EXPLORING PUPILS' CONCEPTUALISATIONS: WHAT MAKES SOMETHING 'MATHEMATICAL'?

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A report of the findings of a study intended to explore year seven pupils' perceptions of the characteristic 'mathematical'. Following a protocol based on the methods of Personal Construct Psychology, participants ranked a set of paintings in terms of 'how mathematical' they felt the paintings were, and were then asked to explain their decisions. The exercise was repeated with a set of five activities. Participants were seen to respond with a not inconsiderable uniformity to the tasks, although the individual explanations of what makes an activity 'mathematical' provided some intriguing insights which indicate possible further exploration. Implications of this finding in terms of future research approaches are discussed with the aim of understanding what determines whether an activity is described as 'mathematical'.

INITIAL OBSERVATIONS

As a mathematics teacher, I have been told that a particular class topic or task is "not maths" (such as transcribing numbers into standard index form, elementary set theory or tessellation), and have been attentive to the surprise in people's voices when they state that a non-classroom task is a "bit mathsy". Turning to the literature for views of what constitutes mathematics provided implicit acknowledgement that different practitioners classify mathematics in different ways. This led me to question how and why they identify their behaviour as 'mathematical'. I felt that Ernest's (1998) classification of different users of mathematics could be further explored in order to understand more about the varying relationships with mathematics. Such an enquiry is also indicated by Evans (2000a), in finding that it is not sufficient to characterise mathematics in a predetermined way for any particular person, who will have a complex relationship with mathematics which cannot be presupposed purely by their academic experience, gender or career. Informal conversations signalled that as first step it would be necessary to investigate characteristics of activities as they appeared to the practitioners involved. This approach rested for its basis on the position that experience is not easily divided into 'mathematical' or 'non-mathematical'. The characteristics of a situation which may point one towards that description and the extent to which those features have any effect will vary between persons and situations. Consequently, we should speak of 'mathematical' as a relative quality, and recognise that the situations in which it may appear are as varied as practitioners themselves. Any predetermined demarcation is illusory and potentially unhelpful, in that it deprives the practitioner of their subjectivity in making sense of the situation for themselves. As Tall *et al.* (1999) state of any mathematical object, "What matters more is not what it is but what we can *do* with it" (p. 229, original emphasis). I would append to that sentence "and how we recount that doing, to ourselves and to others."

RESEARCH BACKGROUND

Popular perceptions of mathematics characterise it as a particular area of human experience which is largely dislocated from most people's everyday lives. It is my view that in studies claiming to have identified mathematics in people's everyday activities participants have often been denied a voice to express their views of mathematics, and mathematics itself has been denied its potential universality. (See Noss *et al.*, 2000, for an overview.) This is due to research designs which deprive these subjects of their subjectivity, presupposing the status of mathematics in their lives. The studies have situated the participants 'outside' mathematics, presenting it as an experiential realm into which they can journey and then return, as and when it is necessary. I see this as a misleading presentation of mathematics and mathematical activity. A particularly pertinent example is provided by Abreu et al (1997). On the issue of problem-solving processes we are informed that "When individuals are confronted with situations outside of school that potentially involve mathematical problems they do not just follow a rigid sequence of cognitive strategies to solve these problems. Since the individuals are more in control they first decide if they will cope mathematically with the problems or follow other routes" (p. 235). I would posit that this formulation conjectures a mode of thought which cannot be guaranteed to take place, it being more likely that people select from a known repertoire of actions for solving like problems. The outcome of this selection might then, in reflection, either retrospective or simultaneous, be described as mathematical. It is in both the choice of means for solving a problem and how those means are classified that the subject asserts their subjectivity.

PERSONAL CONSTRUCT PSYCHOLOGY

This approach echoes a central principle of Kelly's (1970) Personal Construct Psychology (PCP), which focuses on the constructs used by people as they make sense of the world around them. This focus tallied very well with my desire to explore the perceived characteristics of activity, and PCP is a theory which lends itself to educational applications. One can construe the role of the teacher as that of helping the learner to explore and reflect on their constructs and submit them to testing and subsequent change. A central method of PCP, the repertory grid method, was used as the initial basis for the survey method and interviews undertaken. Repertory grid techniques require that a participant opposes the elements under consideration in pairs, or more commonly triads, and explains their differences in order to elicit the constructs with which that participant relates to them. Further methods of qualitative analysis of the data gathered then vary (Fransella & Bannister, 1977). My study adapted repertory grid method to reflect the emphasis on a continuum of experience of increasingly mathematical character. These modifications of the method for my specific purposes fit within the range of modifications used by practitioners of PCP (see Fransella & Thomas, 1988, for examples).

METHODOLOGY – INITIAL STUDY

The fieldwork for my initial study comprised survey and interview stages and was conducted in a boys' independent secondary school in Middlesex. The year seven school group were chosen as suitable participants, for both practical and theoretical reasons. They were taking part in a mathematics-focused fun day, into which the survey fitted well, and they had had the least experience of secondary mathematics education, which may it more likely that their views of mathematics may be less 'fixed' than that of older pupils. It has been noted that pupils of this age seem to be at a critical point in the determination of attitudes towards mathematics. This was a central point in the methodology of Picker and Berry (2000). The method undertaken comprised two stages: A survey of a group of 71 participants, followed by semi-structured interviews of paired participants. The survey stage of the research consisted of the pupils in the group being shown reproductions of five paintings and being asked to rank them in order from 'most mathematical' to 'least mathematical'. (See appendix for details of the paintings. They are not included here for reasons of space.) Part of the intention of this first stage was to explore the validity of relativising the mathematical nature of an object, as a constituent part of a participant's perception of it. The initial focus on looking for characteristics in the paintings was intended as a 'warm-up' for subsequent comparison and discussion of activities, which took place in the interviews. The comparison of activities followed the same format as that of the paintings, with the participants being asked to rank a list of five activities and then explain to each other and the interviewer their rationale behind the ranking.

DATA AND ANALYSIS

The ease with which participants responded comfortably and easily to the ranking task suggests that the notion of a continuum of 'mathematicalness' can not only be understood easily, but that an awareness of it may be of benefit to many. The data provided evidence that pupils are able to make a relative judgement of mathematical content as they see it, thus reinforcing the standpoint that experience is not easily partitioned into the 'mathematical' and 'non-mathematical'.

There is not enough space here to do full justice to the responses given by the participants, so I shall focus on just a few extracts from the interviews. To a strong degree the constructs elicited coincided with traditional distinctions which define mathematical activity, but there were some surprises. Most importantly, automatic rule-following, in particular calculating, was not necessarily seen as mathematical. The commonest references in the interviews were to activities which combined a certain regulation of potential behaviours, but which also involved making informed choices within those regulations. A second prominent feature was the significance of compulsion in thought and action, seen repeatedly in phrases such as "you have to":

HP: For baking a cake you have to think how much, to put in the pan. But I think solving logic puzzles is more mathematical than this one [solving the

> equation], because you do more things, you have to use your head a lot more, whereas that [solving the equation] there's only one way to do it. But with solving logic puzzles you have to think of logic ways, so it's tricky.
>
> [Ranking: F (most mathematical), C, A, E, H]

The conceptualisation of a regular sequence of actions which must be followed contributes to the activity's mathematical character for the practitioner, but for the researcher brings in to question the source of the regulation and the standards against which the choice of actions will be judged. However, following a procedure itself is not quite sufficient to make an activity predominantly mathematical. For HP, solving the equation is not the most mathematical of the activities presented, because there is little or no decision-making involved in the procedure. The implication here is that as a mathematical procedure becomes more practiced and automatic, it loses its mathematical character. This echoes the not uncommon observation that "What I can do is common sense, what I can't do is maths." The number and variety of choices, within a regulated procedure, seem to combine together to make an activity 'mathematical'. The crucial role of complexity also helps to explain why mathematics is perceived as 'hard'. The following extract reinforces this point, but adds further insight into the differences of views between teacher and pupil. In solving the equation, discussing it with his peer and using the calculator, I was told:

> KF: This isn't very hard because you just use the calculator to do it.
>
> DVS: OK, and that's not doing maths?
>
> KF: It is, but no, the computer's brain is doing maths, not yours.
>
> DVS: What are you doing then, if you're using a calculator?
>
> KF: Nothing, just asking, it's like asking the calculator for the answer.
>
> [Ranking: C (most mathematical), E, A, F, H]

The component acts undertaken by KF in the interview all constituted the sort of activity which takes place in the mathematics classroom, sanctioned by the teacher. However, mathematics is seen as an activity which requires intellectual involvement, not just the use of mathematical tools.

MOVE TO DISCOURSE

It became clear through the theoretical and empirical investigation that I should have to review the theoretical basis of the interviews and analysis. Previous experience and the discourse in which the practitioner sites them self have a crucial role in the formation of conceptualisations, as do the mediating objects employed. It was also common to see participants experimenting with words, and in doing so becoming more confident in their conceptualisation. A vital element of expression in the interviews (and in most everyday and classroom language use) was the flexibility granted to the use of terms which may formally have very strict meanings, but in the interviews were seen to inherit 'local' meanings. The employment of the words ‘squares’ and ‘rectangles’ was particularly notable, reflecting their everyday use. This fluidity of language, guided by the ongoing conversation, and the halting, sometimes incomplete character of the interviewees’ articulations caused a critical light to be cast upon the use of PCP and the constant comparison approach, in the analytical dependence of both on language and expression. Consequently, reliable analysis of responses would need to incorporate not only a close attention to what the participants said, but also an awareness of the discourses in which they may site themselves (Evans, 2000a, p. 97). The desire of PCP to be free of any specific culture also means that the theory does not place an individual’s constructs in the context of social structures; hence constructs’ social origins and development are not examined in great detail.

FORMULATION OF FURTHER QUESTIONS

The compulsion referred to above as an external standard against which one’s actions are judged, and against which one is expected to judge oneself, lies at the heart of expression and conceptualisation. Such comparison with versions of reality maintained apart from the individual lies at the heart of the workings of discourse, and was reflected in one of the key observations made of the interview data.

SA: This one is purely squares and rectangles, but this [painting D] has, this has some rectangles but they come out and it doesn’t… and the lines at the top, it just doesn’t really look as mathematical as this one, so I picked G.

DVS: OK, RH can I ask you to explain your thoughts? You don’t necessarily have to agree with SA, you can just tell me what you were thinking.

RH: That one [G] because, it’s like symmetrical,

DVS: Aha, OK, go on.

RH: And you’ve got, like, straight lines, and that, this [A] is all like, wobbly. And this one’s [H] got quite a lot of different shapes in it, but they’re a bit like, you can’t really see a proper pattern.

[Rankings: SA and RH; G (most mathematical), A, D, H, E]

In this extract we see a pattern which was distinct throughout the interviews. When the participants were confident that their impressions matched well with an established reality, their conceptualisations were styled as statements about independently existing objects and activities which acted on the practitioner of their own accord. However, when the thoughts they wished to express were divergent from that reality, the conceptualisations came in terms of their perceptions and rationalisations about the world, asserting their own subjectivity. The transition between these two modes requires further exploration. The next step in my research is to explore how aspects of discourse come in to play in identifying oneself as doing mathematics. In addressing this question, I anticipate that my study will also help us understand about peoples’ need to work with essentialising descriptions of the world and their place in it, as opposed to constructive descriptions.

REFERENCES

Abreu, G.D., Bishop, A.J. & Pompeu, G. (1997) What children and teachers count as mathematics, in: T. Nunes & P. Bryant (Eds) *Learning and teaching mathematics: An international perspective*, pp. 233 - 264, (Hove, Psychology Press Ltd).

Ernest, P. (1998) *Social Constructivism as a Philosophy of Mathematics*, (Albany, New York, State University of New York Press).

Evans, J. (2000a) *Adults' mathematical thinking and emotions: A study of numerate practices*, (London, RoutledgeFalmer).

Fransella, F. & Bannister, D. (1977) *A manual of repertory grid technique*, Academic Press).

Fransella, F. & Thomas, L. (Eds.) (1988) *Experimenting with personal construct psychology* (London, Routledge & Kegan Paul).

Kelly, G. (1970) A brief introduction to personal construct theory, in: D. Bannister (Ed) *Perspectives in personal construct theory*, pp. 1-2, (London, Academic Press).

Noss, R., Hoyles, C. & Pozzi, S. (2000) Working knowledge: Mathematics in use, in: A. Bessot & J. Ridgway (Eds) *Education for mathematics in the workplace*, pp., (Dordrecht, The Netherlands, Kluwer Academic Publishers).

Picker, S.H. & Berry, J.S. (2000) Investigating pupils' images of mathematics, *Educational Studies in Mathematics*, 43(1), pp. 65 - 94.

Tall, D., Thomas, M., Davis, G., Gray, E. & Simpson, A. (1999) What is the object of the encapsulation of a process? *Journal of Mathematical Behaviour*, 18(2), pp. 223 - 241.

APPENDIX

The author will be happy to e-mail copies of the research instrument used. Please contact him at: vosper.singleton@gmail.com

	Paintings			Activities
A	R Delaunay, *Homage to Blériot* (1914).		A	Solving the equation $x^2 = 50$
D	P Klee, *Haupt- und Nebenwege* (1929).		C	Solving logic puzzles
E	E Heckel, *Windmill, Dagast* (1909).		E	Packing a suitcase
G	B Riley, *Movement in Squares* (1961).		F	Baking a cake
H	A da Messina, *Saint Jerome in his Study* (c.1475-6).		H	Reading a book