MATHEMATICAL VISUAL FORMS AND LEARNING GEOMETRY: TOWARDS A SYSTEMIC FUNCTIONAL ANALYSIS

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Mathematics is a multimodal discourse in which mathematical texts use, at least, three different semiotic systems: verbal language, algebraic notations and visual forms. Beside the research that has been done concerning the verbal components of mathematical texts, there is a need to develop tools to describe the non-verbal components. Based on Halliday's SF grammar, Morgan's linguistic approach and multimodality approach. I present a preliminary suggested descriptive framework for analysing geometrical visual forms. My intention is to use this framework in my PhD study which investigates the role of mathematical visual representations in the construction of mathematical meaning. In order to illustrate, aspects of two examples will be analysed using this framework.

BACKGROUND:

A general overview of the status of visual representations, i.e. diagrams, graphs, shapes, etc., in mathematical texts indicates that these representations are: a) limited in representing knowledge with possible misuse of diagrams (Shin, 1994 as mentioned in O'Halloran, 1999, 2005), b) of an 'informal and personal nature' so that the mathematical community will not accept such representations in a research paper even if they are motives and important for the researcher herself/himself (Misfeldt, 2007). One main reason for this view is that the main stream among mathematicians (or even among others) conceives mathematics as '*abstract*, *formal*, *impersonal* and *symbolic'* (Morgan, 2001). At the best, mathematicians consider these representations have or own messages or meanings, even though these messages are limited, which students need to know how to 'grasp' or discover (Shuard & Rothery, 1984). In my prospective study, I consider visual representations as available resources for meaning-making and I intend to investigate what meanings students make when they interact with these representations while solving problems.

THEORETICAL FRAMEWORKS – A STATE OF THE ART:

It has been argued that people communicate by using different modes from the resources available to them, for example spoken and written language, visual representations, gestures, music, etc. (e.g. Kress & van Leeuwen, 2006; Lemke, 1998; Morgan, 2006; O'Halloran, 1999). In order to achieve effective communication, people use what they think the 'best' mode to communicate- 'aptness' of mode (Kress & van Leeuwen, 2001). When people employ visual representations in their communication, visual representations then have a function in representing knowledge, just as language or any other mode. The need to understand and take these representations into consideration when analysing any text is, therefore, salient.

Halliday (1985) argues that any text fulfils three meanings: ideational, interpersonal, and textual. Our ideas about the world are represented in the ideational meaning, the interpersonal meaning is realised by the relationships constructed with others through communication. The textual meaning is realised as these representations get presented in a coherent way. This descriptive framework is called systemic functional linguistics (SFL) or grammar (SFG). Even though this framework was initially developed to account for verbal modes of communication only, it has been extended to include non-verbal modes too. The multimodal approach or the semiotics of visual representations developed by Kress & van Leeuwen (2006) is an example. They have developed a grammar to 'read' images using 'representation, interactive and compositional' corresponding to Hallidyan terms respectively. Other examples are: Lemke's studies in science education and language (e.g. 1998), the semiotics of art (O'Toole, 1990) and application of SFL in mathematics education by Morgan (1995; 1996a; 2006) and O'Halloran (1999; 2005).

Mathematics is a multimodal (or multisemiotic) discourse (Duval, 2000; Morgan, 1995, 1996a, 2006; O'Halloran, 2003) where three semiotic systems, at least, are used: verbal language, algebraic notations (or 'mathematical symbolism'), and visual forms (diagrams, shapes, graphs, etc.). As it has been observed by Morgan, 'the oral discourse of mathematical practices (..) has already been the subject of some research in educational contexts' (Morgan, 2003, p. 112). Furthermore, Morgan (1995; 1996a; 1996b; 2001; 2003; 2006) has opened mathematics discourse for Halliday's systemic functional grammar by adopting it as a framework and an analytic tool to analyse written mathematical texts, thus, developing a linguistic approach to mathematical text. In written mathematical texts, while the ideational meaning is realised by the writer's view to the nature of mathematics and the existence of human agent, the interpersonal meaning is realised through the relationship established between the producer of the text and its reader and the roles of both of them within the text. The textual meaning is –in turn– realised by the role that a coherent text plays, such as developing a mathematical argument, concept or proof (Morgan, 2006).

For non-verbal features of mathematical texts, O'Halloran (1999) develops SFL frameworks for both mathematical symbolism (or algebraic notations) and mathematical visual displays. She uses O'Toole's systemic functional framework to analyse mathematical visual representations. In analogy and accordance with the Halliday's systemic functional linguistics, O'Toole (1990) suggests that 'the semiotic codes of the visual arts (..) are realized through systems of representational, modal, and compositional choices' (p. 187). In turn, O'Halloran (1999; 2003; 2005) adopts this framework for analysis the meanings of mathematical graphs and diagrams. Nevertheless, O'Halloran's framework applies only to graphical forms and her work has not been directed towards geometry, which is the focus of my interest. It is doubtful whether her framework can be applied in a straightforward manner to geometry. I argue that a specialised framework for the grammar of geometrical visual diagrams is needed. Moreover, Morgan (1995; 2006) states that there is a need to develop tools to describe the non-verbal components of mathematical texts from the systemic functional perspective.

Following the efforts of previous research (Chapman, 2003; Morgan, 1995; O'Halloran, 2003), I intend to investigate what meanings visual representations do offer. As a first step towards this aim, I present a ‘first’ draft of a preliminary suggested framework (annex 1) which needs more developing and thinking. This framework is mainly based on Morgan’s linguistic approach (2006) and Kress & van Leeuwen (2006) framework as well.

TWO EXAMPLES:

I will try in this part to ‘apply’ the suggested framework to two examples (annex 2). Because of the limited space available I will focus on one feature: representation of the image of mathematics (ideational meaning) in diagrams in two texts (Examples 1 & 2 in annex 2), one is English and the other is Arabic.

The representational (ideational) meaning in diagrams is realised by determining the nature of the diagram; whether it is a narrative structure or conceptual structure. The main feature is the presence of an action or not, that is, following Kress & Leeuwen (2006), the presence of a vector. Vectors might be a curved arrow, ‘attenuated’ vectors (dotted or solid line) or ‘amplified’ vectors. In both structures, we need to look at the types of processes and participants active in them. Based on Hallidayan functional grammar, Kress & Leeuwen state that in narrative structure, the type of processes is that of ‘happening’, ‘doing’ or ‘going on’ and the participants are active; they are carrying out the identified process. In mathematical discourse, these processes might be generalisation, measurement, naming, etc. In conceptual structures, no actions are being carried out; the participants are, thus, not active. There are three types of processes represent participants ‘in terms of their class, structure or meaning’(Kress & van Leeuwen, 2006, p. 59): classificational, analytical and symbolic.

The diagram in example 1 is an example of a narrative structure. There are some processes occurring here such as generalisation since the diagram uses symbols rather than specific numbers. This process suggests that this is a general situation, not an example, to represent the equation of a circle where the centre is the origin. The dotted line (PN) needs to be drawn in order to prove the equation. This suggests that a human agent exists and, consequently, the image of mathematics is as a human practice rather than being impersonal. The labelling process also emphasises this image; different kinds of labels are presented: measurements (r, O, y), names (y, N), variable (P(x, y)), or property (the right angle symbol at N). It is also significant to observe the position of the diagram as a feature of the compositional/textual meaning. The diagram stands in the middle of the upper section of the page, surrounded by white space. That suggests a certain theoretical or ideal situation; it constitutes a unity that stands on its own and invites for interaction.

In example 2, there are three shapes/figures. The upper rhombus and Venn diagram are, respectively, symbolic and classificatory (conceptual) structures. The upper rhombus's identity is clear since no names, symbols or measurements are on it. The Venn diagram is a classificatory structure presenting the relationships between rhombuses, parallelograms and quadrilaterals. The lower rhombus, on the other hand, is a narrative structure with dotted lines (which represent its diameters) that need to be formed in order to solve the problem. In this case, a human agency is clearly needed; therefore, the mathematical activity is portrayed as human-made.

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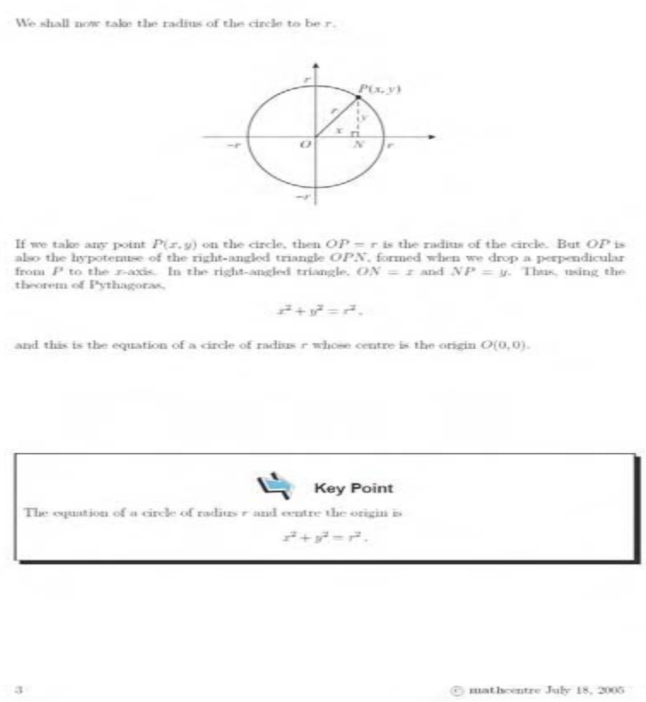
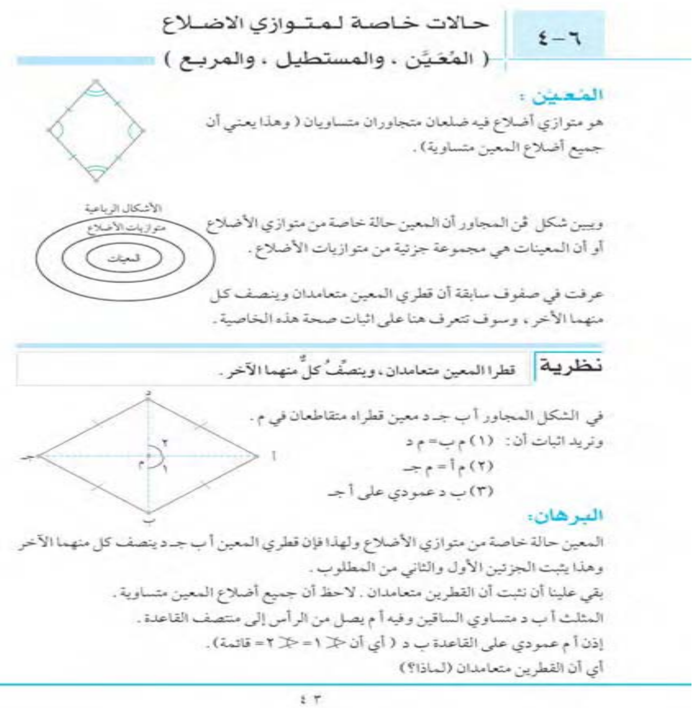
Annex 1:

A preliminary suggested framework

Based on Morgan (2006) and Kress & van Leeuwen (2006) frameworks

Representational/Ideational meaning designing social actions & constructs	**Interactive/Interpersonal meaning** ‘designing the position of the viewer’	**Compositional/Textual meaning** Unity & Coherence
• **Nature/image of mathematics and mathematical activity** The picture of mathematics might be represented through the examination of types of processes and participants acting in them. This meaning (ideational) is realised by determining the nature of the diagram; whether it is a narrative structure or conceptual structure: * <u>Narrative structures</u>: (designing social actions) ‘goings-on’- ‘doing’, ‘happening’, ‘sensing’, ‘meaning’ (vector: action) *Processes*: generalisations, measurements, naming, .. *participants*: active * <u>Conceptual structures</u>: (designing social constructs) *Processes*: 1. Classificational: classify 2. Analytical: part-whole 3. Symbolic: meaning/identity of participants *Participants*: not active	• **Roles and relationships between author/producer and viewer** There are two kinds of participants in the (re)production of diagram, represented participants (‘things’ depicted) and interactive participants (real people, the producers and the viewers). Hence, there are three kinds of relations between these participants. These relations are realised by: * <u>Contact</u>: Does the diagram offer information not mentioned in the co-text? Is the diagram drawn ‘differently’ that demands attention? * <u>(Social) Distance</u>: personal, impersonal. (drawing the diagram neatly vs. roughly) * <u>Attitude/point of view</u>: involvement vs. detachment, relationships (power, equality). (specialty, certainty and authority) * <u>Modality</u> (design the reality/truth) (naturalistic vs. scientific modality). ‘shared truths’, ‘imaginary we’– mathematical community	• **Unity & Coherence** The way that elements are presented/ placed in a text contributes to its meaning. This textual meaning relates the ideational and interpersonal meanings together into a ‘meaningful whole’ or a message by: * <u>Information value:</u> ‘placement of the elements’: left-right, top-bottom, Centre-Margin. * <u>Salience:</u> ‘eye-catching’ or ‘attract the viewer’s attention’: colour, size, perspective (foreground, background, overlap, appearance of human figure) * <u>Framing</u>: separation such as frame lines, white space, colour, etc. What message(s) does the whole/integrated mathematical text present? Examples: ‘instructions for a calculation, argument, new mathematical concept or procedure, proof or a solution to a problem, story’, etc.

Annex 2

Example 1: English text	Example 2: Arabic text
	
Source: Mathcenter. (2005). The geometry of a circle. Available at: http://www.mathcentre.ac.uk/students.php/mathematics/geometry/circle/resources/380. Retrieved 27/12/06.	Source: Palestinian Ministry of Education and Higher Education (2004, in Arabic). Mathematics textbook for 8th grade-part 2. Available at: http://www.pcdc.edu.ps/textbooks/math_G8_p2.pdf.Retrieved 26/12/06.

Example 1 text:

We shall now take the radius of the circle to be r.

If we take any point $P(x, y)$ on the circle, then $OP = r$ is the radius of the circle. But OP is also the hypotenuse of the right-angled triangle OPN, formed when we drop a perpendicular from P to the x-axis. In the right-angled triangle, $ON = x$ and $NP = y$. Thus, using the theorem of Pythagoras,

$$x^2 + y^2 = r^2.$$

and this is the equation of a circle of radius r whose centre is the origin $O(0, 0)$.

Key Point

The equation of a circle of radius r and centre the origin is

$$x^2 + y^2 = r^2.$$

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Example 2 text:

٦-٤ حالات خاصة لمتوازي الاضلاع

(المُعَيَّن ، والمستطيل ، والمربع)

المُعَيَّن :

هو متوازي أضلاع فيه ضلعان متجاوران متساويان (وهذا يعني أن جميع أضلاع المعين متساوية).

ويبين شكل ( ) المجاور أن المعين حالة خاصة من متوازي الأضلاع أو أن المعينات هي مجموعة جزئية من متوازيات الأضلاع .

عرفت في صفوف سابقة أن قطري المعين متعامدان وينصف كل منهما الآخر ، وسوف تتعرف هنا على اثبات صحة هذه الخاصية .

نظرية قطرا المعين متعامدان ، وينصِّفُ كلٌّ منهما الآخر .

في الشكل المجاور أ ب جـ د معين قطراه متقاطعان في م .

ونريد اثبات أن : (١) م ب = م د

(٢) م أ = م جـ

(٣) ب د عمودي على أ جـ

البرهان:

المعين حالة خاصة من متوازي الأضلاع ولهذا فإن قطري المعين أ ب جـ د ينصف كل منهما الآخر وهذا يثبت الجزئين الأول والثاني من المطلوب .

بقي علينا أن نثبت أن القطرين متعامدان . لاحظ أن جميع أضلاع المعين متساوية .

المثلث أ ب د متساوي الساقين وفيه أ م يصل من الرأس إلى منتصف القاعدة .

إذن أ م عمودي على القاعدة ب د (أي أن ∢١ = ∢٢ = قائمة) .

أي أن القطرين متعامدان (لماذا؟)

٤٣