OBSERVING STUDENTS' USE OF IMAGES THROUGH THEIR GESTURES AND GAZES

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In this paper I report on a study observing six year 13 students (18 years of age) working in pairs on a set of mathematical problems, to test the hypotheses related to students' use of imagery. The research was done from a constructivist perspective, looking through a socio-cultural lens. It is concerned with the relationship between mental imagery, thought and action. The results have showed that it is possible to observe students' use of imagery through their gestures and suggest that those students who have access to and are able to manipulate mental images are more successful problem-solvers.

BACKGROUND

I am now in my fifth year of teaching mathematics in a secondary school and I have become particularly interested in the power of mathematical imagery. As a student of mathematics at school, and then at university, I was rarely offered images as a way of accessing mathematical concepts. It was not until I was training to become a teacher that I was introduced to a key mathematical image of a point moving around a circle:

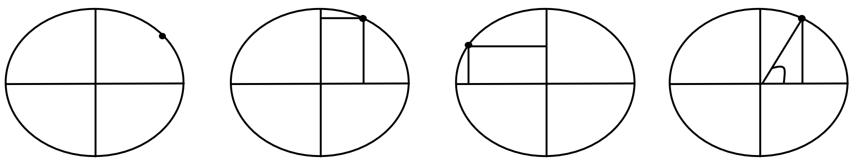


Fig. 1

The images in figure 1 are of a circle with radius of 1, split into four equal regions, and are snapshots of an animation (e.g. see www.mathsfilms.co.uk). A horizontal and a vertical line that meet the point on the circumference are introduced. The horizontal and vertical lines are named Sine and Cosine and their lengths are dependent on the angle that the radius makes with the positive x-axis.

The success that I experienced having used this image – for the first time – as a way of introducing trigonometry with a class of students provoked in me a need to find other such images that I could also work on with students. I had become familiar with the term 'canonical image' when it was introduced to me by Dick Tahta (personal communication) when he was visiting the school where I teach. The term canonical image is used to describe an image that is *economical* in that it gives direct access to the mathematical concept. Using a canonical image in the classroom should lead to an end point that is shared and agreed upon – the essence of a particular mathematical topic (Breen, 1997).

My experience of canonical images and my need to find more of them was my motive behind this piece of research.

THEORETICAL FRAME

The study was done from a constructivist perspective, looking through a socio-cultural lens using research methods associated with a qualitative approach. I briefly set out below what I am taking as the meaning of some key terms.

External Imagery

An *external image* is a *visual* representation that has simple qualities in common with the mathematical object; it is general in nature. By *visual* I am not including text or symbols (e.g. algebraic notation) but rather diagrammatic representations of a mathematical concept or object. The example described (fig. 1) of a dot rotating around a circle is an example of an external visual image, as is any empty number line. A mnemonic such as ‘SOH CAH TOA’ is not an external image.

I am taking *diagram* to mean any 2- or 3-dimensional, static or dynamic, external visual representation of a mathematical structure. It is a purely visual representation that must give some information about the mathematical object that it represents; it is specific in nature. A number line from, say, 5 to -5 would be a diagram rather than an external image.

Mental Imagery

When considering any form of mathematical activity, Gattegno (1958, in Tahta, 1988) presents the triad action-image-thought. A mental image is not purely a visual representation but can share other sensory qualities with the mathematical objects that it represents. A mental image is the mediating link between thought and action in the form of inner mental activity. When we think mathematically we are working with these mental images. Gattegno maintains that although we are able to distinguish between action, image and thought – action is associated with the external world while image and thought are associated with the internal world – each element of the triad happens simultaneously and the mind passes from one to the other.

> The whole of mental activity is internal; hence images and thoughts are not distinguished by virtue of their internal character. It well seems that action calls in an external world on which it is to act, that thought has only virtual recourse to this external world, and that there is a fundamental difference between thought and action only to the degree of one’s interest in this aspect of the problem. (Gattegno, 1958, quoted in Tahta, 1988, p. 27)

In response to this passage from Gattegno, Tahta presents his thoughts on the third element of the triad – the image:

> ..in some sense, the third element of the triad – the image – is a mediating link between thought and action. Our perception of objects yields some inner mental activity which we can refer to as an image. Our awareness that we can not only actually act on objects but can also perform virtual actions, suggests that we can operate on these images in some way. Such images can become sufficiently stable to be independent of the objects from which they arose. (Tahta, 1988, p. 27)

In a seminal article on Mathematics and Imagery, Gattegno (1965) makes a clear distinction between geometry and algebra but at the same time he suggests that the distinction is not a matter of the type of imagery involved:

> In geometry it is visual imagery that is used. But the dynamics of the mind when formalised produces all the conceivable algebras. Algebra differs from geometry in that the first describes mental dynamics while the other uses mental content, imagery. Since images do not function except through the dynamics of the mind, there is no geometry without some inherent algebra that can be noticed per se by a special awareness and made independent of the content. (Gattegno, 1965, p. 22)

So where as geometry is an awareness of imagery, *algebra is an awareness of dynamics* – an awareness of 'the mind at work on whatever content'. These interpretations show how geometry and algebra are complementary and intertwined; offering images can allow students access to geometrical concepts just as much when the mathematical content is overtly algebraic. As Tahta (1988) suggests; the use of Cuisenaire rods and the folding and unfolding of fingers or in general the creation of any *substitutes* can provide an initial 'geometric' situation from which algebra might develop. It follows that "failure to master algebra may be due to neglect of the underlying geometry" (Tahta, 1980, p.6). This notion is consistent with my own experiences of working on mathematics, in particular, the success that I have experienced with algebra having already worked on the geometry and structure of the mathematical concept. Tahta (1980) also suggests that to think algebraically means you must come to an awareness of the mind at work on some mental content. That mental content may be described as geometry, or more directly, as imagery. The crucial role for the teacher is therefore to offer images and encourage students to develop their own power to generate them. In this extract from Gattegno there are some more important implications for teaching:

> A new conscious experience thus leads to a structured image which must be restructured with a greater number of dimensions. In the case of education for abstract thinking, it becomes necessary to avoid beginning with images having a greater number of dimensions, as is the case in the image of the adult as compared to that of the pupils, but instead to increase the number of dimensions of active images. (Gattegno, 1952, quoted in Tahta, 1988, p. 28)

I interpret 'a new conscious experience' to mean any learning experience and hence Gattegno is suggesting that in order to learn you must begin with a structured image which is then adapted and made more complex through the addition of what he calls 'dimensions'. He maintains that the initial image should be one that is not complex with many dimensions but simple and in order to develop abstract or mathematical thinking this simple active image must then increase in its number of dimensions.

An example of a simple active image may be one of a number line consisting of, say, the integers. As we learn more about the number system, then this simple image develops through addition of dimensions, for example, through the inclusion of fractions, decimals, irrational numbers and perhaps even making the transition of the real number line to the complex plane.

RESEARCH DESIGN

The study was based on 6 students; 5 male and 1 female. The 6 students made up the entirety of my year 13 (18 years old) further mathematics class. One of the male students was educated in China until the age of 16 when he moved to Bristol and began attending the school where I teach. The other 5 students were educated solely in the UK.

The students were videoed working in pairs on a set of mathematical problems – one set related to complex numbers and the other set related to trigonometry. The problems were designed so that they may be solved using a variety of methods including visual methods. The students had experience of a variety of methods that they could use to solve the problems in lessons. The students were only given the subsequent question on completion of the previous question and wrote their solutions and workings on sheets provided. Generally, I did not interrupt the students while they worked on the problems but on occasion I interacted with the students in order to probe for further explanation or clarification of the methods being used, for example, by asking a student to draw a picture of what they were describing.

The videos were fully transcribed and analysed using the theoretical frame described.

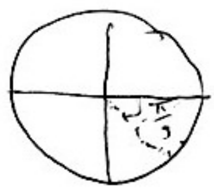
RESEARCH FINDINGS AND ANALYSIS OF DATA

The difficulty in this form of enquiry – attempting to analyse what is going on in the heads of individuals – is obvious. What I have been able to observe however is the external evidence – the students' 'actions' (in the form of gazes and hand gestures).

Having watched the videos a number of times and having transcribed all of the dialogue what I noticed as being particularly significant is the use of hand gestures and the presence of prolonged gazes. I use 'gaze' to signify when a student looks fixedly at nothing in particular (e.g. upwards, at the table, at a spot in the room) but seems to be accessing something in the mind – a mental image.

I was particularly interested in Gattegno's triadic model 'action-image-thought' and decided to report on the data using this triadic model as a framework. The data presented here relates to two particular hand gestures – these have been described under 'actions'. Under 'image' I have inserted any diagrams from the students' work and under 'thought' I have detailed extracts of dialogue from the transcriptions that, for me, provided some insight into the thoughts of the students being considered. For both examples, I have included a comment and the mathematical problem that is being worked on at that time.

By looking at the data in this way, I hoped to see evidence of Gattegno's triad 'action-image-thought'. I have chosen these two particular extracts because I feel that the student speaking is particularly vocal in his apparent use of imagery.

Problem:	Find the general solution to the equation *cos (2x + π/6) = 1/√2*
Action:	Hand gesture - Student F appears to be mimicking the shape of the cosine function. His pen is moving back and forth over the crest of the graph.
Evidence of Image:	
Evidence of Thought:	F: It goes like that **[gestures with hand]** so it's gonna be… no you're gonna have to get the other one is 0 minus whatever the first one is because of the graph **[draws on own sheet]** 'cause the graph will be like that and that point there is say π by 3, for instance, and that's going to be minus… E: Minus π by 3. F: Yep.
Comment:	Student F appears to be verbalising his thoughts as he is manipulating firstly a mental image and secondly a diagram that he sketches. He is relating the images of the cosine function to the algebraic notation and reasoning about the general nature of the solution to the question. He also appears to initially correct himself – using the diagram to justify this change in thought.
Problem:	Write down the value of *(1/2 + √3/2)*^{5}
Action:	Hand gesture - Student F uses pen to gesture the right-hand side of a circle.
Evidence of Image:	
Evidence of Thought:	F: I'm trying to figure out if I can simplify this in any way; I think it's minus π by 3…. 'cause you have the circle, then you go… that's 6π by 3 that's just 2π… 2π, so that's back to the beginning, so that means it's one π by 3 less which is somewhere down here, down here, it's just minus π by 3, so you just get cos… um, π by 3 minus i sin π by 3… yeah…
Comment:	Student F seems to possess a confidence in working with this mental image; using the phrase "then you go" suggests that he is manipulating

	the image in his head as his thoughts are being verbalised. Student F's reference to the image together with his verbalised thoughts seems to support Gattegno's notion of action-image-thought.

CONCLUSION

Gattegno stated that when ever a person is engaged in some form of mathematical activity they are engaged in the triadic process of action-image-thought. By examining the students' behaviour and the dialogue associated with their actions (their hand gestures and gazes) I was able to get a sense of their related images and thoughts. From my data there seems to be some evidence that the triad Gattegno described was occurring. This was particularly evident in the case of student F whose tendency to verbalise his thoughts and demonstrate his use of mental imagery through hand gestures and gazes became especially apparent.

What is difficult to establish is what exactly constitutes a thought. In the case of student F – his verbalisation of a seemingly logical process of reasoning certainly seems to constitute a thought. But in some instances, student F seems to be doing something even more complex – simultaneously he appears to be manipulating mental images, verbalising his reasoning process, gesturing with his hands or recording his workings on paper and performing calculations mentally. There is also evidence here, perhaps, for the important distinction between conscious thoughts (thoughts that we are aware of having) and non-conscious thoughts (those that we are not aware of) – but were does the role of imagery fit with this?

Tahta (1988) talked specifically about one element of the triad – the image – as "a mediating link between thought and action" (p. 27) suggesting that it is these mental images that we can operate on by way of *virtual* actions. The occurrence of these virtual actions may provide an explanation for the hand gestures demonstrated by the students as they worked on the mathematical problems.

Having developed a frame for observing students' use of imagery – through the students' actions (gazes and gestures) – I hope to use this as the beginnings of a larger scale research project and investigate for teaching and learning in school.

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