ADOLESCENCE AND SECONDARY MATHEMATICS

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Abstract: In this paper I outline an argument that the intellectual demands of the secondary mathematics curriculum are fully compatible with adolescence, as a particular phase of life concerned with identity and cognitive development.

INTRODUCTION

In this paper I will show how learning secondary mathematics relates closely to the adolescent project of negotiating adulthood.

My personal warrant for taking this view lies in my work as head of a mathematics department during the late 80s and early 90s in a school which served a socio-economically disadvantaged area. Academic achievement was generally low. The introduction of a national curriculum in 1988 imposed academic demands on students who had not been expected to learn in structured, written, abstract contexts. Nevertheless, in the early days of GCSE our mathematics results were usually the best of all subjects, sometimes ahead and sometimes just behind those achieved in creative arts, and nearly 100% achieved a mathematics grade. We had developed ways of working on mathematics *in* school which were similar to the ways in which quantitative problems arise *out* of school. In retrospect, however, I know we could have done more to support them in engaging with the scientific concepts which can only be deliberately taught (Kozulin 1986, xxxiii).

In this paper, I am going to argue that adolescent motivation can be compatible with 'hard' abstract maths. I shall start with an outline of adolescent concerns.

ADOLESCENT CONCERNS

There is widespread agreement that adolescents are broadly concerned with the development of identity, belonging, being heard, being in charge, being supported, feeling powerful, understanding the world, and being able to argue in ways which make adults listen (Coleman and Hendry, 1990)[1]. Typical development involves the development of abilities to engage in formal-logical thinking, so that they become capable of self-analysis, and analysis of other situations, as internalisations of social consciousness developed with peers (Karpov, 2003). The biological ability to understand things in more complex, abstract, ways is not in itself the most important influence on learning. Rather, it is how adolescents use adult behaviour and interaction as mediators of their activity with peers that influences development.

In adolescence, when examinations become high-stakes, major curriculum topics become less amenable to concrete and diagrammatic representations, full understanding often depends on combining several concepts which, it is assumed, have been learnt earlier[2]. If they have not been learnt earlier students might need to depend on algorithms. While all mathematics students and mathematicians rely on algorithmic knowledge sometimes, learners for whom that is the only option are dependent on the authority of the teacher, textbooks, websites and examiners for affirmation. Since a large part of the adolescent project is the development of autonomous identity, albeit in relation to other groups, something has to break this tension – and that can be a loss of self-esteem, rejection of the subject, or adoption of disruptive behaviour (Coleman & Hendry 1990 pp.70, 155). An alternative way to handle the tension is acquiescence, which often reduces to rule-following.

Changing from proximal, *ad hoc*, methods of solution, derived from elementary understandings, to abstract concepts are hard to make and need deliberate support – indeed this is what is at the heart of Vygotsky's insistence that talk is a necessary aspect of learning scientific concepts (1978 p.131), otherwise one gets stuck with intuitive and everyday notions such as 'multiplication makes things bigger' or 'the bigger the perimeter the bigger the area' (Fischbein, 1987). The shift from horizontal to vertical mathematisation (Treffers, 1987) has to be structured through careful task design; it does not happen automatically. A Vygotskian view would be that this kind of shift necessarily involves disruption of previous notions, challenges intuitive constructs, and replaces them with new ways of thinking appropriated by learners as tools for new kinds of action in new situations.

Realistic tasks can provide contexts for enquiry and often enquiry methods of teaching and learning are recommended for adolescent learning. It is also possible to conjecture relationships from experience with examples, but mathematics is not *only* an empirical subject at school level; indeed it is not *essentially* empirical. Its strength and power are in its abstractions, its reasoning, and its hypotheses about objects which only exist in the mathematical imagination. Many secondary school concepts are beyond observable manifestations, and beyond everyday intuition. Indeed, those which cause most difficulty for learners and teachers are those which require rejection of intuitive sense and reconstruction of new ways of acting mathematically.

SHIFTS OF MATHEMATICAL ACTION

The contradictions between intuitive, spontaneous, understandings and the scientific concepts of secondary mathematics can be the beginning of the end of mathematical engagement for adolescents. If they cannot understand the subject by *seeing* what it does and how it works, but instead have to believe some higher abstract authority that they do not understand, then the subject holds nothing for them. The authority of mathematics does not reside in teachers or textbook writers but in the ways in which minds work with mathematics itself (Freudenthal, 1973 p.147; Vergnaud, 1997). For this reason, mathematics, like some of the creative arts, can be an arena in which the adolescent mind can have some control, can validate its own thinking, and can appeal to a constructed, personal, authority. This was why and how we used to enable students, some of whom were barely literate, to achieve a GCSE in the subject. But to do so in ways which are fully empowering has to take into account the new intellectual tools which simultaneously enable students to achieve in mathematics, and which develop further through supported study of mathematics (Stech, 2007). To understand this further I list below some key intellectual shifts in the secondary curriculum as illustrations of what needs to be appropriated in order to engage with new kinds of mathematical understanding.

- from looking for relationships, such as through pattern seeking, to seeing properties as defined by relationships
- from perceptual, kinaesthetic responses to mathematical objects to conceptual responses
- from intuitive to deductive reasoning.
- from focusing on procedures to reflecting on the methods and results of procedures.
- from discrete to continuous ways of seeing, defining, reasoning and reporting objects.
- from additive to multiplicative and exponential understandings of number.
- from assumptions of linearity to analysing and expecting other forms of relationship
- from enumeration to non-linear measure and appreciating likelihood.
- from knowing specific aspects of mathematics towards relationships and derivations between concepts.

Unsurprisingly, the new intellectual tools described in these shifts turn out to be aspects of mathematics which cause most difficulty and my argument is that it is likely that many teachers do not pay enough attention to these shifts as *inherently* difficult. Extensive literature refers to common erroneous manifestations of some of these shifts as misconceptions, often pathologising the learner rather than the mathematics. Not only do these shifts represent epistemological obstacles, but they are also precisely those changes to new forms of action which constitute the scientific knowledge of mathematics – that which can only be learnt at school. It is inequitable to expect students to bring their everyday forms of reasoning to bear meaningfully on mathematical problems, when everyday forms do not enable these shifts to be made.

SHIFTS OF MATHEMATICAL ACTION COMPATIBLE WITH ADOLESCENCE

I shall now give examples of tasks which generate and nurture mathematical identity in adolescence, while staying focused on the secondary mathematics curriculum as the locus of new forms of thinking.

Learner-generated examples

Students in a lesson were familiar with multiplying numbers and binomials by a grid method. They had been introduced to numbers of the form $a \pm\sqrt{b}$. The teacher then asked them to choose pairs of values for *a* and *b*, and to use the grid method to multiply such numbers to try to get rational answers. Students worked together to explore this arena. At the very least they practised multiplying irrationals of this form. Gradually, students chose to limit their explorations to focus on numbers like 2 and 3 and, by doing so, some realised that they did not need explicit numbers but something more structural which would 'get rid of' the roots through multiplication. Although during the lesson none found a way to do this, many carried on with their explorations over the next few days in their own time (Watson & Shipman, forthcoming)

Tasks in which students gain technical practice while choosing their own examples, with the purpose of finding a particular property or relationship, can be adapted to most mathematical topics. During the work above there were many non-trivial features of thinking that are valuable and could be named, modelled, and praised: exemplifying, controlling variables, conjecturing, limiting the range of variation used, observing and testing special cases, designing spreadsheets to carry out the task, seeing implications of some results, generalising and so on. This confirms the general theory that problem-solving activity in a collaborative working context enhances adolescent identity through recognising agency and granting authority. Yet there was more. This task afforded engagement with structure through reflecting on self-generated examples, the stated goal providing a mediating lens with which to examine evidence, and a tool to guide choice of further examples. Learners found for themselves that merely generating data and looking for patterns in outcomes is not helpful. As Vygotsky says: " neither the growth of the number of associations, nor the strengthening of attention, not the accumulation of images and representations, […] none of these processes, however advanced they might be, can lead to concept formation" (Vygotsky, 1986, p.107)

What they had to do instead is to observe the structure of particular outcomes, and enough students did this for themselves that such observations became the new raw material for learning in the class. This shift of attention came about because " a functional use of […] sign, as a means of focusing one's attention, selecting distinctive features and analyzing and synthesizing them, … plays a central role in concept formation" (ibid. p.106). In this case, 'sign' included the appearance and role of the irrational part in certain fortuitous examples which emerged naturally through students' explorations.

Another and another …..

The following task-type also starts from learners' examples: Students are asked to give examples of something they know fairly well, then asked for more and more until they are pushing up against the limits of what they know, e.g. "Give me a number between zero and a half; and another; and another …", "Now give me one which is between zero and the smallest number you have given me; and another; … and another….". Each student works in a personally generated space of examples, or in a soace agreed by a pair or group. Teachers ensure there are available tools to aid the generation – in this case some kind of 'zooming-in' software, or mental imagery, would help.

This approach recognises learners' existing knowledge, and where they already draw distinctions; it then offers them opportunity to their personal example spaces, either because they have to make new examples in response to prompts, or because they hear each other's ideas (Watson & Mason, 2006). Self esteem comes at first from the number of new examples generated, then from being able to describe them as a generality, and finally from being able to split them into distinct classes.

Putting exercise in its place

If getting procedural answers to exercises in textbooks is the focus of students' mathematical work (whether that was what the teacher intended or not) then shifts can be made to use this as merely the generation of raw material for future reflection. Shifts can be promoted by prompts such as: do as many of these as you need to learn three new things; make up examples to show these three new things; at the end of this exercise you have to show the person next to you, with an example, what you learnt; before you start, predict the hardest and easiest questions and say why; when you finish, see if your prediction was correct; make up harder ones and easier ones.

These questions structure reflective abstraction, and scaffold a newer kind of thinking.

ABSTRACT MATHEMATICS AND ADOLESCENCE

In this paper, I am advancing the idea that ways can be devised to teach all adolescents the scientific conceptualizations, and methods of enquiry, which characterise hard mathematics. I suggest that these cohere with and enhance many features of adolescent development. Moreover this can be achieved without resorting to counter-productive and alienating environments, because the epistemological changes of activity embedded in mathematics are similar to the ways in which adolescents learn to negotiate with themselves, authority, and the world. Agency and identity do not have to be denied, but neither does abstract mathematics have to get lost in the cause of relevance and personal investigation.

In the above task, students create input which affects the direction of the lesson and enhances the direction of their own learning. Classrooms in which these kinds of task are the norm provide recognition and value for the adolescent, a sense of place within a community, and a way to get to new places which can be glimpsed, but can only be experienced with help. To use the 'zone' metaphor – these tasks suggest that mathematical development, relevance, experience and conceptual understanding are all proximal zones, and that moves to more complex places can be scaffolded in communities by the way teachers set mathematical tasks.

NOTES

[1] Naming these as adolescent concerns assumes that Western psychological understandings of adolescence can be taken to be universal, which may not be the case.

[2] I am using this phrase as shorthand for a complex view of conceptual learning which is not elaborated here.

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