VARIATIONS ON A THEME: INTRODUCING NEW REPRESENTATIONS OF FRACTION INTO TWO KS3 CLASSROOMS

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Two teachers in different schools participated in a research project looking at the use of technology based representations of mathematical objects. Each used the same software, incorporating a novel representation of fraction as a dynamic functional relationship between values on two number lines. They planned together, discussing the characteristics of the software, the educational goals and modes of use as well as sharing resources and ideas about student tasks. In practice, the lessons each taught were very different and the ways in which students made use of the software also differed substantially. Influences on the nature of teachers' incorporation of new elements into their pedagogic practice are discussed, including consideration of explicit and implicit theoretical frameworks and of institutional and cultural contexts.

CONTEXT OF THE STUDY

The study reported here was carried out as part of the work of the European Research Team TELMA (Technology Enhanced Learning in Mathematics), part of the Kaleidoscope Network of Excellence[1]. TELMA has brought together six research teams from four countries, working in the area of digital technologies and mathematics education. After initial discussion, sharing and synthesising of the work of the teams, a small-scale methodological experiment was planned and carried out with the aim of making theoretical frameworks and the role of institutional and cultural contexts in forms of use of technology more explicit. In this so-called "cross-experimentation", each team designed and conducted an experiment using a technological tool designed by a team from another country to support learning in the area of fractions. The methodology used for this collaborative work is discussed in Artigue et al. (2007). Some of the outcomes have been reported in the project deliverables[2] and will be published elsewhere in the future. Analysis across national contexts has allowed insights into systemic differences. However, in this paper I will focus only on the conduct and outcomes of the experiment carried out by the London team, based at the Institute of Education.

E-SLATE FRACTION-SLIDERS MICROWORLD

The tool used was a microworld constructed using E-Slate (http://e-slate.cti.gr/), a toolkit for building exploratory environments devised by the Athens team (NKUA-ETL). The Fraction-sliders microworld offers two main types of representation of fraction: "sliders" or dynamic number lines and symbols in the form of Logo. These allow a fraction to be represented (a) as a relationship between values shown by positions on two linked number lines (sliders) and (b) as a number entered into a Logo procedure. The numbers entered in the Logo procedure determine the relationships between the values displayed on the sliders. Dragging the pointer on the 'control' slider causes proportionate changes in position of pointers on one or more dependent sliders. An analysis carried out by the research team at the planning stage of the experiment identified aspects of these representational features of E-Slate that we felt could be significant in learning about fractions. In particular:

- relating visual to symbolic representations provides opportunities to coordinate consideration of relationships between numerator and denominator with an evaluation of the size of a fraction
- students can use fractions with numerators and denominators of any size rather than just those simple enough to manipulate using paper and pencil. While this might take students beyond the demands of the curriculum, it was thought that going beyond the bounds of familiar numbers would support conceptual rather than instrumental learning.
- the slider representation allows a qualitative approach to the size of a fraction and to comparisons between fractions, rather than a computational approach.

DESIGN OF THE STUDY

The IoE team consisted of one university-based researcher (myself) and two teacher-researchers, both of whom were at the time students on the MA in Mathematics Education course at the IoE. We worked face-to face and through email to clarify the aims and design of the study. There was also cross-TELMA web-based exchange of materials and discussion of research questions and theoretical frameworks. In this paper, however, I will focus only on the local study. Our research aim was articulated to investigate representations of fraction and how they are used in lessons with the Fraction-sliders microworld. This aim was conceived at two levels. For the two teacher-researchers, the focus of interest was the representations used by their students. For me there was a further focus on the ways the teachers themselves would make use of the representations afforded by E-Slate or by other available media and the interactions between teachers' and students' employment of representations.

The process of design of the study started with individual and joint exploration of the Fraction-sliders microworld in order to familiarise ourselves with the tool and to consider ways of using it within the limited time available. Through this initial familiarisation, we decided that the visual display of the sliders provided an environment in which it was natural to compare the sizes of fractions. This idea was refined to construct two types of tasks: comparing and ordering fractions - predicting and using the software to check the prediction; finding a fraction between two others and using the microworld to demonstrate the result. An outline plan was agreed for three lessons. However, pedagogical issues were not discussed and it was left to the individual teacher-researchers to plan the details of their lessons. This lack of design of pedagogy was a deliberate choice to allow study of the ways in which the two teachers would make use of the microworld and its representations in their teaching.

Data was collected in the form of student written work, audio-recordings and fieldnotes of the lessons. In order to gain further insight into how the teachers were making sense of the experience and into the theoretical resources on which they were drawing, the teacher-researchers' analyses of the work of their students and their written reflections at the end of the study were also collected as data.

DIFFERENCES IN IMPLEMENTATION

In the rest of this paper, I will describe and attempt to understand pedagogic differences in the series of lessons taught by each of the two teacher-researchers.[3] First it is important to note that there were differences in the organisational and cultural contexts of the two classrooms. Notably, School 1 has a strongly controlled 'traditional' ethos while School 2 is relatively liberal or 'progressive'.

Both teachers used a consistent lesson structure that conformed broadly to the 'three-part lesson'. In the detail, however, there were significant differences between the pedagogies of the two teachers. An important area of difference was in the strength of the control maintained over student participation in whole class interaction. Teacher 1 structured the introduction to each lesson using a PowerPoint presentation displaying the main questions he intended to ask. There was strong asymmetry between his role and that of the students. He adopted a position of authority over what might be considered legitimate knowledge in the classroom. For example, as shown in Extract 1 from early in the first lesson, the teacher-student interaction followed a strong IRE (initiation, response, evaluation) pattern with *explicit* evaluation.

Extract 1: Teacher 1 interaction

Teacher 1: …which is the largest fraction out of those two, D?

Student D: 3/6

Teacher 1: Why do you think it might be 3/6?

Student D: Because three of the … three sixes …one….(inaudible)

Teacher 1: Can anybody explain a little further, she's not wrong, I know she knows what she's talking about. E.

Student E: Because three (inaudible)

Teacher 1: Right, excellent.

In contrast, Teacher 2 introduced each lesson with a discussion whose direction was not pre-determined. Although initial questions were planned, they were more open in nature and concepts introduced by the students became part of the discussion. As may be seen in Extract 2, Teacher 2 legitimated the students' contributions *implicitly* by echoing or revoicing them rather than by making explicit evaluative comments.

Extract 2: Teacher 2 interaction

Teacher 2: What do you think is happening here when you move the top slider? […] What do you think over here girls?

Student A: I don't know. They just all seem to be moving when you move the top one along like that in a diagonal line.

Teacher 2: They're moving diagonally.

Student B: They're moving proportionally, all three of them.

Teacher 2: Can you try and think about what those proportions might be? How would you try and work it out?

Student B: If you move it like that

Teacher 2: Move it right over to the end

The lower level of control in Teacher 2's pedagogy was partly a consequence of her questioning and feedback style but also seemed related to her approach to the mathematical content of the lessons. From the start, Teacher 1 focused closely and explicitly on fractions and calculations, while Teacher 2 focused on qualitative relationships: her introduction to the microworld did not even mention fractions until the word was introduced by one of the students.

Similar differences were apparent in the structuring of student activity during the parts of the lessons when they were working individually or in pairs with the microworld. As well as differences in the types of task set, students were allowed different degrees of control over the software tool. The students in School 1 were provided with exactly the configuration of the software that was required for each task and their interaction with the software was restricted to entering values in the Logo procedure and manipulating the sliders. In contrast, students in School 2 were shown how to change several aspects of the microworld, allowing them to move flexibly between tasks, and apparently legitimating exploration of other features of the software (some, for example, discovered how to change the colour of the sliders and experimented with pink and orange backgrounds).

UNDERSTANDING DIFFERENCE: THEORETICAL RESOURCES AND CONTEXT

As indicated above, one of the aims of TELMA is to understand the roles that theoretical frameworks and cultural and institutional contexts may play in influencing the design and use of technological tools in mathematics education. Reflecting on the differences between the practices of the two teachers involved in this small study, we can speculate how these may relate to institutional differences. In particular, the differences in the strength of framing of the pedagogies of the two teachers, indicated in this analysis of control, appear consistent with my initial identification of 'traditional' and 'progressive' ethos at the level of their schools.

Identifying the role of theoretical frameworks poses further problems as the relation between theory and practice is not straightforward. It is possible, however, to consider at least some of the theoretical resources available to the teachers and how they appeared to deploy these in order to make sense of their practice. Through their participation in the project, reading project materials and discussing the background of the project as a whole and the purposes of the specific experiment, the two teacher-researchers came into contact with a number of theoretical ideas. In particular the aims and design of the study were informed by ideas from socio-cultural theory (especially the idea of semiotic mediation) and social semiotics (focusing on meaning making within situational and cultural contexts). The design of E-Slate was explicitly informed by constructionism, providing an environment in which learners will encounter and construct mathematical ideas. Additionally, the teachers' recent participation in an MA module on the role of digital technologies in mathematics learning and teaching meant that they had been involved in reading and discussions informed by constructionism and constructivism. These ideas were visible in both teachers' descriptions of their planned student activity as "exploration". The following extracts from their reflections on the outcomes of the experiment, however, suggest that the two teachers had very different orientations.

For Teacher 1, despite espousing exploration, it is "exposure" to examples which leads to generalisation. His main focus in reflecting on the outcomes is on student development of specific skills and strategies within the topic domain. The main role of the microworld is presented as facilitating acquisition of traditional forms of knowledge. At this point, his reflection does not appear strongly influenced by any of the theoretical resources made explicit during the project.

Extract 3: Teacher 1 reflections on outcomes

> Pupils were able to get through far more questions … This exposed pupils to far more examples and hopefully enabled pupils to think more generally …
>
> A significant number of pupils began to be able to successfully predict outcomes by rounding fractions, '96/350 is about one quarter but 34/70 is just less than a half, so that must be bigger.' …
>
> It was interesting to see how the pupils felt free to just 'try any old fraction' … This … led to an arbitrary fraction being tried and then the denominator being 'stuck to' and the numerator being altered until successful, this technique was certainly only possible with this software …

In contrast, Teacher 2's focus is on the language and meanings generated in interaction with the representations provided by the tool. This focus is compatible with the original framing of the aims and design of the study, presenting the microworld as a semiotic tool that may structure learning.

Extract 4: Teacher 2 reflections on outcomes

> Initially their talk mainly centred on the 'distance' of the sliders from one another, but some then started to talk about the movement of the sliders …
>
> What I thought was interesting about the replies was that those students who used a 'static' form of language ("the gap is bigger", "more space is taken up") tended to get the answer wrong, whereas those who used a more 'dynamic' language ("it moves faster", "it travels further") tended to get the answer right …

> Finding a fraction between 2/5 and 3/7 was hard as the fractions are so close together. This brought out a confusion about the meaning of 'in between' (does it have to be exactly in the middle?)

The ways in which the two teacher-researchers planned, taught and talked about their work were also influenced by the implicit theories of learning embodied in the official discourse of the National Curriculum and National Strategy guidance for teachers and by the more local discourses of teaching and learning current in their respective schools and departments (cf. Morgan, Tsatsaroni, & Lerman, 2002). As practitioner researchers, the teachers were engaged in research but simultaneously engaged in their professional role as teachers. Where they used resources from discourses of research, including those of explicit theory, these were thus recontextualised (Bernstein, 1996), acquiring new types of meaning as they were put to new purposes within the practice of teaching in their institutional contexts.

This study perhaps raises more questions than answers about the complex interaction between institutional context, pedagogic orientation and teachers' use of available theoretical resources. Nevertheless, analysis of teachers' practices allows us to think about consistencies and inconsistencies and to seek their sources. In the context of the design of technological tools, their introduction into mathematics classrooms and dissemination across different contexts of use, it is important to recognise that, while the alternative representations of mathematical objects offered by such tools provide possibilities for new ways of thinking, the meanings actually available to teachers and students are highly contingent on the contexts within which they are working.

NOTES

[1] Kaleidoscope http://www.noe-kaleidoscope.org is funded by the EC under FP6 (IST–507838).

[2] See the TELMA website http://telma.noe-kaleidoscope.org/outcomes/

[3] There were also differences in the ways the teachers configured and interacted with the Fraction-sliders microworld itself, with consequences for the nature of the mathematical knowledge encountered but there is not space to discuss this in the present paper.

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