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# TEACHERS' BELIEFS ABOUT MATHEMATICAL PROBLEM SOLVING, THEIR PROBLEM SOLVING COMPETENCE AND THE IMPACT ON INSTRUCTION: THE CASE OF MS ELECTRA, A CYPRIOT PRIMARY TEACHER

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*This paper refers to the case of Ms Electra, a Cypriot primary teacher, with respect to her beliefs about problem solving, her problem solving competence and the teaching of a problem solving activity. Ms Electra was interviewed about her problem solving beliefs. After the interview, she was given a mathematical problem and asked to solve it and explain her thought. Ms Electra prepared a lesson based on that problem and taught the problem solving activity in her classroom.*

## INTRODUCTION

### What is a mathematical problem?

In mathematics education, the term 'problem' is probably one of the most commonly used terms. Despite the large body of research in this area, many researchers dealing with mathematical problems have used the term quite differently. Indeed, it is remarkable that when people refer to a problem in mathematics, it is possible that they do not have the same thing in mind and what is a problem for someone, may not be a problem for another (Borasi, 1986; Blum & Niss, 1991; Wilson, Fernandez & Hadaway, 1993).

According to Blum and Niss (1991), there are two kinds of mathematical problems. The first kind is applied mathematical problems, which refer to real world situations, "the 'rest of the world' outside mathematics" (p. 37). The second kind is purely mathematical problems, which are entirely connected to the world of mathematics. I consider this distinction of problems to be both logical and functional, since the same problem can exist in both domains, due to the way it is posed (context).

### Mathematical problem solving

Problem solving in mathematics education, reports Chapman (1997), means different things to different people. Polya (1981) asserts that "solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems is the specific achievement of intelligence, and intelligence is the specific gift of mankind: problem solving can be regarded as the most characteristically human activity" (p. ix). Polya (1945) outlines four phases in problem solving: understanding the problem, devising a plan, carrying out the plan, looking back.

Mason, Burton and Stacey (Mason *et al*, 1985) analyze three phases for the process of tackling a question; Entry, Attack and Review. Entry begins when the problem solver is faced with a question. The Entry phase work is largely in formulating the question precisely in deciding exactly what should be done. Thinking, according to Mason *et al* (1985), enters the Attack phase, when the problem solver feels that the question has moved inside her/him and become her/his own. The phase is completed when the problem is abandoned or resolved. During Attack, several different approaches may be taken and several plans may be formulated and tried out. The final phase is what Mason *et al* call the Review. When the problem solver has reached a reasonably satisfactory resolution or when he/she is about to give up, it is essential to review the work made. Mason *et al* (1985) assert that in this phase it is time to look back at what has happened in order for the problem solver to improve and extend her/his thinking skills and try to set her/his resolution in a more general context.

The fact that there is limited research on problem solving from a teacher's perspective was observed by Chapman (1997) a decade ago, while he noted that the general focus is on teachers' instructional effectiveness rather than teachers' problem solving competence. A similar comment was made by Thompson (1985), according to which, "research related to instruction in problem solving has centered on the effectiveness of instructional methods designed to develop global thinking and reasoning processes, specific skills, and general, task-specific heuristics" (p. 281). Thompson argued that the disproportionately small amount of attention that researchers have given to the role of teachers is disturbing.

## METHODOLOGY

For the purposes of my research, I used a case study approach, with three Cypriot primary teachers. All three teachers were individually interviewed about how they viewed themselves as mathematics teachers, the nature of problem solving, their competence as problem solvers and about the management of problem solving in classrooms.

Immediately after interviewing, each participant was given a mathematical problem and was to solve it and to talk aloud explaining every step. I made use of this method in order to examine the problem solving competence of my participants. I informed them that it was their way of thinking I wanted to observe and for that I would not interfere and would not give any further explanations apart from what was written on the paper.

The problem was presented in Greek. Each teacher was given a piece of paper with the problem and a grid paper. Its accurate translation into English is the following:

"On the grid paper you have been given, each little square is equal to one square unit. How many isosceles triangles can you make which will satisfy all of the following three criteria?

1. The area must be nine square units.
2. One of the vertices is at the given point.

3. The other two vertices are on grid points too.”

There are 36 isosceles triangles which satisfy all three criteria. The number 36 seemed to me like a ‘good’ number of solutions, enough to make the problem particularly challenging and at the same time not so big so as to render a solution of the problem unattainable.

Participants were asked to prepare a lesson based on the problem and teach it to their classrooms. I acted as a non-participant observer, sitting in the classrooms and watching the lessons. My observations are thought to be unstructured, because I did not have a list of predetermined points that I wanted to examine. The purpose was not to observe if the teachers did something specific. It was to find out ‘what’ they did. Nevertheless, classroom observations were focused on the teachers’ actions during the lesson, the way they presented the problem, how they managed the classroom during problem solving and how they approached students’ questions and solutions.

For the purpose of analysing and presenting teachers’ actions during solving the given problem, I chose to incorporate the three-phase model proposed by Mason and his colleagues (Mason *et al*, 1985). After collecting the classroom-observation set of data, I have noticed a convergence between my participants’ actions and the mathematical didactics of Andrews’ (2007) work. Andrews reports a comparative study between Flemish Belgium, England, Finland, Hungary and Spain. Their cross-national research has revealed ten mathematical didactics or, in other words, ten “teaching strategies that teachers use to facilitate their learners’ ability to understand and use mathematics” (p. 28).

**The case of Ms Electra**

Ms Electra is a 22-year old teacher and she is teaching for her first time. She has always been an excellent student but with low self-esteem as regards her competence in mathematics. However, she has always been successful in every mathematics module she has attended, from high school to university. As she says, “mathematics and I are not good friends”. Despite the fact that she has always received excellent grades in mathematics, in high school she chose to study Classical Greek, History and Latin, and at university she specialised in language education. She is currently a graduate student, studying for a master’s degree in Curriculum Development and Assessment.

As Ms Electra points out, mathematics is incorporated in the curriculum of every educational system around the world and that is enough to justify its importance. I assume that the connection she makes between mathematics, its importance and curricula derives from the fact that she is currently undertaking a master’s degree in Curriculum Development and Assessment; it does not seem like a personal opinion based on experiences and positive attitudes towards mathematics. I am quite convinced that her opinion derives from her status of post-graduate student.

She calls herself an insecure problem solver and makes no effort to hide her low self-confidence in teaching mathematics and problem solving activities without being well prepared. Nevertheless, during the process of solving the problem, she managed to control her initial anxiety and successfully navigated through the problem's difficulties. As soon as she realised that the way she was following was not appropriate, because of the three criteria, she revised her thinking and applied a new way. After discovering a pattern, she easily generalised her ideas and found all 36 solutions.

Generally, Ms Electra's espoused beliefs were consistent with her classroom practices. Her lesson was mostly based on students' sharing and explaining their ideas to their classmates. Also, Ms Electra was particularly interested in offering individual and group feedback and assessing her students' understanding. This was achieved either through a group discussion, where the teacher was managing rather than explaining, or through questioning, where the teacher used to challenge her students with questions. Analytically, selected incidents confirming the mathematical didactics employed by Ms Electra are presented below.

<u>Sharing</u>

(a) A student argued that the height that begins from the vertex between the two equal sides of an isosceles triangle divides the triangle into 2 equal parts. The teacher asked the student to demonstrate his ideas on the board and explain it to his classmates.

(b) At a point where most of the students had found all of the triangles with base 2 and height 9 and base 6 and height 3, the teacher asked if there was anyone who would like to explain analytically her or his way of thinking to the others. One student went to the board (where a grid paper was presented though an overhead projector) and explained to her classmates how she found all the triangles with base 6 and height 3. In the meantime, the other students asked that student questions on some of her steps.

<u>Questioning</u>

The teacher asked the students to number the characteristics of isosceles triangles.

Student: The sum of their angles is 180˚.

Ms El.: Is this a characteristic that only isosceles triangles have?

Student: No, it applies to all triangles.

Ms El.: How important do you think this information is for our problem?

Student: We don't need it.

<u>Assessing and coaching</u>

When students were working either individually or in groups, Ms Electra went round the class and asked them to explain what they were doing. Her help and feedback was very encouraging. When the students faced difficulties, she challenged them with hints that would lead them to discover their own way.

A very important finding is the inconsistency between Ms Electra's low self-esteem as a problem solver and her problem solving competence. The wider literature highlights the strong interconnections between students' self-esteem and competence in mathematics (i.e. Nicolaidou & Philippou, 2003). However, Ms Electra seems to comprise the exception to the general 'rule'. Presumably, a number of prior negative experiences about mathematics have led her to the belief that she is not mathematically competent. I should point out that Ms Electra's high performance in mathematics as a high-school and a university student contradict her anxiety as regards mathematic teaching.

## CONCLUSION

Thompson (1985) highlights the need for teachers (1) to experience mathematical problem solving from the perspective of the problem solver before they can adequately deal with its teaching, (2) to reflect upon the thought processes that they use in solving problems to gain insights into the nature of the activity and (3) to become acquainted with the literature on research on problem solving and instruction in problem solving. According to Cooney (1985), studies suggested that teachers may not possess rich enough constructs to envision anything other than limited curricular objectives or teaching styles and hence may be handicapped in realising a problem solving orientation. The use of a problem-solving approach demands not only extensive preparation but also the development of ways to maintain at least a modicum of classroom control and, perhaps most importantly, the ability to envision goals of mathematics teaching in light of such an orientation.

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