ICT on mathematical concepts

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The rapidly increasing use of ICT in the mathematics classroom clearly influences the ways in which teachers teach and students learn. Computers can offer dynamic visual images that may open up some areas of mathematics to a much wider audience. But can ICT affect not just *how* we teach, but also *what* is taught and *what* is learnt? This paper considers aspects of the possible impact of the use of computers on the mathematical concepts that students develop.

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Instructional software and tool software

There is a growing range of mathematics teaching software available for use in schools. Weist (2001) suggests that this may be classified into two types of program, and she distinguishes between what she calls *instructional* software and *tool* software.

> Instructional software is designed to teach students skills and concepts….[while] Tool software is used as an aid towards another goal. It does not teach but rather performs a function that facilitates attainment of some objective. (Weist, 2001, p. 47)

As Papert put it, in the first case "the *computer is being used to program* the child", while in the second, '*the child programs the computer*' (Papert, 1980, p. 5). So this distinction dates back at least to the nineteen eighties – we may contrast, for example, the largely topic-specific Micro-SMILE instructional programs with Papert's more generic tool software, *Logo*. These are two ends of a continuum rather than an absolute divide – so a *Logo* micro world, or a set of examples created by a teacher using a spreadsheet to demonstrate a specific concept, for instance, will have some of the qualities of an instructional program. Similarly, some topic-specific programs allow an exploratory approach that may be akin to that encouraged by the use of tool software. Nonetheless, the two broad categories may be identified, and specific pieces of software may be classified in relation to these.

There is a good range of tool software available to schools, including, for example, data handling packages such as *Furbles*; dynamic geometry software such as *Cabri-Geometre, Geometer's Sketchpad,* or *Cinderella*; procedure based software such as the free *MSW Logo*; computer algebra systems such as *Derive*; and some wider tools that offer a combination of functions, such as *Autograph*, or the free *GeoGebra.* However, apart from *Logo*, few of these software tools were designed specifically for the secondary mathematics curriculum. Some, such as *Furbles*, are aimed primarily at younger students. Others, such as *Cabri-Geometre* and *Autograph*, were designed for use at a tertiary level, and have been more or less cut down to make them more accessible to secondary level students. All of these programs require a significant commitment of time and effort on the part of both the teacher and the students if they are to be used effectively in the mathematics classroom.

While the range of mathematical tool software that is available to teachers is impressive, and growing, the sheer quantity of instructional software may be quite overwhelming. There is far too much to list here, but, for example, the websites of the Department for Children, Schools and Families' Standards, Maths Net, Nrich, the Association of Teachers of Mathematics, and the *Times Educational Supplement* all offer free instructional software for use in the mathematics classroom. A more comprehensive list of software, including a section on 'Freebies off the Net', has been compiled by the ICT Training Centre at Oundle, Peterborough.

Opening doors with mathematical software

Many teachers have described how a familiarity and facility with the ever-growing range of mathematics teaching software can help them to make key concepts accessible to a much wider range of students. Some of the most commonly reported examples of the use of ICT in the mathematics classroom relate to dynamic geometry software such as *Cabri-Geometre*, *Geometer's Sketchpad, GeoGebra,* or *Cinderella.* Experience with dynamic geometry software may help students to develop a deeper understanding of the principles that underlie geometric reasoning, enabling many more students to engage meaningfully with a range of geometrical concepts that were formerly restricted to a much smaller group. Computer graphics allow us to share mental images of structures, patterns and working systems that have hitherto been hidden away inside each individual's mind. They put into motion the flat, static 'snapshot' views of drawn or printed diagrams. As Kate Mackrell and Peter Johnston-Wilder remark,

> One of the ironies of trying to describe motion and its effects in text is that one necessarily has to miss out on *all* of the essential ingredients. Not least among these is the sense of surprise and wonder that animating mathematical diagrams and images can bring, externalising and setting back in motion images that have been held static in the pages of textbooks for over 2000 years. (Mackrell and Johnston-Wilder 2005, 82)

So the dominance of print as the defining factor of 'proper' mathematics is, at last, being challenged in the classroom. Furthermore, the support that the computer gives to these internal mental visualisations encourages their development. With time, many more students whose visualisation skills might have atrophied and faded in a traditional, text-book based environment are likely to improve their ability to think visually.

But this example of the impact of ICT seems to relate, not so much to *what* students are taught, but rather to *how* they are taught. Our main interest as test developers, however, is on the content rather than the pedagogy – on the *what*, not the *how*. We need to ask whether students who study mathematics in an ICT-rich environment actually learn something different to those who follow a totally print-based curriculum – or whether they learn the same things, more quickly and effectively perhaps, but with no real difference to the concepts being learnt. This is significant, because if the nature of what is learnt is essentially the same in the two cases then there is no reason why we should not go on using the same assessment methods – the same kinds of tests, examinations and teacher assessment activities – that we have always used, however our students have been taught. On the other hand, if the students in the ICT-rich classroom are learning something that is qualitatively different – so that they develop different key mathematical concepts – then these different concepts should be assessed.

But here the picture suddenly becomes much more complicated. It is hard to define what it is that students learn with, or without, computers. Rather, there is a continuum, with a lot of overlap. However, several areas may be worth exploring – including visual imagery, angle and ways of working.

Developing visual imagery

We have seen that a significant aspect of an ICT-rich mathematics curriculum is the ready availability of dynamic graphics which enable us to share mental images and thus encourage their development. But dynamic images are nothing new. Some – though certainly not all – teachers have been encouraging their students to use them for years. The approach to transformational geometry in the first School Mathematics Project 'O' level course in Britain in the late 1960s, for instance, laid great stress on the development of what was then called students' 'geometrical intuition' – or mental imagery. Students were introduced to mathematical translations in Book 2, for example, with the explanation,

> When an object is translated each point of the object undergoes the same change of position because it moves the same distance in the same direction as every other point. When you see a platoon of soldiers drilling, a formation team dancing, or a pair of ice skaters figure skating and see them moving as a single body it is because each person involved moves in exactly the same way. (SMP 1970, 120-121)

This picture of a group or a pair 'moving as a single body' is a strong dynamic mental image that supported students' understanding of the concept of a mathematical translation. In later sections of the book copious exercises with tracing paper, mirrors, scissors and drawing pins were used to give students plenty of experience of the movements involved in such transformations as translations, reflections and rotations. These dynamic images fostered the mental visualisations that students needed if they were to make sense of transformational geometry.

The manipulable images of dynamic geometry software are a direct descendent of all those practical and kinaesthetic exercises of forty or more years ago. What the computer does is not something completely new. It just makes it much easier to create, manage and share such images. ICT gives the student and the teacher greater access and more control, but some teachers, at least, have always offered powerful images to enable students to develop their understanding of mathematical concepts. For example, one very experienced teacher described her use of *Activ-Studio* with an interactive white board. She wrote,

> Students ...tessellated four-sided shapes – trying to find one that doesn't tessellate, moving to a proof where it is possible for students to illustrate dynamically a shape rotating around the mid-point of an edge. This is an activity I have used over the generations but has felt static. Now it's dynamic because of the technology. So geometric images are at the heart of what I'd be doing anyway. It's the dynamic nature of the [computer-based] images that matters. (Laurinda Brown, personal communication)

So yes, computers can encourage the use of dynamic visual images – but this is not an entirely new approach. Some teachers were using them long before the advent of the digital mathematics curriculum.

Angle

Another area where the use of computers might – or might not – have an impact on the mathematical concepts that students develop is Angle. Angle is a complex construct, with at least two interrelated meanings. On the one hand it is a dynamic concept – a measure of turn. But alternatively, an angle may be thought of as a property of a shape. So a square or a rectangle, for example, is defined as having four right angles, while a triangle with three equal angles is equilateral.

It is the latter concept of angle, as an aspect of a shape, that is emphasised in the National Curriculum in the UK. This may be because we cannot show the movement of a rotation on the printed page – and both the curriculum and its assessment are predominantly print-based. Similarly, students' understanding of the angle properties of polygons focuses on the angles' static, not their dynamic, nature. For example, students commonly explore the angle sum of a triangle first by tearing the three corners off a number of cut-out triangles, and finding that when they are arranged to meet at a point they always form a straight line. They may go on to consider the more generalisable image of a tessellation of triangles, where the equality of alternate angles, and the supplementary nature of three angles on a line, leads to a more formal proof of the angle sum of a triangle. But in both of these activities the angles are always viewed as properties of a static triangle – not as measures of dynamic movement.

The dynamic graphics that can be provided quite easily with a computer, on the other hand, offer an alternative way to develop students' understanding of Angle as a measure of turn rather than as an aspect of a shape. Turning a pointer through the three angles of any triangle leaves it facing in the opposite direction to its start – so it has turned though a half turn. A similar approach, in which angles are regarded as measures of turn rather than as properties of shapes, can lead to a proof of the angle sum of any polygon (Clausen-May 2008).

Rather surprisingly, however, the concept of Angle that students will meet if they use a dynamic geometry package may not, perhaps, be regarded as fully 'dynamic' in this sense. When students click and drag the vertex of a shape to change the angle they are not considering the angle as a measure of turn. They are only changing an aspect of the shape. So although clicking and dragging one vertex of a triangle, for example, gives a feel for the way in which the three angles are interdependent – so if one is changed, either or both of the other two must change as well – there is nothing here to draw attention to the rotations that the three angles represent.

Ways of working

So one or two specific topics or concepts which might be different in print-based and in ICT-rich mathematics curricula may, perhaps, be identified. But another difference, which relates to a way of working rather than to a particular area of mathematics, should also be noted.

The use of 'trial and improvement' to solve simultaneous equations is often specifically prohibited in conventional test papers, and may lead to a loss of marks even when the final answer is correct. This being the case, teachers may discourage their students from adopting such an approach. However, trial and improvement is a perfectly reasonable way to solve simultaneous equations with a spreadsheet, using successive refinements rather than algebraic manipulation to home in on the solution.

Furthermore, a similarly exploratory approach may be useful in a range of different computer-based contexts. In their study of responses to questions in the World Class Tests of mathematics, Threlfall and Pool (2004) observed that students approached some of the 'dynamic and open' questions in a way which, they suggest, was significantly different from their responses to more closed, paper-based activities. As they explain, students took advantage of

> the opportunity that the dynamic and interactive computer context offers to 'play' with the situation…. [This] enables an exploratory approach to possibilities, trying this and that to see whether it 'works' rather than needing to develop a strategy for determining the answer. (Threlfall and Pool 2004, 8)

This 'exploratory approach' may, perhaps, be similar to the 'trial and improvement' solutions of simultaneous equations that students commonly use (and may be penalised for) when working on written test questions, but the writers observed it in the context of a much wider range of topics. So, they argue,

> success on some kinds of computer-based assessment items can arise from different skills and abilities than those required for success in related paper and pencil items. (ibid, 11)

These skills appertain to process rather than to content – to mathematical thinking as opposed to specific mathematical techniques. Mathematical thinking may be highly valued, at least in the rhetoric of the school mathematics curriculum, but, as Threlfall and Pool point out, its assessment may none the less be in conflict with traditional assessment criteria. As they explain,

> For the questions that do offer exploratory potential, there is… the issue of whether the qualities and skills that are used to answer them are felt to be legitimate criteria for the assessment being considered. Judged against the current conventions of mathematics assessment, the means by which answers can be arrived at … could be considered inappropriate. (ibid, 14)

If such a judgement is made, so that students continue to be penalised for the use of trial and improvement methods in a wide range of contexts, then there can be little real progress in the use of ICT for the teaching and learning of mathematics. This may be particularly relevant to the use of generic *tool* software, whose primary purpose is to allow students to explore, experiment, and, as Threlfall and Pool put it, see what 'works'.

Mathematics teaching software, and particularly tool software, offers a magnificent promise. Programs such as *Furbles*, *GeoGebra*, *Excel* and of course *Logo* could allow students to develop their capacity for mathematical reasoning by exploring in an experimental environment. But this promise might not be met unless the assessment criteria – what it is that tests are designed to assess – changes to allow students to 'play' with mathematical ideas on the computer in the course of the assessment.

References

Clausen-May, T. 2008. Another angle on angles *Australian Primary Mathematics Classroom* 13(1): 4-8

Mackrell, K. and P. Johnston-Wilder. 2005 Thinking geometrically. In *Teaching Secondary Mathematics with ICT* eds. S. Johnston-Wilder, and D. Pimm, 81-100, Milton Keynes: Open University Press

Papert, S. 1980. *Mindstorms: Children, Computers and Powerful Ideas* Brighton: Harvester Press

School Mathematics Project. 1970. *SMP Book 2 (Metric edition)* Cambridge: Cambridge University Press

Threlfall, J. and P. Pool. 2004. How Might the Assessment of Mathematics through Dynamic Interactive Computer Items be Different from that in Conventional Tests?. Available online at: http://www.icme-organisers.dk/tsg27/papers/03_Threlfall-Pool_fullpaper.pdf (accessed 20 June, 2008)

Weist, L. R. 2001. The role of computers in mathematics teaching and learning. In *Using Information Technology in Mathematics Education* eds. D.J. Tooke and N. Henderson, 41 – 55. Binghampton, NY: Haworth Press