BEGINNING TEACHERS' USE OF REPRESENTATION

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The effectiveness of representations used by elementary school teachers, in the first three years of their teaching, is the focus of this paper. I report on findings, from a study which makes use of the Knowledge Quartet framework. I draw on data from the first two years of the study which indicates that the use of representations is a key feature of lessons. Beginning teachers recognised that their use of representations were not always appropriate. There is some evidence that focused reflections have facilitated the development of the teachers' pedagogical knowledge in relation to the use of representations.

INTRODUCTION

From the early years of primary school, mathematics becomes an abstract subject in which an understanding of symbols and their manipulation is a central feature. Physical and pictorial representations are widely used in order to support teaching and scaffold learning of mathematics, acting as intermediaries between the concrete and the abstract. The way in which teachers represent mathematical ideas and processes has been shown to impact on how well children learn (Iliada, Gagatsis and Delivianni, 2005). Jerome Bruner (1974) saw the role of representations as important mediators in developing abstract understandings. Bruner referred to such mediators as 'scaffolds' which support learning and which may be removed once ideas are internalised. Bruner proposed a theory of learning development in which being able to hold a representation, or picture, in the mind freed the learner of the restraints of their physical world, enabling them to perform abstract operations. Bruner suggested three hierarchical but complementary modes of learning. The most concrete of these he termed *enactive* in which learning takes place through physical action. At the next level, *iconic* representation enables the learner to make use of mental images in understanding the world while at the highest level *symbolic* representations allow mental manipulation.

Curriculum guidance for England, Wales and Northern Ireland (DfES 1999) recommended the use of a number of forms of representation for the teaching of primary mathematics including number lines. Number lines may be seen as examples of both iconic and symbolic representations. They involve numerals and are therefore symbolic and the way in which these symbols are put together to form number lines presents a mental image or an iconic representation.

A teachers' ability to use such representations appropriately is a key aspect of what Shulman (1976) termed pedagogical content knowledge (PCK). A team of mathematics educators at Cambridge University observed how subject matter knowledge (SMK) and (PCK) can be seen to underpin the pedagogical decisions made by prospective teachers (Rowland, Huckstep and Thwaites, 2003). One

important aspect of such decisions was found to be the teachers' choices of representations. This research led to the development of a framework, which might be used for the identification and discussion of teachers' mathematics content knowledge as evidenced in their teaching (Rowland, Huckstep and Thwaites, 2005). This framework, termed the Knowledge Quartet, formed the basis for reflection on and discussion of mathematics lessons in my study of beginning teachers. The use of representations by teachers in their mathematics lessons, along with the related reflections and discussions facilitated by the Knowledge Quartet framework, are the focus of this paper.

METHOD

The study is ongoing and investigates the way in which beginning teachers' understanding of mathematics content knowledge needed for teaching might be developed through reflection on their teaching (Schon, 1983). It is a longitudinal study working with beginning teachers from their initial training year to their third year in post. The Knowledge Quartet framework was used as a tool for analysis of lessons and as a means to focus reflections and discussions of lessons on mathematical content.

The participants in the study were students on a one year post graduate elementary teacher education course at the University of Cambridge. Twelve initial participants were selected from 36 volunteers. One mathematics lesson of each student teacher was observed and video-taped during their final teaching practice placement. This was analysed in terms of the four dimensions of the Knowledge Quartet, and relevant issues were identified for discussion with the trainee. Within the same day, the trainee teacher watched the video-tape and was invited to comment on the lesson. This use of stimulated recall, along with focused prompts was employed to facilitate reflection on issues of mathematical content. These discussions were audio-taped for later transcription and analysis. At the end of their training the participants all met with me to discuss their feelings about the study so far and the way in which they would like it to continue.

Nine of the 12 participants who had obtained posts teaching children aged between 4 and 11 years took part in the second year of the study. During this year the participants were observed and video-taped on two occasions. Issues of content knowledge were again identified using the Knowledge Quartet and these were discussed in feedback sessions shortly after the lessons. As soon as possible, DVDs of the lessons were sent to participants for further reflection. Participants were asked to complete regular written reflections on their mathematics teaching in relation to the Knowledge Quartet. Eight of the nine teachers remaining in the study met together at the end of the year to discuss the impact of this project on their teaching.

All the lessons observed during the first two years of this study were analysed according to the Knowledge Quartet framework. In my analyses I found many instances where a teacher's choice of representation impacted on their teaching.

There is not space in this paper to discuss all the use of representation observed therefore I focus on one instance each of the use of number lines and hundred squares.

FINDINGS

Number lines

0 1 2 3 4 5 6 6 7 8 9 10

Number lines are often used by teachers to help children carry out addition or subtraction calculations, making jumps forwards or backwards along the line to represent the numbers to be added or subtracted. However, the choice and use of number lines by the beginning teachers proved not always to be appropriate. When teaching a reception class (4 – 5 years) Sally chose a number line showing the numerals from zero to ten with a picture of a frog over each numeral including the zero. In the Introduction part of the lesson 'Freddie the Frog' puppet was made to jump along the number line to represent the number of frog friends he had. Sally began by telling the children that they were going to use the number line to show how many friends Freddie had. A child shouted out "ten", it would seem that he had looked at the last numeral and assumed this was the number of frogs. Sally then asked a child to make Freddie jump from zero to three on the number line to show that he had three friends. Though the puppet ended up at the interval at which the numeral '3' was written there were actually four pictures of frogs to this point on the line, five frogs if we count Freddie.

Sally went on to use this representation to demonstrate addition and subtraction. She asked 'What would you have to do if Freddie has two more friends? A child demonstrated jumping the puppet two intervals to land above the '5'. Sally then asked 'What if one of Freddie's friends has jumped into the pond. Which way are we going to go? The children responded that Freddie should make jumps to the left. A child demonstrated this but seemed to land Freddie above the '3' and Sally had to 'nudge' him to finish up above the four. Sally told the children that if Freddie's friends all jumped into the pond he should be placed above the one to show this and then quickly corrected herself to above the zero. She moved Freddie to the zero.

Sally then asked a child to make Freddie jump the appropriate number of intervals to show that seven frogs had jumped out of the pond. Ellie-May began her count on zero and ended up at '6'. She realized that this was not right and immediately moved the puppet to the '7'. Sally did not comment on this. Ellie-May seemed to know that the puppet should end up above the numeral '7', but her jumps had only reached the '6'. She had made the common mistake of counting the start point as one jump on the number line ending up in the wrong place. Her understanding of counting would seem to have been confused rather than aided by the use of the number line.

Superficially, the number line with pictures of frogs had seemed to be ideal for a teaching activity on addition and subtraction in which the context was frogs jumping

in and out of a pond. However Sally had not considered the confusion that might be caused by the number of frogs pictured being different to the number represented by the place on the number line. This activity, in which the frog puppet jumped along the number line to model the subtraction, represented the ordinal relationships between the number of frogs after each jump. However the children may have focused on the cardinal aspect represented by the number of pictures of frogs visible in the set, and this included a frog beneath the zero.

Linda's lesson with her reception (4-5 years) class illustrates the problematic nature of using a hundred square as a representation when teaching addition and subtraction of ten. Linda placed a large hundred square on the floor and the children were directed to sit in a horseshoe shape around it. In the first part of the lesson, Linda used a kangaroo toy on a stick to move around the grid demonstrating adding or subtracting first one and then ten. When moving on to add and subtract ten she asked the children "Does he have to jump a long way or just a short way to add ten?" A child responded that he just has to move down one square.

In the main part of the lesson Linda introduced a 'Tigger' puppet and initially used it to demonstrate adding ten to single digit numbers on the hundred grid. Linda also focused on recording these 'additions' as number sentences of the type '1 + 10 = 11'. She demonstrated how to write an addition number sentence on the white board and discussed the symbols '+' and '='. When Tigger was 'bounced' to demonstrate adding ten, Linda referred to this as 'one step downwards or forwards'. She emphasised that he only had to make one jump in a forwards direction. When later 'Tigger' was bounced to demonstrate taking away ten, it was referred to as moving 'backwards. Children were asked to choose where on the hundred square 'Tigger' should start and Linda asked them where he should be bounced, either to add or subtract ten. The accepted responses were either forwards or backwards.

This activity was problematic on two counts. Firstly, it suggested that the distance travelled by a move of one was the same as that travelled by a jump of ten, albeit in a different direction. The focus on direction was also problematic as the children were all sitting facing the 'hundred square' from different directions so that the jump did not appear 'down' to all of them. At no time did Linda either demonstrate, or get the children to explore, what happens when ten individual moves are made along the grid. This may have helped the children to understand why moving down one square on a hundred grid has the effect of adding ten. Rather, it was demonstrated to them as a procedure, almost like a magic trick, requiring a kangaroo or a 'Tigger' puppet and a hundred square.

DISCUSSION

The choice and use of representations by beginning teachers in this project was seen to be somewhat problematic. The teachers seem to have chosen representations based on superficial appropriateness. They did not fully consider the relationship between what they wanted the children to learn and the mathematics inherent in the

representation. Sally chose a number line with pictures of frogs because she was using the context of frogs jumping into ponds to teach addition and subtraction. The pictures however proved problematic as did her use of jumps on the number line. Sally seemed not to recognise the complexity of the relationship between the ordinal and cardinal aspect of the numbers in her use of the number line. Linda used the hundred square in very procedural fashion and failed to develop any understanding of why bouncing just one square might show an addition of ten.

It may not be surprising that beginning teachers do not always realise the complexity involved in choices to be made about which representations to use in their teaching. It would seem however that this is an important aspect of pedagogy and that they should be helped to focus on their choice and use of representations in order to become more effective teachers of mathematics. In the first two years of my longitudinal study the teachers were given feedback on their lessons using the Knowledge Quartet framework to focus the feedback on mathematical content knowledge and provide a basis for dialogue. The teachers' use of representations was often a key aspect of this feedback and dialogue. In the following two years of the project the teachers' own reflections using the Knowledge Quartet framework are taking over as the driver of their development.

It is hoped that by focusing on their use of representations, initially in dialogue and then in reflections, the teachers will give more consideration to choosing appropriate representations and to the way in which they use them. There is some evidence that this is happening. In her written reflections on her mathematics teaching in the first term of her second year in post Kate wrote:

> When teaching place value I am now trying to be less reliant on a 100 square. While I am still using 100 squares as visual aids and I think the way the numbers are arranged makes them very useful, I tried this week to start with real objects and grouping them in 10s, and we did a lot of 'making' the written number by noting the number of groups of 10 and then the number of ones left over and seeing that this information is given in the number which is written down. When we did work with the 100 square this week we talked about the patterns which can be found in it and why these patterns are there, and some of the Year 2s were able to explain that there were 10 numbers in each row so the tens numbers were at the end of each row.

In the final two years of the study I will continue to consider teachers' use of representation as a key aspect of development. It will be interesting to see whether the teachers will continue to use the Knowledge Quartet framework and focus on their use of representations once they are left to reflect more independently on their teaching.

REFERENCES

Anghileri, J., Beishuizen, M. and van Putten, K. (2002) From Informal Strategies to Structured Procedures: Mind the Gap! *Educational Studies in Mathematics* 49(2) pp. 149-170.

Bruner, J. S. (1974) 'The growth of representational processes in childhood', In *Beyond the Information Given*, London: George Allen and Unwin.

Department for Education and Employment (DfEE) (1999) *The National Numeracy Strategy: Framework for teaching mathematics*, Sudbury: DfEE Publications.

Glaser, B. G. and Strauss, A. L. (1967) *The discovery of grounded theory: Strategies for qualitative research*, New York: Aldine de Gruyter.

Iliada, E., Gagatsis, A. and Delivianni, E. (2005) 'A review of the effects of different modes of representations in mathematical problem solving' MEDCONF: Fourth Mediterranean Conference on Mathematics Education. January 2005, Palermo, Italy.

Rowland, T., Huckstep, P. and Thwaites, A. (2003) 'Elementary teachers' mathematics content knowledge and choice of example' CERME 3: Third Conference of the European Society for Research in Mathematics Education. March 2003, Belaria, Italy.

Rowland, T., Huckstep, P. and Thwaites, A. (2005a) 'The Knowledge Quartet: Considering Chloe' CERME 4: Fourth Conference of the European Society for Research in Mathematics Education February 2005. Sant Felue de Guixols, Spain.

Rowland, T., Huckstep, P. and Thwaites, A. (2005b) 'Elementary teachers' mathematics subject matter knowledge: The Knowledge Quartet and the case of Naomi'. *Journal of Mathematics Teacher Education*, (8) pp. 255-281.

Shulman, L. (1986) Those who understand, knowledge growth in teaching. *Educational Researcher* 15 (2), pp4-14.

Schon, D.A. (1983) *The Reflective Practitioner*, New York, Basic Books.

Wagner, J.: 1997, 'The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation.' *Educational Researcher*, 16 (7) pp. 13-22.

Thompson, I. (2003) 'Place value: the English disease' in Ian Thompson (Ed) *Enhancing primary mathematics teaching*. Maidenhead: Open University Press.

Threlfall, J. (1996) 'The role of practical apparatus in the teaching and learning of arithmetic', Education *Review*, vol. 48, no. 1, pp. 3-12.