PROSPECTIVE MATHEMATICS TEACHERS' PRACTICES OF TECHNOLOGY INTEGRATION: A CASE OF DEFINITE INTEGRAL

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This study focuses on prospective mathematics teachers' practices of integrating technology into instruction for the case of the definite integral. We are particularly interested in how prospective teachers use technology to address student difficulties concerning the limit process to define definite integral. For that purpose, we selected two prospective mathematics teachers and investigated their pedagogical use of technology. The data comes from micro-teaching videos and lesson plans of these prospective teachers, interviews with them and teaching notes. In this paper, we will discuss implications of our data to diagnose prospective teachers' difficulties and to identify the areas in need of development for a successful integration.

LITERATURE REVIEW

Recently, the question of what teachers of mathematics need to know in order to integrate technology appropriately into their teaching has received much attention (ISTE (2000) as cited by Mishra & Koehler, 2006). This study focuses on prospective mathematics teachers' practices of integrating technology into instruction for the case of definite integral. Our investigation is situated within the Shulman's (1986) pedagogical content knowledge (PCK) theoretical framework. He defines pedagogical content knowledge as the "subject matter for teaching". For the last two decades, mathematics education literature has focused on teachers' and prospective teachers' pedagogical content knowledge (Carpenter et al,1988; An, Kulm, & Wu, 2004; Cha, 1999). These studies investigated various aspects/components of PCK as described by Shulman (1987) and Grossman (1990). Shulman (1987) puts forward two components in his definition of PCK: knowledge of students' understanding, and the use of representations; and strategies for teaching particular topics. Grossman (1990) proposes four components of PCK: knowledge of strategies and representations for teaching particular topics; knowledge of students' understanding, conceptions, and misconceptions of these topics; knowledge and beliefs about the purposes for teaching particular topics; and knowledge of curriculum materials available for teaching.

THE RESEARCH

In this study we will focus on a particular component of PCK: *knowledge of students' understanding of and difficulties with specific mathematics topics*. This aspect is particularly important for PCK since it is defined as the knowledge of how to represent and formulate the subject that makes it comprehensible for students (Shulman, 1986) and this requires an understanding of how students conceptualise various mathematical concepts and the difficulties they might have. We are particularly interested in how prospective teachers use technology to address student difficulties concerning the limit process to define definite integral.

Limit is one of the important concepts in calculus because of its use to construct other mathematical ideas. Bezuidenhout (2001) states that students' failure to express meaningful ideas in calculus is, to a large extent, due to inappropriate and weak mental links between knowledge of 'limit' and knowledge of other calculus concepts such as 'continuity', 'derivative' and 'integral'. For the concept of definite integral, limit process is essential for constructing the limit of the Riemann sum. Orton (1983) investigated students' understanding of the limit process to define the area under a curve. He asked whether it was possible to obtain an exact answer for the area under the curve $y=x^2$ by taking more and more rectangles under the curve. Out of 110 students, only ten students stated that a limiting process was required. 69 students stated that by taking more and more rectangles under the curve they could obtain better and better approximations but such a procedure would never produce the exact answer. Considering the literature about pedagogical use of technology and students' difficulties reported on the limit process for definite integral, the following research question is formulated: *How do prospective teachers use technology to address student difficulties concerning the limit process to define definite integral?*

METHODOLOGY

This case study is a part of a wider study which sets out to investigate the development of prospective secondary mathematics teachers' PCK during a mathematics teacher education program in Turkey. This one and a half year program admits mathematics graduates, and participants take general and content specific pedagogy courses and do teaching practice in schools. Content specific pedagogical courses include "Instructional Methods in Mathematics-I and II", "Instructional Technologies and Material Development".

This study focuses on two prospective teachers (called Pelin and Sevgi) who taught an introduction to definite integral during micro-teaching activities. After the first micro-teaching sessions, a workshop was conducted in which a Turkish version of Graphic Calculus software was used. The potential of the software in terms of providing multiple representations and links between them were discussed. After the workshop prospective teachers taught the same topic again but this time using the software.

RESULTS

In this section, we present the analysis of data obtained from two prospective teachers' micro-teaching videos, interview transcripts, lesson plans and teaching notes. A detailed account of data obtained from the first micro-teachings of prospective teachers was presented in a previous BSRLM conference (Akkoç, Yeşildere and Özmantar, 2006).

Case of Pelin

Pelin used the same teaching notes she prepared for her first micro-teaching lesson where she did not use the software. Pelin started her lesson by drawing the graph of $f : [0,5] \rightarrow R$, $f(x) = x^2$ on the board, divided the interval into five sections and calculated the area under the curve using the lower sum of the rectangles she constructed on the board. She pointed out to the remaining areas under the curve. She then calculated the upper-sum and emphasised that there were still remaining areas left above the curve. She then told to students to run the software to be able to divide the interval into more rectangles. At that stage she did not start a discussion on why more rectangles are needed to be able to obtain a closer value to the actual area under the curve. She rather directed students to do that. After that, she used the software to divide the interval into five, ten and twenty for choosing both upper and lower sum options and pointed out the values of area that were calculated by the software:

Pelin: Let's divide the interval into 10. This time, the software finds 35 (lower-sum) and 48 (upper-sum), so it's in between, it's more sensitive now. Now, let's divide into 20. As you can see, the remaining area gets smaller, it's between 34 and 44.

She then wrote the mathematical definition of upper and lower sums. After giving the formal definition of Riemann integral, she explained the graphical meaning of Riemann sum on a graph on the board and noted that it is between upper and lower sums. She then wrote the mathematical expression of Riemann sum. She asked the following question: "What should we do to find the exact value?". One of the prospective teachers responded: "we make them finer and finer". After that response, Pelin did not encourage an argument on the need for limiting process. Instead, she continued and made an explanation as follows:

Pelin: Well, how can we do that? Let's make the intervals approach to 0. Then, what is the number of the intervals? Infinite. Then, Δx goes to zero as the number of intervals goes to infinity.

As can be seen in her approach, although she mentioned that the lengths of the intervals approach to zero, she did not encourage her students to explore why a limiting process is needed. As can be seen from this case, mere technical knowledge of how the software calculates the areas is not enough on its own unless teachers have related pedagogical knowledge and skills.

To explore Pelin's pedagogical use of technology, we interviewed her during which she reflected on her lesson. She was asked how she planned to use the software in her lesson and stated that she used the software to divide the intervals into more pieces. Although she believed that dividing the interval into more pieces may help students to understand limiting process, she could not address this difficulty successfully in her micro-teaching as described above.

During the interview, Pelin stated that although using software was useful for visualisation, she believed that she taught better on the board. She said the following:

Researcher: You also taught the concept of definite integral before technology workshop. Did graphic calculus software enrich your lesson when you compare this lesson to your previous lesson?

Pelin: I was better when I didn't use the software. But, actually we could approach more closely to the area under a curve using it. It added visualisation to the learning process.

Researcher: Did you confront with any difficulties during your teaching?

Pelin: Teaching at board is easier for me. Actually one could expect it must be easier using the technology. But, when using the software verbal explanations were used frequently so I feel as if taught mathematics verbally.

As can be seen in her responses, she had difficulties in using the software and she felt she had to make verbal explanations when using the software. This might be because she had difficulties in coordinating between the board and the computer. For instance, she did not write on the board what was explored and observed using the computer e.g. preparing a table which presents the number of intervals and the areas that were calculated so that the number to which the values of the area approach can be more easily observed. In summary, Pelin used technology for only teacher-demonstration in a teacher-centred approach without having students to try and discover that they need a limiting process to find the exact area under a curve using their own computers.

Case of Sevgi

Sevgi prepared a new lesson plan for her second micro-teaching using the graphic calculus software. After introducing the notion of partition of a closed interval, Sevgi drew the graph of $f:[0,3]\to R$, $f(x)=x^2$ on the board, divided the interval into three and then into five, and calculated the area under the curve using the lower sums of the rectangles she constructed on the board. She found the areas by constructing the lower sums and emphasised that the area was getting close to nine.

After emphasising that the value of area she found did not give the exact value, she said they needed to use the software. She told the students that she was going to divide the interval into three then six, twelve, a hundred and a thousand. She then found the upper sum for the first of these on the board by dividing the interval into three and then used the software to find the other upper sums.

After that she said she would also find the area mathematically and evaluated the lower sum as follows:

$$A(f,p)=\frac{3}{n}f(0)+\frac{3}{n}f(\frac{3}{n})+\ldots+\frac{3}{n}f(\frac{3n-3}{n})=\ldots\ldots\frac{18n^3-27n^2+9n}{2n^3},\quad \lim_{n\to\infty}\frac{18n^3-27n^2+9n}{2n^3}=9$$

After evaluating the upper sum in a same manner, she emphasised that the upper sum approaches nine from the right and the lower sum approaches to nine from the left. Finally, she wrote the definition of Riemann sum on the board.

After her micro-teaching, she was interviewed during which she reflected on her lesson. During the interview, she mentioned that her understanding of definite integral improved as she prepared for the lesson:

Researcher: You said you were better in your second lesson. What was the difference?

Sevgi: I felt I knew the content much more conceptually. Actually I knew it in my first lesson too but I realised that I could explain better with graphic calculus software. And I also understood the concept of definite integral better. For example, I know more conceptually where to use limit, why do I need to calculate upper and lower sums... I had already known the visual representation but I didn't know its proof, I mean I didn't know the limiting process. But now I know very well.

Researcher: What else can you say about the software? How did graphic calculus software affect your teaching?

Sevgi: I asked questions to students such as 'let's divide the interval into more pieces and see what would happen' and students realised that rectangular areas became smaller and the sum of areas approach to the exact value.

Researcher: How do you think the use of technology overcome the difficulties about limiting process?

Sevgi: I practised the software before the lesson. It divides the interval into maximum 10000 pieces. We could make use of software at that point and ask students: 'what would happen if we divide the interval into 'infinite' pieces?'. I expect students to say that 'definite integral is the area under the curve and upper and lower sum approach to real area more and more'.

As can be seen from her responses, her experiences with the software improved both her content knowledge and her teaching as compared to her first micro-teaching experience. During her first micro-teaching, she divided the interval into only four and defined the Riemann sum afterwards. As she mentioned, during the second micro-teaching lesson, she was more aware of the limiting process to find the area under a curve. She asked students what would happen if the interval is divided into more and more pieces. This way, she used the software to help students to explore more precise values of areas. However, she did not provoke a discussion on why a limiting process is needed.

DISCUSSION AND IMPLICATIONS

The data in this study indicated the importance of pedagogical knowledge to be able to integrate technology into instruction. As in the case of Pelin, technology was used in a teacher-centred manner without having students to try and discover that they need a limiting process to find the exact area under a curve using their own computers. As the other case, Sevgi started a discussion on how to obtain closer values of area using the computer. However, she did not provoke a discussion on why a limiting process is needed. These results indicate that mere technical knowledge of technology is not enough for successful technology integration. Pedagogical content knowledge is crucial for an effective use of technology. As exemplified in this study, the software has the potential of addressing students' difficulties (as one component of PCK) with limiting process. At the same time, it has a limitation e.g. intervals could be divided into 10000 at most. Awareness of these affordances and limitations and pedagogical implications is crucial for technology integration.

The analysis of data provided implications for mathematics teacher education. PCK could be a useful theoretical framework to monitor prospective teachers' development of technology use. In this study, we focused on one component of PCK. Future studies should explore the pedagogical use of technological tools considering the other components of PCK as described by Grossman (1990).

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