CAN MATHS IN A TEST BE 'FUNCTIONAL'?

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This paper argues that the assessment of functional mathematics as currently being developed at GCSE level should go beyond testing mathematical skills and using word problems. Drawing on the literature about mathematics in the workplace, criteria for functional mathematics assessments are offered, and the constraints of tests considered, before answering the question, with: "Yes, but it won't be easy".

1. WHAT IS FUNCTIONAL MATHEMATICS?

Functional mathematics is about mathematics being useful, and what can occur in school that enables that, but the scope of what is meant by the phrase varies considerably. Some authors use it to refer to a re-conceptualisation of the mathematics curriculum, a complete rethink about what is done in school to make the mathematics that is learned more useful, responding especially to the changes in the workplace occasioned by advances in technology (e.g. Forman and Steen, 2000).

A narrower view of functionality, and the one that is the focus of this paper, is the set of skills that enables a person to cope with everyday life. In this vein, functional mathematics is defined in the White Paper "14-19 Education and Skills" as the "maths that people need to participate effectively in everyday life, including in the workplace" (DfES, 2005, page 35). This functional mathematics is going to be examined both as part of GCSE, and separately as a series of stand-alone qualifications. Standards for these assessments can be found on the QCA website: http://www.qca.org.uk/downloads/QCA-06-2932_Functional_skills_standards_maths.pdf

However, there is ambiguity in the meaning of "the maths that people need". It is possible to interpret that as the 'core skills' of basic mathematical competences that are applied when mathematics is functional. This interpretation sees 'functional mathematics' as the subset of mathematics knowledge that is useful (e.g. arithmetic and interpreting graphs), contrasted with the mathematics that is learned as a step towards more mathematics (e.g. algebraic manipulation and Pythagoras' theorem).

While acknowledging that knowing some mathematics is essential to being functional, a more helpful interpretation of "the maths that people need" is a broader view of mathematics that includes the processes that are involved in actually applying mathematics. This interpretation of functional mathematics encompasses more of what a person does when they operate successfully with mathematics in real settings.

While less easy to pin down than a list of mathematics 'content', it seems important that an assessment of functional mathematics – and the teaching that precedes it – should engage with this broader view of functional mathematics.

2. WHAT IS INVOLVED IN BEING FUNCTIONAL WITH MATHEMATICS?

A review of research on mathematics at home and at work (e.g. Lave, 1988; Nunes et al, 1993; Pozzi et al, 1998, Resnick, 1987) reveals a number of ways in which out-of-school mathematics differs from what is usually done with mathematics in school.

Purpose and meaning in the activity

What is done at home and at work is almost always done with a purpose, which adds meaning to the mathematics that is deployed. In school, of course, the purpose is to learn, and the mathematics is done just as an exercise towards learning it.

Understanding and reasoning linked to the context

In school there is an expectation that whatever understanding accompanies mathematical activity it is mathematical in nature, and mathematical reasoning is about drawing mathematical conclusions from mathematical premises. In situations in which mathematics is being used, however, the understanding and reasoning is derived from features of the context, with the mathematics subordinated to that.

Tool use

Although school does to an extent permit the use of tools like calculators in restricted circumstances, the use of tools in work is much more pervasive, with tables and charts to support calculations commonplace. This no doubt reflects the contrasting purposes in the two settings, which in school are about learning the mathematics, and at work are about getting the right result using whatever means are available.

Situation specific competence

The emphasis of much school mathematics is in developing general purpose competences. However, the competences of work and at home are usually specific to the context, again reflecting the purposes of mathematical activity out of school.

Error-free performance

There is no benefit and some cost to getting an incorrect outcome in out of school settings. For that reason, circumstances evolve that minimise that risk, and people's performance out of school is for the most part without error. For example, in most work settings, people are matched to the level at which they will operate successfully, so some individuals are not permitted to undertake certain tasks. In school, however, much is made of the benefits of making a mistake, and everyone is asked to undertake tasks that are beyond them, which of course is because of the purpose – promoting learning.

3. CRITERIA FOR THE ASSESSMENT OF FUNCTIONAL MATHEMATICS

A consideration of mathematics out-of-school suggests that assessment of the 'functional' ability to deal with contexts mathematically should require that students engage with problems-in-context in a way that anticipates real-life engagement, and:

- include contexts and questions that are realistic enough to invoke a sense of purpose, even though that purpose cannot be real;
- have what is in effect an assessment of process that does not include any attention to the actual process used;
- avoid unreasonable obstacles to success in the mathematical aspects of the assessment.

4. WORD PROBLEMS IN THE ASSESSMENT OF FUNCTIONAL MATHS

Word problems are conventionally seen as the place where mathematics and the real world meet. They are the 'obvious' first candidate for assessment of functional mathematics, particularly in a test context, and so need to be considered in relation to these criteria. Will this kind of thing do?

> "An electrician earns a basic rate of £11.20 per hour for a 35 hour week. For each hour worked over 35 hours, he earns 1½ times the basic rate. One week he works for 40 hours. How much does he earn?"
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> (Key Skills Application of Number, Level 2, 15 March 2004)

> "I wish to paint the outside walls of my house. A tin of paint covers $25m^2$ correct to the nearest $5m^2$. The outside walls of my house have an area of $320m^2$, correct to the nearest 10m2. Calculate the maximum number of tins of paint I may have to buy"
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> (OCR GCSE Specimen Assessment Materials, Mathematics A)

The first criterion for a functional mathematics assessment is that it brings a sense of purpose, which is contingent on some kind of engagement with the context that is described. However, Reusser and Stebler (1997) say that there is:

> "ample evidence that many students in mathematics lessons 'understand' and 'solve' mathematical word problems without considering the factual relationship between real-world situations and mathematical operations."

Students commonly consider word problems as a kind of mathematical exercise, and work with the numbers without thinking about the context much at all. For example, little of the context needs to be thought about in the first example above to arrive at 11.2 x 35 plus (40 – 35) x 11.2 x 1.5.

Wyndhamm and Säljö (1997) do not feel that students' unwillingness to engage with contexts in word problems represent a lack of ability to do so.

> "The student … is not necessarily lacking in skills or competencies. Rather s/he is operating within a communicative contract of how to interpret problems, which in all likelihood has developed in response to how schools 'do' mathematics."

As a result, whether word problems are answered correctly or not does not reflect functionality with mathematics, and they are not a good way to assess it.

5. MODELLING AND FUNCTIONAL MATHEMATICS

A different perspective on what it means to be functional is found in the notion of 'modelling'. In dealing with a word problem, it is intended that the mathematical features of the described context are identified and abstracted from the situation, and the mathematical relationships worked with, to arrive at a mathematical result which is applied back to the context. This can be shown in diagrammatic form as follows:

$$\begin{tikzcd} \text{Situation} \arrow[r] & \text{Model} \arrow[d] \\ \text{Solution} & \text{Result} \arrow[l] \end{tikzcd}$$

This reflects the structure of 'mathematical modelling' that is taught at higher levels of mathematics, and it is commonly supposed that this sort of modelling is what is needed for the everyday application of mathematics as well. However, Gravemeijer (1997) reports that two conceptions of modelling can be found in the literature. The one described above is modelling as a form of translation between the real and mathematical worlds, and originated in the work of Polya. A contrasting view is that of modelling as a form of organising of the real world using mathematics, which started with the ideas of Freudenthal.

It can be argued that word problems fit the first kind of modelling, but most actual mathematical functionality is better described in the second way. In ordinary settings people take mathematics to the context, rather than abstracting mathematics from it, and the mathematics usually remains attached to the context, as observed by the researchers cited in the first section. This is not to say that modelling of the first kind cannot be used in real settings: it can and it is. However, for most people in most contexts it is the second account of modelling that is more accurate, and assessment of functional mathematics is better served by an approach which reflects that, rather than operating on an assumption of the first kind of modelling, as word problems do.

6. BEYOND WORD PROBLEMS

In order to have demands that are more likely to bring an engagement with the context, which is necessary to assess mathematical functionality, Verschaffel et al (1997) say that what is needed is "more complex and more authentic problem situations" - but what does that mean? What has to be changed?

Van den Heuven-Panhuizen (2005) boils it down to two criticisms of word problems:

> "the context is not very essential – it can often be exchanged for another without substantially altering the problem"; and "the reality that is presented is often not in tune with the real situation of the actors in the problem"

Reusser and Stebler (1997) add a third:

> “Students not only know from their school mathematical experience that all problems are undoubtedly solvable, but also that everything numerical included in a problem is relevant to its solution, and everything that is relevant is included in the problem text”

7. HOW EACH OF THESE MIGHT BE OVERCOME

(i) The ‘essential’ context

A word problem starts in the mathematical world, with a piece of mathematics to be ‘applied’, and the problem setter ‘translates’ this into a context, contrived for the purpose. Another context would indeed have done as well, and students understand that what they have to do is concerned with mathematics, not the situation as such. The alternative is to start with a context, not the mathematics. The mathematically related questions that are then asked must be more genuinely about that situation.

(ii) The reality of the ‘actors’

An obviously ‘mathematics lesson’ question is likely to get similar responses to a word problem, even if the situation being referred to is more complex and authentic as a context. The alternative is to choose questions that might actually be asked.

(iii) The closed context

In word problems all the required information about a specific event is given, and all that is given is required. Giving the information ‘on a plate’ reduces the need to engage with the situation. The alternative is to describe a broader context that includes data relevant to a range of events, only some of which will be asked about.

8. THE CONSTRAINTS OF A TEST

An assessment that is based on authentic contexts, described in ways that are broader than the questions asked about them, and where the questions are realistic in the sense of being genuine questions that might actually be asked, can be addressed in different ways, and perhaps more easily in contexts other than in a test. In a test there are constraints that impact on the means by which these ambitions can be realised.

(i) What it is reasonable to ask in a time limited test.

Because of reading issues and time to process, there is only so much complexity that can be included in a test – and would it be enough to assess functionality? Introducing more complexity can be helped by the inclusion of ‘pre-release’ material that candidates can spend time with before the test, but this too is bound to be limited.

(ii) Mark schemes

In a test, assumptions have to be made about the understanding about a context that reading the description of it will provoke. A test cannot afford to have students drawing in realistic considerations from their own experience, because it would be impossible to allow for that in the mark-scheme. The answer needs to use the given information, but not any information that a more ‘active’ reading of the description of the context would bring. As Cooper and Harries (2003) say, a test of functional mathematics must look for “a *particular* realistic consideration, but not realistic considerations *in general*”.

9. CONCLUSION

The challenge of getting students to act functionally in a simulated reality in order to assess whether they have the capacity to cope out of school is not easily met, although some straightforward suggestions can be made to improve the prospect of it. However, a test seems to make it that bit harder, both through the constraints on describing a realistic situation, and through the limited information a test gives about the thinking that informed a response.

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