A SYNTHESIS OF TAXONOMIES/FRAMEWORKS USED TO ANALYSE MATHEMATICS CURRICULA IN PAKISTAN

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The paper reports the development of a framework for analysing written and tested curricula. Most of the existing taxonomies/frameworks (e.g. proposed by Bloom, 1956; Smith et al 1996; Biggs, 1995; and Porter, 2002) recognize overlapping as well as discrete categories of cognitive processes. Anderson & Krathwohl (2001) have added another dimension, i.e. categories of knowledge. This paper synthesises these existing taxonomies/ frameworks in order to develop an integrated framework which is appropriate for the mathematics curriculum. The paper will also discuss how the new framework is being used in analysing examination curricula of secondary school mathematics in Pakistan. Purpose of this analysis is to develop a common 'non-routine' test paper for the students who have studied different curricula at secondary school level.

THE REVIEW OF TAXONOMIES/FRAMEWORKS

Recognizing some limitations of Bloom's taxonomy (1956), Smith et al (1996) have suggested modifications to it in order to make it compatible with the purpose of assessing students' understanding in mathematics. They have elaborated the category of 'application' of Bloom's taxonomy by replacing it with the three new application categories 'routine use of procedure', 'information transfer', and 'application in new situation'. Also they grouped first three categories — 'knowledge' (factual), 'comprehension' and 'routine use of procedure'— into a first level of thinking and 'information transfer' and 'application in new situation' into a second level of thinking. They replaced the categories of 'analysis' and 'synthesis' of Bloom's taxonomy with mathematics specific categories of 'justifying and interpreting' and 'implication, conjecture and comparison' and put these categories with the category of 'evaluation' into a third level of thinking.

In order to analyse content of instruction (curriculum, textbooks, and teaching), Porter (2002) has used a framework in which categories of 'knowledge and comprehension' are merged into a single category of 'memorize (facts, definitions and formulas)'. However the next categories of his framework, 'solve routine questions' and 'solve non-routine questions', are similar to Smith et al's categories of 'routine use of procedure' and 'application in new situation' respectively.

In the light of Smith et al's level of thinking, if one reviews Bloom's and Porter's taxonomies/frameworks it will be noticeable that there is some compatibility among them. Overall it may be implied that the first level of mathematical thinking is 'assimilation of knowledge' which involves knowing about mathematical facts, formulas and definitions; drawing meaning from them in familiar contexts/situations; and solving routine questions using them.

Next level is 'transformation of knowledge' which is required when one has to recognize meaning of the assimilated knowledge in unfamiliar contexts or in solving non-routine questions. It may require transferring of information from one form to another, recognizing the applicability of a formula or method in a different and unusual context or recognizing non-applicability of a general formula/ definition into a particular context (Smith et al, ibid).

The most advanced level is 'generation or validation of knowledge' which is deployed when one has to conjure, prove, justify, generalize, evaluate and "restructure the information into a new whole" (Smith et al, ibid, p.71).

The above synthesis of Bloom's, Smith et al's and Porter's taxonomies and frameworks, based on levels of thinking, makes them comparable to Biggs' (1995) SOLO (Structure of Observed Learning Outcomes) taxonomy.

Assimilation involves knowing isolated bits of information which goes along well with 'Pre-structural level' of the SOLO taxonomy. The next layer of assimilation is pertinent to drawing meaning of the assimilated discrete facts in familiar context or solving routine questions using them. At this stage one has to make some basic connections among isolated bits of information to draw meanings from them. This is called the 'Uni-structural level' of the SOLO taxonomy.

For transformation one uses the assimilated knowledge in unfamiliar situations or in solving non-routine questions (the questions for which the learners need to construct novel methods to find solutions). In order to perform at this level more interconnected schemata of knowledge are required which is why this stage is compatible with the 'multi-structural level' of the SOLO taxonomy.

For generation and validation students have to have a well-structured knowledge schema to be able to conjure, prove, justify, generalize and evaluate. This level matches up with the 'relational level' of the SOLO taxonomy. However, for going beyond this level to restructuring the information into a new whole requires another layer of thinking which is compatible to 'extended level' of the SOLO taxonomy.

Apart from taxonomy/framework discussed above, Anderson & Krathwohl (2001) have suggested another dimension in Bloom's original taxonomy. The new dimension has four types of knowledge which are factual, procedural, conceptual and meta-cognitive. They argue that each piece of curriculum, item of instruction or assessment can be placed at the intersection of the cognitive process domain and the knowledge domain.

Following on Anderson & Krathwohl (2001), my ongoing study attempts to use a two-dimensional framework to analyse exam questions, textbook tasks and instructional objectives of the secondary mathematics curricula in Pakistan. One dimension of the framework has categories of cognitive processes specific to mathematics obtained through the synthesis of above mentioned frameworks/taxonomies. The other dimension of the framework includes types of knowledge suggested by Anderson & Krathwohl (2001). The reason for inclusion of these types of knowledge is their relevance with mathematics. For example, in mathematics education factual knowledge could be taken to mean something about which the community of mathematics educators have consensus.

Procedural knowledge is all about procedure which is a set of actions and an accepted ways of doing mathematics. It is assumed that the person who knows and performs a particular mathematical procedure (e.g. solving simultaneous equations or carrying out geometrical construction) does not necessarily know the underlying principle (concept) of the procedure.

Conceptual knowledge is the knowledge of the concepts (generalization, principle, or theorem). In mathematics this type of knowledge has been viewed as distinct but integrated from/with procedural knowledge (Hiebert, 1986).

Meta-cognitive knowledge includes knowledge of general strategies used in doing different tasks, conditions under which these strategies might be used, extent to which the strategies are effective (Pintrich et al., 2000). This type of knowledge is considered as the core of problem solving in mathematics (Schoenfeld, 1992)

This paper presents few examples of tasks, from written and tested curricula of public examination board and O-level examinations in Pakistan, that require students to generate different levels of cognitive processes using different types of knowledge.

In the tables below Examination 1 refers to the questions which have appeared in the Public Board examinations and Textbook 1 to the textbook which is used for the preparations of those examinations whereas Examination 2 refers to the O-level examinations and Textbook 2 to the textbook which is often used for its preparation.

Table 1 gives examples of assimilation tasks. The main activities in these tasks are recalling (Q1, Q4); understanding (Q3); or applying (Q2) in a familiar situation.

Table 1:	Cognitive process: Assimilation
Factual	*Q1: Pick out the correct answer from the brackets:* *The circle passes through the vertices of triangle is called ---------. (Circumscribed, inscribed or escribed) (Examination 1, 2001)*
Procedural	*Q2: Draw a tangent from a given point A to the circle with centre O, using only straight edge and compasses. (Textbook 1)*
Conceptual	*Q3: What is meant by similarity? In what conditions are two triangles considered to be similar? (Curriculum content)*
Meta-cognitive	*Q4: Introduction of some of the problem solving strategies such as 'using tabulations', 'making a supposition' and 'eliminating unlikely guess', 'making logical deduction'. (Textbook 2)*

Table 2 presents examples of Transformation of knowledge tasks. Q5 is based on knowledge of scientific notation which appears in the textbook as factual knowledge. The question expects students to go beyond the factual knowledge and recognize applicability or non-applicability of that information into a particular context.

Q6 is based on a cubic expression. Curriculum 1 indicates that the students are provided with experience of solving quadratic equations. They are also familiar with the ways of manipulating and factorizing algebraic expressions. Although Q6 demands students to solve cubic equations which would not be familiar to the students, it is assumed that they might be able to transform the cubic expression into the products of a linear and a quadratic expression and be able to the find values of x. It is expected that they will notice that the linear equations have one solution and the quadratic equations have two solutions. On the basis of this observation, I infer that the students will realize that the cubic equations have three solutions. So it is assumed that Q6 is a transformation task for students appearing for Examination 1.

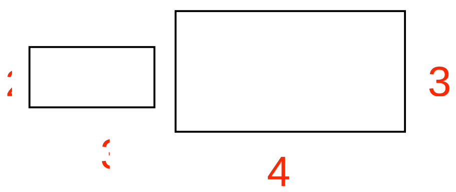
Table 2:	*Cognitive process: Transformation*
Factual	*Q5: Consider each of the following and determine whether it would be appropriate or convenient to express them in the standard form. (a) the monthly salary of a teacher (b) the number of grains of sand on a beach.(c)…(d)…. (Textbook 2)*
Procedural	*Q6: Solve the equations:* $x^3 + x^2 - 9x = 9$ *(Could be seen as a 'transformation' question)*
Conceptual	*Q7: Which of the following rectangles is more "like a square"? Give mathematical reasons for that. (Could be seen as a 'transformation' question)* 
Meta-cognitive	*Q8: There are worked out examples of using meta-cognitive strategies. After each example there are questions to help students reflect on the strategies they used. For example: Can you solve the problem using another problem solving strategies? (Textbook2)*

Table 3 provides examples of tasks of creation/validation which may include restructuring the knowledge into new whole. Q9 is chosen to model the situation where restructuring seems to be possible. This question appeared after providing students with practice in simplifying mathematical expressions involving indices, which presumably required only knowing and applying factual knowledge. This question also involves different cognitive processes associated with this level like generating a pattern of repetitive multiplication, observing the pattern, inferring and making conjectures, checking those conjectures and finally developing a generalization.

Students working at this level are also expected to have developed their expertise of using procedures and to be able to identify flaws in a routine procedure. Q10 offers an example how this opportunity is provided to the students.

Table 3:	Cognitive process: Creation and validation
Factual	*Q9: What is the last digit of the number* 9^{1997} *? (Textbook for O-level examinations)*
Procedural	*Q10: Consider the following simultaneous equations:* $2x + y = 6$---(1) $\qquad x = 1 - \frac{1}{2}y$----(2) *Substitute (2) into (1) ;* $2(1 - \frac{1}{2}y) + y = 6$ $2 - y + y = 6$ *Therefore 2=6!* *Do you know where the problem lies?* *(worked example in Textbook 2)*
Conceptual	*Q11: If Rectangle A is more like a square than rectangle B so it can be considered that the property of "square- ness" is more in rectangle A, than what it is in rectangle B. How could you define mathematically the property of "square-ness"? (Could be seen as a 'Creation' question)*
Meta-cognitive	*Q12: Discuss the merit of the problem solving strategies (given earlier) with reference to the particular problem. (Textbook 2)*

CONCLUSION

In the second part of this paper I have identified typical examples in order to understand the applicability the new framework described in the first part. However, it is acknowledged that the nature of knowledge which is required to solve rich mathematical tasks is integrated and multifaceted. At the level of assimilation different types of knowledge, to some extent, could be recognized but as one goes towards the more advance cognitive level the boundaries among the types of knowledge get blurred. Particularly working satisfactorily at the Construction level students needs to bring in not only different types of knowledge but also knowledge from different mathematical content. However the knowledge dimension of the framework is still useful in inferring which type of knowledge is more emphasised in the curriculum.

In Pakistan textbooks and past examination papers play a dominant role in classroom teaching and learning; that is why in this analysis I have only considered the experiences which students receive from written and tested curricula. However there are always individual differences among students in their ways of interpreting a task.

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REFERENCES

Anderson, L. W., & Krathwohl, D. R. (Eds.). (2001). A taxonomy for learning, teaching, and assessing: A revision of bloom's taxonomy of educational objectives. New York: Longman.

Biggs, J. (1995). Assessing for learning: Some dimensions underlying new approaches to educational assessment. The Alberta Journal of Educational Research, 41(1), 1-17.

Bloom, B. S. (1956). Taxonomy of educational objectives, the classification of educational goals–Handbook I: Cognitive domain. New York: McKay.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum.

Pintrich, P. R. (2000). The role of goal orientation in self-regulated learning. In: M. Boekaerts, P. R. Pintrich and M. Zeidner, Editors, Handbook of self-regulation: Theory, research, and applications, Academic Press, San Diego, pp. 451–502.

Porter, A. C. (2002). Measuring the content of instruction: Uses in research and practice. [Electronic version]. Educational Researcher, 31(7), 3.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), Handbook for research on mathematics teaching and learning (pp. 334-370). New York: MacMillan.

Smith, G., Wood, L., Coupland, M., & & Stephen, B. (1996). Constructing mathematical examination to assess a range of knowledge and skills. International Journal of Mathematical Education in Science and Technology, 21(1), 65-77.