ASSESSING THE STRUCTURE AND SENSITIVITY OF THE BELIEF SYSTEMS YIELDED BY THE REVISED MATHEMATICS-RELATED BELIEFS QUESTIONNAIRE

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This is the third of three papers in which we describe how the mathematics-related beliefs questionnaire (MRBQ), developed at the University of Leuven (Op't Eynde and De Corte, 2003), can be adapted for use in Spanish and English educational contexts. In earlier papers we showed that the MRBQ could be refined to yield four reliable scales and ten subscales for both Spanish and English Students (Diego-Mantecón et al., 2007), and that the four scales highlighted a number of differences related to culture, age and gender (Andrews et al., 2007). In this paper, we examine the structure of student belief systems as reflected by the interrelations of the subscales. In so doing we present further evidence of the revised MRBQ's sensitivity to nationality, age, and gender.

INTRODUCTION

The aim of our study is to adapt the mathematics-related beliefs questionnaire (MRBQ) to the Spanish and English educational contexts. The original MRBQ was devised by Op't Eynde and De Corte (2003), from a warranted theoretical perspective, for assessing student beliefs about mathematics in Flanders, but there was no evidence suggesting it was transferable to other contexts. Moreover, only two of the four scales generated by the original factor analyses provided satisfactory levels of reliability, and no factor analytic had been made to determine any subscales. Our first paper (Diego-Mantecón et al., 2007) details how the questionnaire was amended for Spanish and English students in order to yield four reliable scales and ten reliable subscales. Our second paper furthers this research (Andrews et al., 2007) and describes the degree of sensitivity of the four identified scales to nationality, age and gender. In this current and third paper we aim to examine the structure of the ten subscales to observe how they are interrelated, and we analyse the subscale scores in order to provide further evidence of the revised MRBQ's sensitivity to nationality, age, and gender.

THEORETICAL FRAMEWORK

Despite agreement on the influence of socio-cultural factors on beliefs, the extent to which mathematics beliefs are culturally distinguished is still unclear. On the one hand, studies carried out in the USA (e.g. Garofalo, 1989), Finland (e.g. Pehkonen, 1995), and Hong Kong (e.g. Lam et al., 1999) suggest that students from different countries share some beliefs (e.g. mathematics is the practising of rules); indeed, results are transferred from one culture to another without question (Pehkonen, 1995). On the other hand, without identifying clear patterns, international comparative studies indicate some beliefs may be universally held while others may be culturally located (e.g. Berry and Sahlberg, 1996; Pehkonen and Tompa, 1994). These contradictory results seem to be associated with the fact that comparative studies have failed to acknowledge the structural aspect of beliefs. Berry and Sahlberg (1996), for instance, grouped item scores according to predetermined categories which they described as factors, rather than outcomes of systematic factor analyses. Pehkonen and Tompa (1994) used factor analysis to reduce large numbers of items to ‘compact’ proportions, but they essentially ignored the structural implications and focused on a comparison of individual item scores. From another standpoint, those studies which have focused on the systematic analysis of belief structures were undertaken in a single nation context and with students of a specific age (e.g. Op’t Eynde and De Corte, 2003; Lazim et al., 2004), without therefore attending to explicit comparative evaluations. In this light, our study aims to use the MRBQ to identify belief structures sensitive to different cultural contexts, ages and genders.

METHOD

The original instrument, comprised of 40 items, was augmented by a further 33 drawn from various sources in the literature. All the items were closely examined by several colleagues in both England and Spain in order to establish conceptual and linguistic equivalence. The revised questionnaire was administered to 12- and 15-year-old students in one school near Cambridge, and three near Santander.

RESULTS

As we presented in our first paper (Diego-Mantecón et al., 2007), a principal component analysis with ‘varimax rotation’ generated four main factors: **teacher’s role** (factor 1, $\alpha = 0.921$), **mathematical competence** (factor 2, $\alpha = 0.915$), **mathematical relevance** (factor 3, $\alpha = 0.875$), and **mathematics as an inaccessible and rote-driven subject** (factor 4, $\alpha = 0.764$). A second analysis of each of the four factors identified ten further subfactors. Factor 1 yielded two reliable subfactors; the first subfactor ($\alpha = 0.924$) alludes to the ways teachers attend to their students’ meaningful learning, and the second subfactor ($\alpha = 0.734$) addresses affective dimensions of teacher interest in the student.

Factor 2 suggested three subfactors: the first subfactor ($\alpha = 0.917$) concerns student perception of enjoyment in mathematics, the second subfactor ($\alpha = 0.812$) relates to absolute mathematical competence (e.g. I will do well in maths this year); while the third subfactor ($\alpha = 0.667$) addresses the issue of comparative mathematical competence (e.g. by doing the best I can in maths I try to show my teacher that I’m better than other students).

The analysis of factor 3 showed a three subfactor solution. The first subfactor ($\alpha = 0.821$) relates to the personal relevance of mathematics (e.g. maths has no relevance to my life), while the second ($\alpha = 0.814$) asserts the collective relevance of mathematics (e.g. maths is used all the time in people’s daily lives). The third subfactor ($\alpha = 0.741$) concerns student perception of the different strategies in the learning of mathematics and problem-solving (e.g. time used to understand why a solution works is time well spent).

Factor 4 generated two negatively orientated subfactors: the first subfactor ($\alpha = 0.741$) was labelled ‘inaccessible knowledge’, because it emphasises mathematics as something inaccessible to all but the able child. The second subfactor ($\alpha = 0.615$) concerns a view of mathematics as a fixed body of knowledge which requires little but a good memory, and thus it was named ‘rote knowledge’.

Assessing the structure of the belief systems

In accordance with our objective of examining the structure of student belief systems, Spearman correlations were undertaken between factor scores. Table 1 shows high correlations: in particular, factor 1 correlates with factor 2 (r = 0.516), factor 3 (r = 0.563) and factor 4 (r = -0.310). These indicate that students with favourable beliefs about the role of their teacher (factor 1) are more likely to believe themselves competent (factor 2), to acknowledge the relevance of mathematics (factor 3), while rejecting the notion of mathematics as inaccessible rote (factor 4). There is also a high correlation (r = 0.664) between factors 2 and 3, which suggests that students who feel positive about their mathematical competence are also those who perceive mathematics as relevant. All these correlations compared favourable with those in the original study (Op't Eynde and De Corte, 2003), which also showed the highest correlation as being between factors 2 and 3 (r = 0.480).

Table 1. Spearman correlations calculated between the four factor scores, and between the ten subfactor scores. All the correlations, apart from those denoted with*, were significant at $p < 0.005$. The figures in bold show correlations above ± 0.3.

	Teacher facilitator of learning	Teacher interest	Enjoyment	Absolute competence	Relative competence	Personal relevance	Collective relevance	Strategies for learning	Inaccessible knowledge
	Factor 1		Factor 2			Factor 3			Factor 4
Teacher interest	**0.661**					Factor 2	Factor 3	Factor 4	
Enjoyment	**0.588**	**0.354**				**0.516**	**0.563**	**-0.310**	Factor 1
Absolute competence	**0.387**	0.179*	**0.815**				**0.664**	-0.210	Factor 2
Relative competence	0.223	0.049*	**0.438**	**0.579**				**-0.358**	Factor 3
Personal relevance	**0.535**	**0.379**	**0.618**	**0.452**	0.296				
Collective relevance	**0.530**	**0.313**	**0.646**	**0.472**	0.297	**0.707**			
Strategies for learning	**0.425**	0.279	**0.471**	**0.335**	0.164*	**0.538**	**0.527**		
Inaccessible knowledge	**-0.333**	**-0.403**	**-0.425**	-0.265	-0.007*	**-0.482**	**-0.394**	**-0.330**	
Rote knowledge	-0.076*	**-0.316**	-0.084*	0.026*	0.254	-0.127*	-0.122*	-0.129*	**0.552**

To provide further insights into the belief systems structure, Spearman correlations were calculated between subfactors. These show that some subfactors are highly correlated even though their actual factors are not (see Table 1). For instance, although factors 2 and 4 show a low correlation (r = -0.210), subfactor 2.1, 'enjoyment', reasonably correlates (r = -0.425) with subfactor 4.1 'inaccessible knowledge'. This negative correlation indicates that those students who perceive mathematics as inaccessible are also those who do not enjoy the intellectual challenge of mathematics.

In general, the Spearman correlation test provides strong consistency for the structural aspect of mathematics-related beliefs, as it indicates high correlations between almost all the subfactors; particularly, there are high correlations between 'teacher facilitator of learning', 'teacher interest', 'enjoyment', 'absolute competence', 'personal relevance', 'collective relevance', 'strategies for learning' and 'inaccessible knowledge'. There are only two subfactors, 'relative competence' and 'rote knowledge', which show low correlations. 'Relative competence' is only correlated strongly with 'enjoyment' ($r = 0.438$) and 'absolute competence' ($r = 0.579$), which are subfactors belonging to the factor 'personal relevance'. This lack of correlation could be explained by 'relative competence' being a subfactor which refers to particular personal goals (e.g. I try hard in maths to show the teacher and my fellow students how good I am), which may not be directly associated with mathematics or the teacher's role. Further research is required, however, to explain the lack of correlation of the subfactor 'rote knowledge', which shows only two correlations above ± 0.3; one with 'inaccessible knowledge' ($r = 0.552$) and the other with 'teacher interests' ($r = -0.316$).

Evaluating the subfactors

In accordance with our objectives of presenting further evidence of the revised MRBQ's sensitivity to nationality, age, and gender, subfactor scores were calculated for each student. On a six-point scale a score of 3.5 would correspond to neutrality.

Regarding nationality, our previous analysis of the factors (Andrews et al., 2007) showed that Spanish students, irrespective of age and gender, were significantly more positive on the three positively directed factors and significantly more negative on the negatively directed factor than English students. This tendency is also held for the subfactors, apart from that of 'relative competence'. As Table 2 shows there is no significant nationality-related difference for this subfactor; indeed both Spanish and English students tend to be equally negative regarding their 'relative competence'. The analysis of the subfactors produced other interesting results; for example, Spanish students were of the order of a point more positive than their English counterparts regarding 'enjoyment', and of the order of half a point more positive about their 'personal competence'.

With respect to student age, the analysis of the factors showed that across three of the four factors students at age 12 were significantly more positive than at age 15. The only factor which indicated no significant age-related difference concerned factor 4, 'mathematics as an inaccessible and rote-driven subject' (Andrews et al., 2007).The analysis in Table 2 reveals, however, that there is no age-related difference for subfactor 4.2 'rote knowledge', and that significant differences are only held by subfactor 4.1 'inaccessible knowledge'. Students at age 12 are significantly ($p < 0.0005$) more negative ($M = 4.81$) than at age 15 ($M = 4.53$) concerning their perception of mathematics as inaccessible. 'Rote knowledge' is thus the only one out of ten identified subfactors with no age-related differences. Regarding the remaining subfactors, younger students are always of the order of half a point more positive than older students.

Table 2. Means, standard deviation, t scores and associated probabilities for each subfactor by student nationality, age and gender. The figures in bold show significant correlations at p < 0.005.

	Country	Mean	SD	t / p	**Age**	Mean	SD	t / p	**Gender**	Mean	SD	t / p
Teacher facilitator of learning	S	2.42	1.03	5.272/	15	2.99	1.10	8.786/	F	2.61	1.00	0.634/
	E	2.87	0.98	**0.000**	12	2.27	0.86	**0.000**	M	2.56	1.06	0.525
Teacher interest	S	2.40	1.21	3.092/	15	2.71	1.27	3.376/	F	2.42	1.24	1.795/
	E	2.73	1.27	**0.002**	12	2.37	1.20	**0.001**	M	2.60	1.23	0.073
Enjoyment	S	2.57	0.90	12.37/	15	3.21	1.02	6.836/	F	3.10	1.07	4.705/
	E	3.52	0.95	**0.000**	12	2.66	0.96	**0.000**	M	2.72	0.94	**0.000**
Absolute competence	S	3.08	0.96	7.085/	15	3.54	1.01	5.940/	F	3.49	0.96	5.286/
	E	3.62	0.90	**0.000**	12	3.08	0.90	**0.000**	M	3.09	0.95	**0.000**
Relative competence	S	3.67	1.13	0.726/	15	3.95	1.04	6.222/	F	3.92	1.05	5.904/
	E	3.60	1.04	0.468	12	3.41	1.09	**0.000**	M	3.41	1.09	**0.000**
Personal relevance	S	2.06	0.84	3.877/	15	2.52	0.97	7.994/	F	2.23	0.96	1.267/
	E	2.40	1.14	**0.000**	12	1.92	0.89	**0.000**	M	2.13	0.97	0.206
Collective relevance	S	1.99	0.73	6.961/	15	2.48	0.87	8.487/	F	2.22	0.82	1.408/
	E	2.48	0.86	**0.000**	12	1.94	0.68	**0.000**	M	2.13	0.80	0.160
Strategies for learning	S	2.06	0.68	5.396/	15	2.36	0.74	5.528/	F	2.20	0.73	0.689/
	E	2.40	0.79	**0.000**	12	2.04	0.71	**0.000**	M	2.16	0.74	0.491
Inaccessible knowledge	S	4.79	0.82	4.420/	15	4.53	0.77	4.262/	F	4.69	0.75	0.416/
	E	4.49	0.78	**0.000**	12	4.81	0.84	**0.000**	M	4.67	0.88	0.678
Rote knowledge	S	3.94	0.82	3.293/	15	3.92	0.72	1.606/	F	4.00	0.72	3.903/
	E	3.73	0.64	**0.000**	12	3.82	0.81	0.109	M	3.75	0.79	**0.000**

Finally, concerning gender, our previous analysis showed that girls are significantly less positive in their beliefs about ‘personal competence’ than boys, but also they are more negative than boys for the negative subfactor ‘mathematics as inaccessible rote’. Table 2 confirms that for the three subfactors of ‘personal competence’, girls are significantly less positive than boys; that is, they enjoy mathematics less, and feel less positive about their personal and relative competence. However, the results show significant differences for only one of the two subfactors of factor 4. There are differences at $p < 0.0005$ regarding ‘rote knowledge’; significantly more than boys ($M = 3.75$), girls ($M = 4.00$) reject the belief that mathematics is a fixed body of knowledge which requires good memory. But there is no gender-related difference regarding ‘inaccessible knowledge’; both girls and boys, irrespective of age and nationality, equally reject the notion of mathematics as an inaccessible and elitist subject.

This analysis of the subfactors has allowed us to present more precise results than those of the factors. However, caution should be exercised in interpreting such data as the combined effects of the background variables, which are not presented here, may offer more compelling explanations.

CONCLUSIONS

Our study has shown that the original instrument devised by the University of Leuven (Op't Eynde and De Corte, 2003) could be further developed. As a consequence of this development, the revised instrument has allowed us to identify a number of common factors and subfactors in individuals from different educational contexts, ages and genders. In particular, this paper has provided further evidence of the structural properties of beliefs, since the findings show high correlations between the systems. No studies, as far as we are aware, have identified such consistent structures. We have also demonstrated the effectiveness of the systems in detecting differences related to nationality, age, and gender. Notably, these results suggest that students' mathematics-related belief systems appear to transcend European cultural boundaries: this has not been acknowledged before given the lack of attention to belief structures in international comparative studies. Nevertheless, a confirmatory analysis is necessary to provide further consistency for the revised MRBQ.

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