# PROSPECTIVE MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE OF DEFINITE INTEGRAL: THE PROBLEM OF LIMIT PROCESS


| Hatice Akkoç | Sibel Yeşildere | Fatih Özmantar |
|---|---|---|
| Marmara Üniversitesi | Dokuz Eylül Üniversitesi | Gaziantep Üniversitesi |
| TÜRKİYE | TÜRKİYE | TÜRKİYE |


*This study investigates prospective mathematics teachers' pedagogical content knowledge (PCK) of definite integral. Considering the notion of PCK as described by Shulman (1986, 1987), we will investigate prospective mathematics teachers' knowledge of student difficulties in relation to the limit process to define definite integral. For that purpose, four prospective mathematics teachers were observed during their micro-teaching and were interviewed afterwards. Micro-teaching videos, interview transcripts, prospective teachers' lesson plans and teaching notes were analysed. In this presentation, we will discuss how prospective teachers addressed student difficulties for the limit process when constructing the area under a curve from the sum of rectangular areas and consider the implications in terms of PCK.*

## LıTERATURE REVIEW

In this paper, we focus on the prospective teachers' pedagogical content knowledge (PCK) of definite integral. PCK is one of three types of content knowledge that Shulman (1986) describes, the other two being subject matter knowledge and curriculur knowledge. In another paper, Shulman (1987) proposes pedagogical content knowledge as an important domain of teachers' knowledge together with content knowledge, general pedagogical knowledge, curricular knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of the philosophical and historical aims of education. Shulman (1987) emphasises that "pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (p. 8). He defines pedagogical content knowledge as the "subject matter for teaching".

For the last two decades, mathematics education literature has focused on teachers' and prospective teachers' pedagogical content knowledge (such as Carpenter, Fennema, Peterson & Carey, 1988; An, Kulm, & Wu, 2004; Cha, 1999; Winsor, 2003). These studies investigated various aspects/components of PCK as described in different ways by Shulman (1987) and Grossman (1990). Shulman (1987) puts forward two components in his definition of PCK: knowledge of students understanding, and the use of representations and strategies for teaching particular topics. Grossman (1990) proposes four components of PCK: knowledge of strategies and representations for teaching particular topics; knowledge of students' understanding, conceptions, and misconceptions of these topics; knowledge and beliefs about the purposes for teaching particular topics; and knowledge of curriculum materials available for teaching.

## THE RESEARCH

In this study we will focus on a particular aspect of PCK: *knowledge of students' understanding of and difficulties with specific mathematics topics*. This aspect is particularly important for PCK since it is defined as the knowledge of how to represent and formulate the subject that makes it comprehensible for students (Shulman, 1986) and this requires an understanding of how students conceptualise various mathematical concepts and the difficulties they might have. This aspect of PCK will be investigated in the context of definite integral with special attention to the limit process when constructing the area under a curve from the sum of rectangular areas.

Limit is one of the important concepts in calculus because of its use to construct other mathematical ideas. Bezuidenhout (2001) states that students' failure to express meaningful ideas in calculus, to a large extent, is due to inappropriate and weak mental links between knowledge of '*limit*' and knowledge of other calculus concepts such as '*continuity*', '*derivative*' and '*integral*'. For the concept of definite integral, limit process is essential for constructing the limit of the sum of rectangular areas under a curve. Orton (1983) investigated students' understanding of the limit process to define the area under a curve. He asked whether it was possible to obtain an exact answer for the area under the curve $y = x^2$ by taking more and more rectangles under the curve. Out of 110 students, only 10 students stated that a limiting process was required. 69 students stated that by taking more and more rectangles under the curve they could obtain better and better approximations but such a procedure would never produce the exact answer. Considering the students' difficulties as reported in the literature on the limit process for definite integral, the following research question is formulated:

*What are the knowledge of prospective mathematics teachers about students' understanding of and difficulties with the limit process when constructing the area under a curve from the sum of rectangular areas to define definite integral?*

## METHODOLOGY

This paper is part of a wider study which investigates the development of prospective mathematics teachers' PCK during a teacher education program in a university in Turkey. This one and a half year program admits students graduated from mathematics departments. Participants take general and content specific pedagogy courses and do teaching practices in schools. Content specific pedagogical courses include "Instructional Methods in Mathematics-I and II", "Instructional Technologies and Material Development". The aim of these courses is to help prospective teachers develop their pedagogical content knowledge. In these courses, prospective teachers have opportunities to explore theories of mathematics teaching and learning, do microteaching activities, examine their own teaching, observe and examine peer teaching. The data was collected during prospective teachers' micro-teaching activities each of which lasted for forty minutes. Four prospective teachers (3 female and 1 male) prepared lesson plans and teaching notes and taught definite integral as their peers followed their teaching taking a role of a student. They were also interviewed after their teaching.

## Analysis of Data

The data comes from the following sources: micro-teaching videos, interview transcripts, written documents such as lesson plans and teaching notes. The lesson plans and teaching notes were analysed to investigate how prospective teachers addressed the limit process in their planning. The analysis of micro-teaching videos was carried out by taking detailed descriptive observation notes as the researchers watched the videos. These notes were then analysed inductively and themes which are related to the limit process were identified.  Interviews lasted around forty-five minutes. There were two main aims of these semi-structured interviews: to investigate how did prospective teachers prepared for their lessons and to give them an opportunity to reflect on their lessons. Interviews were audio-recorded and verbatim transcripts of the audio-records were open-coded and memos were written considering the research question. Themes were identified considering the memos.

## RESULTS

All of four prospective teachers mentioned limit as a prerequisite knowledge in their lesson plans. However, when their activities in their lesson plans and their micro-teaching videos were examined, it was found that they could not ‘properly’ address the limit process when constructing the area under a curve from the sum of rectangular areas. Prospective mathematics teachers attempted to show that the upper and lower sums are coming closer to each other when the number of rectangular areas is increased. Activities they used aimed to obtain better and better approximations of the area under the curve. However, they did not discuss if it could be possible to obtain an exact answer by taking more and more rectangles under the curve.

All prospective teachers used the same activity which was given in the national curriculum materials as an activity example (MEB, 2005). The aim of the activity is to obtain the upper, lower and Riemann sums for the area under a parabola by considering different partitions. Eren, one of the prospective teachers, gave the definitions of *upper sum, lower sum* and *Riemann sum.* After that he explained it using a specific example “ $f:[0,3]\to R$ ,  $f(x)=3x^2$ ”, he divided the interval into three parts and calculated the sum of the rectangles. He said that the lower sum gets bigger and the upper sum gets smaller as the number of rectangles increases. However, he did not encourage a discussion to explore how an exact area can be obtained and why the limit process is necessary.

Other prospective teachers, Sena, Sevgi and Pelin, used the same activity. However they did not discuss whether the sum of rectangular areas equals to the area under the curve. They just stated the necessity of the limit process.  Such discussions could be important because of the misconception addressed by Orton (1983): taking more and more rectangles under the curve gives better and better approximations but such a procedure would never produce the correct answer. Similar misconceptions were reported in the literature on the limit concept. For instance, one of the epistemological obstacles for the limit concept is related to whether the limit attained or not (Cornu, 1991). It is also found that there is a tendency to view the limit concept as getting closer and closer to a number without actually reaching it (Tall & Schwarzenberger, 1978). Such misconceptions could make it harder to decide if it could be possible to get the exact value of area under a curve by summing up the areas of rectangles. Shulman (1986) states that “if preconceptions are misconceptions, which they so often are, teachers most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates” (p. 9-10). In that sense, prospective teachers’ understanding of preconceptions of students which may reveal itself as misconceptions for the limit concept is crucial.

Interviews with prospective teachers were useful to gain more insights on their PCK in relation to student difficulties with the limiting process. In the interviews, prospective teachers were asked possible student difficulties for the definite integral. Two prospective teachers (Pelin and Sevgi) stated that students might have difficulties to realise that upper and lower sums approaches to the same value. Eren suggested that calculation of area is the most difficult part of the topic. Sena determined the most challenging part of the topic as the lack of preliminary knowledge of limit and students’ prejudice about the difficulties of the topic. Although they were able to state the difficulty in a general manner, they could not clearly explain why the limit process is necessary for defining definite integral.

In the interview, prospective teachers were also asked to reflect on the choice of their lesson activities. Three of prospective mathematics teachers, Sena, Sevgi and Pelin, used the activity in the national curriculum which aims to show that the upper and lower sums are coming closer to each other when the number of rectangular areas is increased and limit of the sum gave the area under the curve. However, they stated that although this approach was useful for students, it was time consuming in practice. For instance, Sena said the following:

Researcher: What do you think about the approach of the curriculum to definite integral?

Sena: Generally I found curriculum’s approach useful but it’s not something applicable.

Researcher: Why is it useful to find the areas of rectangles under a curve?

Sena: It could help students not to forget it, later they can remember easily, but I don’t know if it’s necessary to spend such a long time just for that. I find it useful but it may take so long in practice.

Pedagogical content knowledge includes an understanding of what makes the learning of specific topics easy or difficult (Shulman, 1986, 1987). Prospective teachers appeared to be unaware of the importance of the limit process to construct the area under a curve and of students’ possible misconceptions that might hinder conceptual understanding. They found such an approach time consuming and not worthy of spending so much time to dwell upon limit process to construct the area under a curve from the sum of rectangular areas. Eren also stated that the approach in the curriculum was not helpful to students for conceptual understanding, and therefore the best thing to do is to introduce the limit directly by the formal definition of definite integral by Riemann sum:

Researcher: What kinds of difficulties students may confront about definite integral?

Eren: Students want to learn the definition and the theorem directly (applying the Sandwich theorem to show that the Riemann sum is between the upper and lower sums)

Researcher: How do you decide what students prefer?

Eren: Students always want to learn the short cuts, not the long way. They say: show me the easiest way to solve it… the longer you make it, the more likely you lose the students.

Clearly Eren is in favor of teaching the definite integral by introducing its definition. At this point it should be noted that the sandwich theorem for the limit concept that prospective teachers used is not covered in the national mathematics curriculum. They reported that they prepared their lessons mainly based on the content of calculus course rather than curriculum materials. Curriculum scripts are an important part of the content delivered during lessons as it provides a structure on basis of learning goals (Leinhardt et al, 1991 cited in Utter, 1997). Further to this, the conceptions and preconceptions that students of different ages and backgrounds bring with them is important for PCK (Shulman, 1986). However, prospective teachers did not consider the curriculum scripts to get an idea as to necessary preliminary knowledge students should have.

## DISCUSSION AND IMPLICATIONS

In this study, we investigated prospective mathematics teachers PCK in relation to student difficulties. Although prospective teachers mentioned limit as a prerequisite knowledge in their lesson plans, they could not ‘properly’ address the limit process when constructing the area under a curve by encouraging a discussion to explore how an exact area can be obtained and why the limit process is necessary.

We believe that understanding of students’ difficulties as one of the components of PCK is crucial and reflective teaching experiences in real settings are needed for its development. In preparing prospective mathematics teachers during teacher education programs, micro-teaching activities is a starting point to discover prospective teachers’ existing knowledge of student difficulties. To develop this knowledge in teacher education programs we propose two suggestions not only for PCK of definite integral but also for PCK of other topics. Firstly, after determining prospective teachers’ existing knowledge of student difficulties using micro-teaching activities, they could be given specific student difficulties reported in the literature and could be asked to prepare lessons which aims to address these difficulties. This, we believe, is necessary since the prospective teachers have no teaching experience with students. Secondly, the same approach could be used during their school placements in which they teach topics in the real school settings. A study is being conducted by the authors, investigating the development of various components of PCK during school placements.

## REFERENCES


An, S., Kulm, G. & Wu, Z. (2004) The pedagogical content knowledge of middle school, mathematics teacher in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145–172.

Bezuidenhout, J. (2001). Limits and continuity: some conceptions of first-year students. *International Journal of Mathematical Education in Science and Technology*, 32, 487-500.

Carpenter, T. P., Fennema, E., Peterson, P., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, 19(5), 385-401.

Cha, I. (1999) Prospective secondary mathematics teachers' conceptions of function: Mathematical and pedagogical understandings, *Unpublished PhD Thesis*, University of Michigan.

Cornu, B. (1992). Limits. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 153-166). Dordrecht, The Netherlands: Kluwer Academic.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education.* New York: Teachers College Press.

Milli Eğitim Bakanlığı, Talim ve Terbiye Kurulu Başkanlığı (2005). *Orta Öğretim Matematik (9, 10, 11 ve 12) Sınıflar Dersi Öğretim Programı*, Ankara.

Orton, A. (1983). Students' understanding of integration, *Educational Studies in Mathematics*, 14, 1-18.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.

Tall, D. O. & Schwarzenberger, R. L. E. (1978). Conflicts in the learning of real numbers and limits, *Mathematics Teaching*, 82, 44-49

Utter, F. W. (1997) Relationships among AP calculus teachers' pedagogical content beliefs, classroom practice, and their students' achievement (Oregon State University). *Dissertation Abstracts International*, 57,5091A.