SOME REFLECTIONS ON THE PHILOSOPHY OF MATHEMATICS EDUCATION

Stuart Rowlands

Centre for Teaching Mathematics, University of Plymouth

Much reference has been made to Paul Ernest's 'philosophy of mathematics education' to legitimise current trends in mathematics education. This session presents the argument that 1. The 'philosophy' is more a sociology than it is philosophy. 2. The very basis of the 'philosophy' contains a contradiction – that mathematics cannot be separated from its social origins, yet mathematics has a logical necessity that is independent of origin. 3. The 'philosophy' downplays mathematics as a formal, academic discipline in the attempt to promote a child-centred pedagogy. 4. The 'philosophy' makes unwarranted assumptions that have been taken as 'given'. For example, that 'absolutist' or 'Platonist' views of mathematics necessarily implies the transmission model of teaching mathematics.

INTRODUCTION

There lies an assumption in mathematics education that a Platonist, absolutist or objectivist philosophy of mathematics, basically the view that mathematics consists of a body of truths that cannot be doubted, necessarily implies a transmission model of teaching. For example, "adopting an objectivist stance within mathematical philosophy means accepting that mathematical 'truths' exist and the purpose of education is to convey them into the heads of the learners" (Burton, 1995, p.210). Is this assumption warranted? Consider, for example, Plato, the exemplary Platonist and absolutist who advocated and used the dialectic – the Socratic method of questioning leading to the target concept. This is hardly a transmission model of teaching, but more to the point: why should the certainty of the angle property of the plain triangle, for example, imply the teaching of that *truth* as a formal statement with proof as something to recite? I suspect Platonism and absolutism are 'boo' words because they are seen to imply the transmission model with the declaration that the pupil is either right or wrong. For Ernest (Guardian, 2007), being either right or wrong does not allow for shades of opinion.

The learner may be right or wrong, but it is a matter of good teaching practice to enable the learner to realise *why*. This is a pedagogical point, but what has been asserted is the epistemological claim that the learner is neither right or wrong. For example, Jaworski (1994) uses fallibalism re. the angle property of the triangle to justify why the teacher should not proclaim the learner wrong, simply because we can never proclaim with *certainty* that the learner is wrong (see Rowlands and Carson, 2001). If this is merely a question of how the learner sees the problem then fair enough, but the philosophical justification is that we can never be absolutely certain of the truth of a theorem or the falsity of a learner's answer. But can we not proclaim the truth of the angle property of the *plain* triangle, that the three angles equal two right-angles, with certainty? If there is any doubt concerning the angle property then reasons have to be given as to why the angle property may not be true. Scepticism requires justifying the doubt and the giving of criteria for the doubt to cease. Any reference to the failure of the foundationalist programme to establish with certainty the foundations of mathematics is not relevant here (see Rowlands et al, 2001), but this failure has become the reason for the scepticism. Fallibilism, the notion that mathematics is fallible and open to revision, becomes the blanket claim that we can never be certain of anything. But fallibility regarding mathematical proof has to do with domain specificity (Rowlands et al. 2001), such as the angle property of the triangle is restricted to plane triangles, it is not a license for unrestrained scepticism concerning any theorem. Is Euclid's proof of infinite primes merely a belief? Unfortunately, fallibilism in mathematics education has not given any justifiable reason to suppose such knowledge to be mere belief, apart from the failure of the foundationalist programme. Such 'fallibilism' lacks specificity with regard to its scepticism and relies on the failure of foundationalism so as to make the point that we should not regard mathematics as a special body of certain truths. By denying its special status, mathematics as a formal, abstract body of knowledge has been downplayed so as to push for a child-centred pedagogy (e.g. Burton, 1995). This article is an examination of the philosophical basis for such 'fallibilism'.

A CRITIQUE OF ERNEST'S (1991; 1998) TWO BOOKS

This critique is very brief because it has already appeared in Rowlands et al. (2001).

For Ernest (1991), objectivity "*itself* will be understood to be social" (p.42, emphasis added) in that publication is a necessary condition (p.43) and social acceptance a sufficient condition (p.49) for subjective knowledge to become objective mathematical knowledge. This is not objectivity in the sense of logical necessity *from which* the objectivity can be recognised; rather, subjectivity *becomes* objectivity through consensus. The rationale for this is the failure of the foundationalist programme to establish certainty in the foundations of mathematics: take away the certainty of mathematics then you can take away logical necessity as having any role in establishing what is to be accepted – objectivity merely becomes part of that which is accepted (even 'if-thenism' is merely a matter of convention). If the "absolutist philosophies of mathematics have failed to establish the logical necessity of mathematical knowledge" (p.13), then objectivity and consensus become *synonymous*, they are not separate entities from which the former may play a part in establishing the latter. This argument is extremely weak for several reasons:

- What 'absolutist' philosophies (the foundationalist programme) have failed to establish is not logical necessity but *absolute* certainty re. the foundations.
- With perhaps the exception of the most rigorous sceptic, concerns over the foundations of mathematics have little bearing on whether we can be certain of the truth of a theorem that is not subordinate to the foundations (e.g. Pythagoras' theorem). We may follow the protocols of proof, protocols that have been

established and agreed upon in mathematics as a social practice, but the question as to the validity of the proof also entails the proof *itself* - independently of social considerations such as the origin of the protocols or of the consensus itself.

- If publication and agreement constitutes objectivity, then we may have a published 'proof' of a theorem that is actually flawed. Any agreement that the 'proof' is valid will not affect the 'proof' being flawed (just as the change in consensus that the world is flat did not make the world round). Any subsequent discovery that the 'proof' is flawed *presupposes the flaw existed in the first place*.
- If by publication we also include that which has been firmly established, such as the mathematics written in textbooks, then the truth of a theorem is not synonymous with being in a textbook, it is in the textbook because it is true.

Although consensus and authority do play a part in *establishing* what is to count as truth and logical necessity, a distinction has to be made between the former and latter. That distinction has been blurred by Ernest (1991), and so his position is *interesting* due to its tenability. However, a U-turn has been made in Ernest (1998) whereby logical necessity becomes something over and above consensus. For example, "the adoption of certain rules of reasoning and consistency in mathematics mean that much of mathematics follows, without further choice or accident, by logical necessity" (p.248). Why wasn't this said in the first place? Here, logical necessity is not something that is merely social (although the issue has become fudged in the attempt to maintain his earlier position. See Rowlands et al. 2001).

A REVIEW OF ERNEST'S (2007) "THE PHILOSOPHY OF MATHEMATICS, VALUES, AND KERALESE MATHEMATICS"

According to Ernest, the legacy of foundationalism has over-valued the philosophical significance of axiomatic mathematics and undervalued calculation and problem-solving. His argument is that although all three involves deductive reasoning, axiomatic mathematics is held to be the supreme achievement of mathematics; so-much-so that we have a purist 'ideology' (signifying a 'boo') that regards the other two as insignificant and belonging to lesser mortals.

Ernest's objection to this 'purist ideology' is that it is based on a set of values, choices and preferences which have no logical compulsion. As Ernest states, this 'ideology' began with the Greeks, but if we reflect upon the possible reasons why the Greeks elevated abstract mathematics over and above practical mathematics, we might then see a logical compulsion to do so. Although somewhat ontologically indefensible, Plato's Forms may well have been a heuristic to emphasise the distinction between abstract concepts with concepts that are contextually bound. Here was the formation of theoretical objects (such as the geometric straight-line) distinct from the concrete exemplars that once served to represent them (such as the stretched rope, See Carson and Rowlands, 2007). The Forms emphasised a distinction that could have easily collapsed (and still could!). If there was an anxiety that this newly formed mathematics could vanish into something more concrete, then there would have been the logical compulsion to 'elevate' this mathematics.

To strengthen his argument against the purist 'ideology' that elevates proof above calculation, Ernest attempts to formally show similarity in structure by revealing both as a production of sequences of signs. The attempt is problematic. Formal proofs cannot be reduced to sequences of signs because they also contain symbols and deductive rules. Symbols and the concepts to which they refer do not feature in the attempt, and his treatment of deductive rules is suddenly, *without warning*, introduced and with an ambiguity that seemingly saves the attempt from becoming *interesting:* "such transformational sequences [of signs] can represent a deductive proof for a theorem. In this case it consists of a sequence of sentences, each of which is derived from its predecessors by the deductive rules of the system (including the introduction of axioms or other assumptions)" (p.5). Either the deductive rules of the system are separate from the sequences of signs, or they are sequences of signs and nothing more. If the former, then we have something over and above a system of signs, and so the attempt fails with the 'purist ideology' left intact. If the latter, then it remains to be shown how the deductive rules are merely sequences of signs.

The attempt is to show that if both proof and calculation are sequences of signs, then the former should not be elevated above the latter: "The very strong analogy and structural similarities between proof and calculation, including their inter-convertibility, challenges the preconception often manifested in philosophical and historical accounts of mathematics that proof is somehow intellectually superior to calculation in mathematics" (p. 6). Who claims that proof is *intellectually* superior? It is altogether quite a different claim, however, to say that deductive proof is perhaps the most remarkable and brilliant form of human achievement. This claim is not undermined by showing that proof is a sequence of signs, and the attempt to undermine has the same logical form as the following analogy: Suppose A is a doctoral thesis and B a third-year undergraduate dissertation. We cannot compare A with B and say A is a higher form of achievement precisely because both use language. Similarly, we cannot compare proof with calculation because both are sequences of signs. Proof is thus seen to be overvalued and (references not forthcoming) this constitutes a prejudice (and the ideology 'racist' p. 11). However, valuing one thing above all else does not necessarily mean undervaluing anything else.

Valuing formal, abstract mathematics is seen to be racist because it appears to undermine cultural values. The mistake here is to confuse epistemological claims with equity issues, and the sociology of Ernest does just that. Consider the following critique of Ernest (1991) by Matthews, who is criticising the moral superiority of constructivism:

> The well elaborated social constructivism of Paul Ernest certainly includes ethical and political dimensions. For instance, he says

> Each culture, like each individual, has the right to integrity. Thus, the system of values of each culture are *ab initio* equally valid. In absolute terms, there is no basis for asserting that the values of one culture or society is superior to all others. It cannot be asserted, therefore, that Western mathematics is superior to any other form because of its greater power over nature. (Ernest 1991, p. 264)

> It is important that the ethical and political arguments for different multiculturalist positions not be confused with epistemological arguments. In the foregoing quotation, the epistemological conclusion that different values, much less systems of mathematics, are equally valid simply does not follow from the ethical premise that each culture has a right to integrity. The ethical, or political, premise can be agreed to without any commitment at all to the relativistic epistemological conclusion. The right to individual or cultural integrity is simply not dependent upon individual beliefs, or cultural norms, being right. Being silly does not nullify one's right to respect from others. (Matthews, date unknown, p. 2).

Ernest's unwillingness to separate the theoretical objects of mathematics with social concerns, interests and practitioners has created a sociology of mathematics in which epistemological claims become confused with equity issues and language. This isn't to say that a sociology of mathematics isn't possible or undesirable, but the reduction of the philosophy of mathematics to a sociology of knowledge is part of the agenda to undermine the learning of formal, academic mathematics for something deemed more relevant:

> "I disagree with people who think that mathematics is neutral and value-free," he [Ernest] says. "It is human made, therefore culturally influenced, and this makes social justice central and relevant in mathematics. We need to think of different ways of contextualising maths to take multi-culturalism, racism and sexism into account. . ." (Guardian, 2007).

This isn't simply about the cultural domination by western societies, equity issues in the history of the profession or differences in learner cognition; this is about downplaying formal, academic mathematics and replacing it with something else deemed less racist and sexist. If we take this argument in the context of the whole article in which it appears, then the learning of formal, academic mathematics for many 14-16 year olds ought to be replaced with something like soduku. This gives fuel to the impression that many mathematics educationalists simply do not like formal, academic mathematics.

CONCLUSION

There is a sense in which, once created, the theoretical objects of mathematics exist independently of the creator or the consensus of the community. The angle property of the plane triangle is independent of any one person, and appears in textbooks because it is true. It is also independent of ideological persuasion. It is not masculine, Eurocentric or oppressive. Ernest (1991; 1998; 2007) argues that a philosophy of mathematics should not and cannot separate mathematics from its social origins, but it can! Yes, mathematics is a social, historical and socio-cultural product, but it is also much more than this. It is a subject without a knower. Ernest's relativism reduces philosophy to a sociology (of 'knowledge'): attempting to formally discredit the certainty of mathematics (with reference to philosophy) so as to establish mathematics as a social product and nothing more.

Mathematics is necessary knowledge. Why downplay this fact or fudge the issue with concensus? Why not embrace this fact as a supreme human achievement? Why not embrace the theoretical objects of mathematics as tools of the mind that can enable the child's mind to develop independently of everyday or concrete considerations, providing an educative break from the turmoil of the everyday? Why not mediate these tools of the mind in a way that can enable the most concrete thinkers to think purely in the abstract in imaginative but rule governed ways? Why not induct learners into the exotic landscapes of the mind that formal, academic mathematics offers (by this I do not mean the pedestrianised mathematics of the National Curriculum, such as Shape and Space or mind-numbing Data Handling)? Why not arouse the mind to life with formal, academic mathematics to which all learners have a right to know?

REFERENCES

Burton, L. (1995) Moving towards a feminist epistemology of mathematics, in: P. Rogers and G. Kaiser (Eds) Equity in mathematics education (London, Falmer).

Carson, R. & Rowlands, S. (2007) Teaching the conceptual revolutions in geometry, Science & Education, 16, 921-954.

Ernest, P. (1991) The philosophy of mathematics education. London, Falmer.

Ernest, P. (1998) Social Constructivism as a philosophy of mathematics. New York: SUNY.

Ernest, P. (2007) The philosophy of mathematics, values, and keralese mathematics. Available online at: http://www.people.ex.ac.uk/PErnest/pome20/index.htm (accessed November 2007).

Guardian (2007) Who needs maths? Available online at: http://education.guardian.co.uk/schools/story/0,,2209769,00.html (accessed Nov. 07)

Matthews, M. R. (date unknown), Constructivism in Science and Mathematics Education. Available online at: http://wwwcsi.unian.it/educa/inglese/matthews.html (accessed Nov. 07).

Rowlands, S., & Carson, R. (2001) The contradictions in the constructivist discourse. Available online at: http://www.people.ex.ac.uk/PErnest/pome14/rowlands.pdf

Rowlands, S. Graham, E. & Berry, J. (2001) An objectivist critique of relativism in mathematics education, Science & Education, 10, 215-241.