

UNDERSTANDING THE STRUCTURE OF A SURFACE BY PLOTTING VIA CAS

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The use of computers and especially Computer Algebra Systems in mathematics education offers new opportunities in teaching and learning the relationship between mathematics and the real world. Visualization is also the process of using geometrical illustrations of mathematical concepts. In this paper, an algorithm, into how visualization is used to improve students' comprehension of the structure of a surface which is a two dimensional graphic, has been described. According to the constructivist learning theory, we tried to determine a road from the picture of a curve to a surface. The plotting method, described in this paper, is an electronic plotting algorithm which makes the structure of a surface more comprehensible.

INTRODUCTION

The use of computers and especially Computer Algebra Systems in mathematics education offers new opportunities in teaching and learning the relationship between mathematics and the real world.

Visualization is also the process of using geometrical illustrations of mathematical concepts. Over the past centuries, a decline in the geometric element in mathematical instruction has been observed. Computer Algebra Systems have given mathematics the opportunity to adjust the balance between the various modes that constitute the basis of mathematical education (Kabaca and Mirasyedioğlu, 2006).

In this paper an algorithm, into how visualization is used to improve students' comprehension of the structure of a surface which is a two-dimensional graphic, has been described. According to the constructivist learning theory, we tried to determine a road from the picture of a curve to a surface.

An ordinary Computer Algebra System may easily plot a surface represented by a two variable function. This method of plotting may only make us excited and provide us a chance to see a two dimensional graph. In this paper, we aimed to suggest a procedure constructing a comprehensive understanding of the fundamental structure of a surface.

PROBLEM SITUATION

A two dimensional curve, which is a graph of a one variable function, can easily be plotted by applying an analytic procedure which has several steps. These steps' aims are finding out some critical point of the curve, like intersection points of curve and axis, extreme points, inflection points.

This method of plotting a two dimensional curve is very well-known by every calculus student and easily applied by most of them. When we encounter the problem of plotting the graph of a two variable, which is a surface, our work is getting a bit hard. Even if the properties of derivative of two variable functions give us some clues about how to find critical points of a surface, it is not as easy to use these clues to construct the graph of three dimensional surfaces as plotting a two dimensional curve.

A formal procedure, to construct the graph of a two variable function, has already declared formally. According to this procedure, let $z = f(x, y)$ be a two variable function, for every constant value c of z a relation is obtained between x and y . The graph of this relation is called as “*level curve*”. By plotting level curves in order to construct the structure of surface, you can obtain the general outline of the graph of surface. However, this is a very time-consuming and difficult procedure, because during the process, you may have to plot a lot of level curves and some of these curves may be implicitly defined.

AN EDUCATIVE USE OF MAPLE

Using a Computer Algebra System can help to make the plotting process clearer and more comprehensible. us at this point. Maple, which is a sort of CAS, has been already used to teach mathematics in the most of recent educational researches (Aktümen, 2007; Kabaca, 2006; Aksoy, 2007; Tuluk, 2007).

Maple can plot a two variable function’s graph using a special command directly. But this method of plotting only provides a nice three dimensional picture. In this paper, our aim is more than obtaining the picture of a surface. Let’s choose a specific function as an example to clarify what we mean exactly. Maple can plot directly a three dimensional graph via below command:

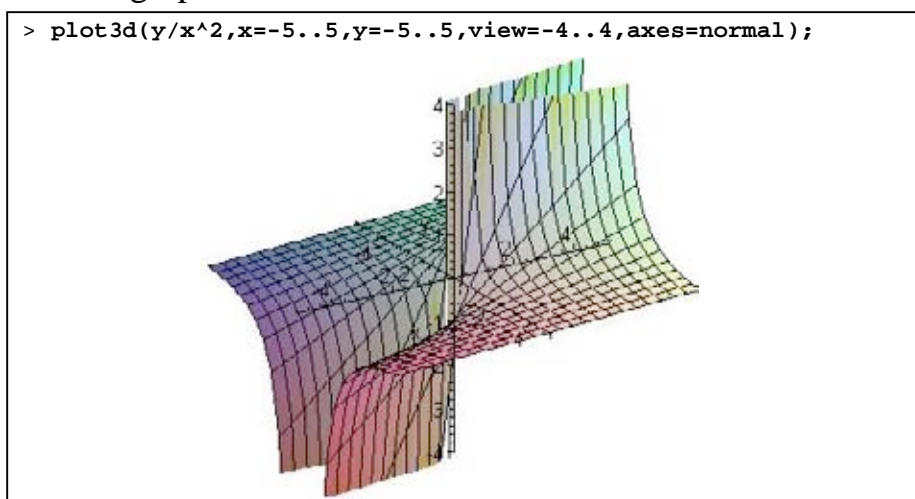


Figure 1

Although we have obtained the surface simply and quickly, this does not mean anything more than an excited picture for a beginner calculus learner. Computer technologies should be used as an imaginary teacher. And a constructivist teacher never provides the final result directly.

Constructing the Surface

Let's give a real constant c for z . This constant defines the height of the two dimensional curve, which is a picture of relation between x and y . First, the function has been defined to Maple.

> $z := y/x^2;$

$$z := \frac{y}{x^2}$$

Then, for the value of constant c from -5 to 5, some level curves have been plotted implicitly on \mathbb{R}^2 plane. In Figure 2 (below), the level curves and every level curves' height are displayed [1]. We also produced an animation to highlight which curve is represented for the value of constant c .

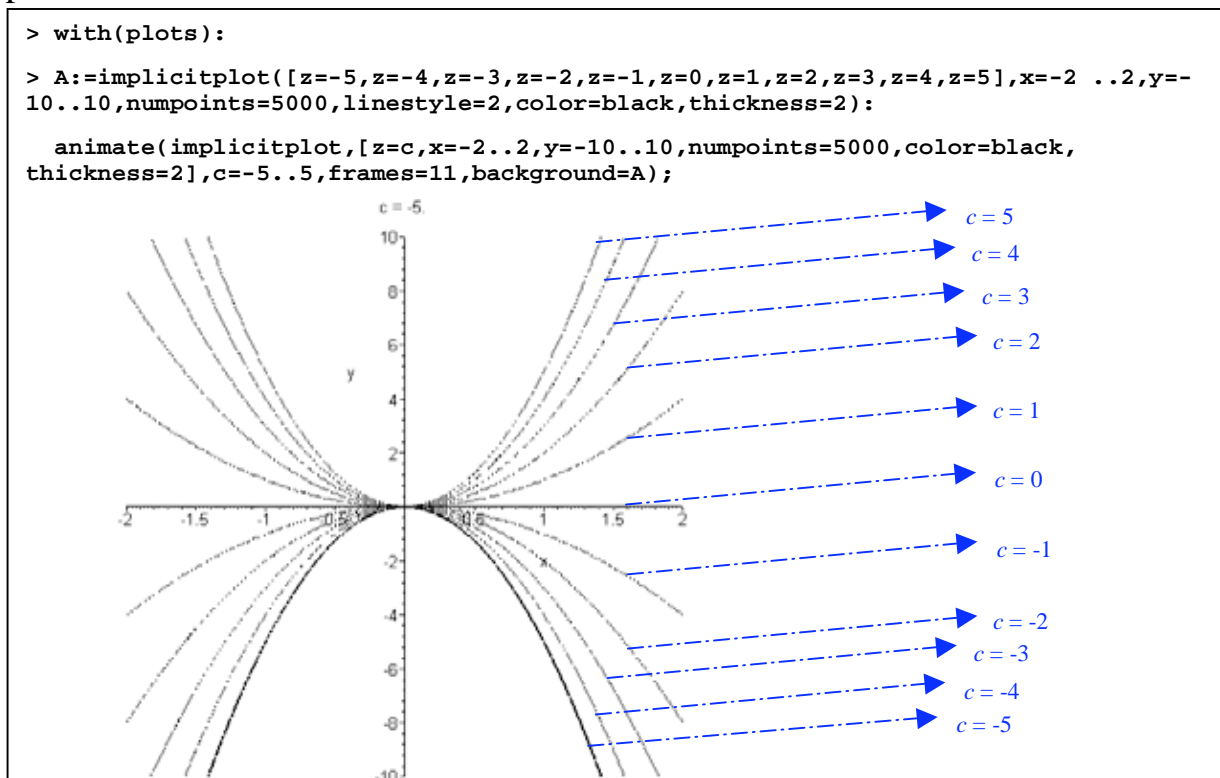


Figure 2

Note that, we have also chosen the values of the constant c with equal intervals. In this way, these curves can be considered as contour lines of a topographic map. The animation also helps students while examining which curve was plotted for which value of c .

We should direct our students to estimate the surface by asking their own reasoning. Then, we can also apply to the technology again. Let's locate all level curves to their original height $z = c$ on three dimensional environment by using the Maple command "spacecurve". By this way, we can obtain a real surface skeleton.

Before using "spacecurve" command we have to make a parameterization of the function $z = y/x^2$. It will be better to encourage students to make this parameterization.

We will choose a constant value c for z . So, we can give the parameter t for y . Then, after rearranging the function, $x_1 = \sqrt{t/c}$ and $x_2 = -\sqrt{t/c}$ are obtained.

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> A1:=spacecurve([sqrt(t/(-5)),t,-5],t=-10..10,color=black,numpoints=5000, axes=normal):
A2:=spacecurve([-sqrt(t/(-5)),t,-5],t=-10..10,color=black,numpoints=5000, axes=normal):
A3:=spacecurve([sqrt(t/(-4)),t,-4],t=-10..10,color=red,numpoints=5000, axes=normal):
A4:=spacecurve([-sqrt(t/(-4)),t,-4],t=-10..10,color=red,numpoints=5000, axes=normal):
A5:=spacecurve([sqrt(t/(-3)),t,-3],t=-10..10,color=yellow,numpoints=5000, axes=normal):
A6:=spacecurve([-sqrt(t/(-3)),t,-3],t=-10..10,color=yellow,numpoints=5000
,axes=normal):
A7:=spacecurve([sqrt(t/(-2)),t,-2],t=-10..10,color=blue,numpoints=5000, axes=normal):
A8:=spacecurve([-sqrt(t/(-2)),t,-2],t=-10..10,color=blue,numpoints=5000, axes=normal):
A9:=spacecurve([sqrt(t/(-1)),t,-1],t=-10..10,color=red,numpoints=5000, axes=normal):
A10:=spacecurve([-sqrt(t/(-1)),t,-1],t=-10..10,color=red,numpoints=5000, axes=normal):
A11:=spacecurve([sqrt(t/(1)),t,1],t=-10..10,color=blue,numpoints=5000, axes=normal):
A12:=spacecurve([-sqrt(t/(1)),t,1],t=-10..10,color=blue,numpoints=5000, axes=normal):
A13:=spacecurve([sqrt(t/(2)),t,2],t=-10..10,color=yellow,numpoints=5000, axes=normal):
A14:=spacecurve([-sqrt(t/(2)),t,2],t=-10..10,color=yellow,numpoints=5000, axes=normal):
A15:=spacecurve([sqrt(t/(3)),t,3],t=-10..10,color=green,numpoints=5000, axes=normal):
A16:=spacecurve([-sqrt(t/(3)),t,3],t=-10..10,color=green,numpoints=5000, axes=normal):
A17:=spacecurve([sqrt(t/(4)),t,4],t=-10..10,color=red,numpoints=5000, axes=normal):
A18:=spacecurve([-sqrt(t/(4)),t,4],t=-10..10,color=red,numpoints=5000, axes=normal):
A19:=spacecurve([sqrt(t/(5)),t,5],t=-10..10,color=black,numpoints=5000, axes=normal):
A20:=spacecurve([-sqrt(t/(5)),t,5],t=-10..10,color=black,numpoints=5000, axes=normal):

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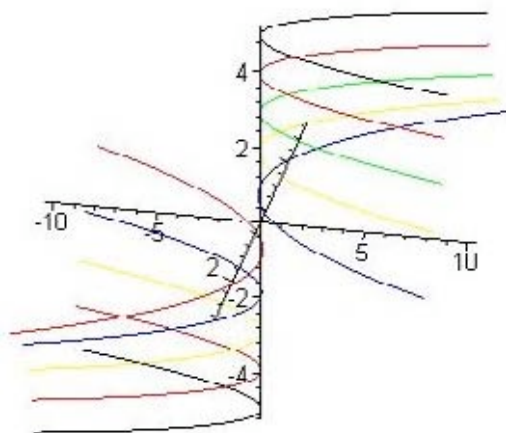


Figure-3

In Figure 3, you can see a surface skeleton, obtained by locating all the level curves shown in the Figure 2, to their original height according to the z -axis. If you look this shape from the top, you can obtain Figure 4 which is exactly the same as Figure 2.

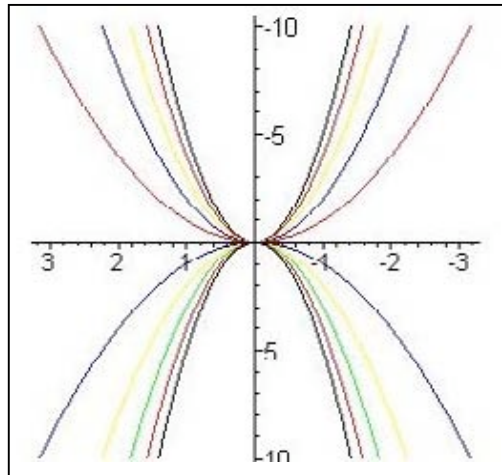


Figure 4

Now, let's display the graph of the surface in Figure 1 and the surface skeleton in Figure 3 together.

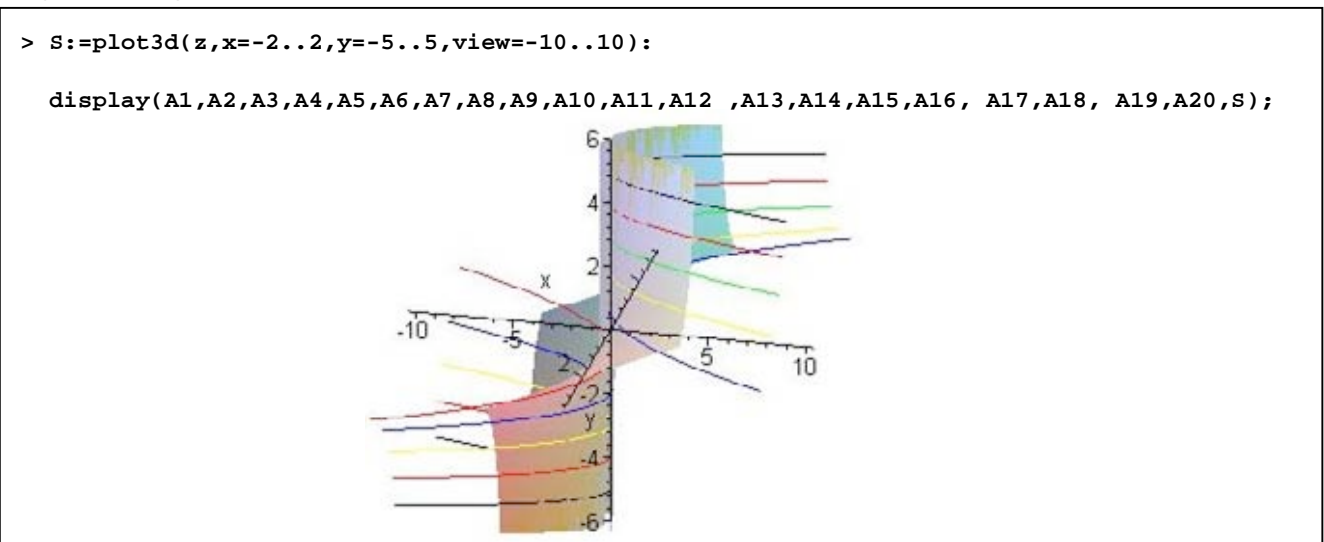


Figure-5

In the above figure, you can see that the surface skeleton and the surface coincide exactly.

RESULT AND DISCUSSION

Thanks to Computer Algebra Systems like Maple, many mathematics educators try to show some geometrical illustrations to their students. A CAS has serious capability of visualizing. While organising a visualization application, the most important issue should be constructivist approach (Baki, 2002; Dubinsky et al, 1996). In this paper, a constructivist alternative has been presented instead of obtaining a graph directly. By this way, students have been provided a chance to make reasoning.

This application has also a secondary functionality which is the advanced usage of several Maple commands like “plot3d”, “implicitplot”, “display”, “spacecurve” and “animate”. Students may develop their own learning application thanks to these examples.

We have developed this instructional environment for use on a calculus course. Even if a positive influence has been observed on students, a formal observation has not been reported. Future studies, which are conducting formal educational experiments, should be designed to gain more meaningful results.

The most interesting result of this study may be considering the level curves as contour lines of topographic map. An educative project may be organised for the students related with geographical maps.

NOTES

1. An animation has been used while obtaining Figure 2. If this document is saved as “Web page” or the commands are used in a Maple worksheet, the animation can be observed. Blue indicators do not exist in the original Maple graph. These indicators are placed artificially only for this paper to determine the heights of the curves.

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