CODING STRATEGIC BEHAVIOUR IN MATHEMATICAL PROBLEM SOLVING

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The crucial role that strategic behaviour plays in achieving mathematical problem-solving success has been well documented by research into learning mathematics. Strategic behaviour is also important for self-regulation, a goal for mathematical problem solving (MPS). A coding scheme that consists of cognitive, self-regulatory, resource management and task strategies allocated at each stage of MPS was developed to code and analyse primary students' self-regulated MPS behaviour. The coding scheme was piloted after three studies which involved carrying out clinical interviews with students of year 4, 5, and 6 while working on word mathematical problems. A coded transcript of a clinical interview is presented as an example to illustrate the applicability of the coding scheme.

THEORETICAL FRAMEWORK

The importance and relevance of strategic behaviour to the theories of Self-Regulated Learning (SRL) and Mathematical Problem Solving (MPS) has been well documented by research into educational psychology and mathematics education (Pintrich, 1999; Verschaffel *et al.*, 1999). SRL, or taking charge of one's learning, is closely related to the ability of effectively using a diversity of learning strategies which can be used for a lifetime and monitoring the progress of the use of these strategies (Hamilton & Ghatala, 1994). Zimmerman and Martinez-Pons (1986) first used and defined the term 'self-regulated learning strategy' as the "actions directed at acquiring information and skill that involve agency, purpose (goals), and instrumentality self-receptions by a learner" (p. 615). They identified fourteen classes of SRL strategies such as self-evaluation, organizing and transforming, goal-setting and planning. Besides psychologists, many mathematicians have pointed the crucial role of strategic behaviour (Polya, [1945] 1957; Posamentier & Krulik, 1998), though by the term they generally mean mathematics strategies, like finding patterns or intelligent guessing and checking (Posamentier & Krulik, 1998) whereas others term these strategies as 'heuristics'(Verschaffel *et al.*, 1999). Pape and Wang (2003) admitted that strategic behaviour is central to processing mathematical word problems and identified twelve categories of strategic behaviour by studying middle school students while working on word MPS such as self-evaluating, organising and transforming, planning and seeking information. As one can observe, those are very similar to the categories of Zimmerman and Martinez-Pons (1986) although were observed in a more domain-specific context, word MPS.

Pintrich (1999) has grouped the various SRL strategies into cognitive, metacognitive and resource management. Cognitive learning strategies are the rehearsal, elaboration and organisational strategies and can be applied to simple memory tasks, such as recall of information, and to more complex tasks such as trying to understand a piece of text. In particular, rehearsal strategies help the students to focus on and select important information from texts and keep this information active in working memory; however they do not reflect a very deep level of processing. Some examples are the recitation of information and saying of words aloud as reading a piece of text. Elaboration strategies help the students understand the new information by establishing relationships between the new and the pre-existing information. Some examples are the paraphrasing or summarizing a piece of text and explaining ideas. Finally, the use of organizational strategies declare a deeper level of processing and understanding of the material since the employment of these strategies include behaviours such as selecting, outlining and organizing the main ideas using various representations like drawings or diagrams.

Metacognitive or self-regulatory strategies include planning, monitoring and regulation behaviours (Pintrich, 1999). Planning strategies help the students to plan their use of cognitive strategies and trigger their prior knowledge in order to make the organization and comprehension of the task easier. To be more precise, they encompass behaviours such as setting goals and doing a task analysis in order to activate any relevant prior knowledge. Monitoring strategies are probably the most crucial aspect in SRL since they "… alert the learner to breakdowns" (Pintrich, 1999, p. 461). They include self-testing through the use of questions about the task to check understanding and monitoring comprehension while reading a text. These strategies inform the learner whether or not she is on the way to achieve a goal. If the student realizes that something is going wrong she will enable the use of regulation strategies in order to 'restore' behaviour towards the achievement of a goal. In this respect, regulation strategies are closely related to monitoring strategies and they can be extremely important in that they improve students' understanding and "… bring the behaviour back in line with the goal" (Pintrich, 1999, p. 461). Examples of regulation strategies include going back and rereading a piece of complicated text and reviewing aspects of one's work. Finally, examples of resource management strategies are the help seeking strategies which enable students to identify when they are not able to proceed further and so find the appropriate source of assistance.

THE CODING SCHEME

As mentioned before the aim of the paper is to present a coding scheme as a way of analyse students' strategic behaviour in terms of self-regulated mathematical problem solving. What is innovative about the scheme is that it emerged after the transfer of Pintrich's theory of SRL to the domain-specific context of word MPS and after conducting three studies of observing and clinically interviewing (Ginsburg, 1997) year 4, 5, and 6 students working either individually or in groups on routine or process problems (LeBlanc, 1982). The first study, conducted in UK in 2005, involved observing one group of five students working on process tasks whereas the second study, carried out also in 2005, involved clinically interviewing five groups of two or three students working collaboratively on similar tasks. Finally, the third study, carried out in 2006, concerned individual clinical interviews with 33 students while working on both process and routine tasks. The second and third studies were conducted in Cyprus where teaching MPS focuses on routine and process tasks. The clinical interviews of the second and third studies were video-taped.

The new-born scheme (see Table 1) consists of hierarchical stages, similar to Polya's ([1945] 1957) model but only with three stages, the fewest possible. The name of each of the three stages, 'forethought', 'performance' and 'self-reflection', respectively, was 'borrowed' from Zimmerman's (2004) self-regulated learning model in order to establish a connection with SRL theory and stress that the goal is not just successful problem solving but also student-regulated problem solving. In forethought stage students are expected to read and analyse the text of the problem (F1 to F8), in 'performance' stage they are supposed to carry out their plans (P1 to P6), and in 'self-reflection' students are opt for verifying the answer obtained (S1 to S8). Each stage consists of various cognitive, metacognitive and resource management strategies whereas the 'performance' stage includes additionally the mathematics-task strategies. The allocation of each strategy at the three stages was decided by the frequency of its occurrence that was observed during the three studies[1]. As one can observe, there are eight strategies in forethought and self-reflection stages and six in performance stage. This indicates the importance of analysing, planning and verifying procedures. Furthermore, as one moves from forethought to self-reflection stages the number of metacognitive strategies increases whereas the cognitive strategies decrease. In addition, certain strategies can be found in more than one stage of the scheme due to their critical role in the process of MPS. Examples of these are the 'talking to herself' and 'explaining ideas'. 'Talking to herself or self-verbalization is considered as a key learning strategy due to the crucial role language plays in self-regulation processes. Furthermore, 'explaining ideas' is also important since this can aid students become critical listeners in classroom discussions and also help them to become familiar with strategies used by their more sophisticated peers (Pape & Wang, 2003).

AN EXAMPLE

To illustrate how one can use the above coding scheme to analyse students behaviour in word MPS, I present an example from the third study with a year 5 student (S22) who is being clinically interviewed by me-the researcher (R) while working on the two-step routine task: *Three friends are counting the stamps of their collections during school break time. Andri has triple the stamps Nina has. Maria has triple the stamps Andri has. If Andri has 99 stamps how many stamps do Nina and Maria have?* In brackets one can see the code attributed to each excerpt.

1 [As the student is reading the task quietly, she underlines the sentences 'Andri has triple…the stamps Nina has. Maria has triple the stamps Andri has.'] (F1)

2 [She then circles the question: 'If Andri has 99 stamps how many stamps do Nina and Maria have?'] (F1)

3 [S22 re-reads it quietly.] (F5)

4 S22 [writing at the same time]: I will do 99 …, that’s Andri’s divided by 3… (P3)

5 S22 [looking at the R]: Because it says triple than Nina. (P4)

6 R: To find what?

7 S22: To find how many stamps Nina has. (P4)

8 [She does the calculation vertically, silently.] (P2)

9 S22 [looking at the text]: And I find 33 for Nina. (P3)

10 [She gets down Nina equals 33 stamps.]

11 S22 [looking at the text, whispering]: Andri, it says to us that Andri has 99. (F6)

12 [She gets down Andri equals 99.] (F1)

13 S22 [looking at the text]: Now I will find… (F6)

14 [S22 re-reads from the forth to the sixth lines of the text.] (F5)

15 S22 [confident, to the R]: Maria…because it says that she has triple the number Andri has, meaning that I will do 99 times 3. (F3)

16 [She does the calculation vertically.] (P2)

17 S22 [looking at the R]: ..and I found that Maria has 297. Do I need to find the number of stamps Maria and Nina have together? (P6)

18 R: No, the question is only about their stamps separately. You did very well so far. Explaining your thought is what matters to me. Keep talking aloud.

19 S22: Now I check to see whether I did these correct… (S4)

20 S22 [re-reading]: Andri has triple than Nina…(S1) (S7)

21 S22 [looking at her notes]: Correct [she ticks the sentence at the text]. (S2)

22 S22 [re-reading]: Maria has triple the stamps Andri has… (S1) (S7)

23 S22: Correct [she ticks the sentence at the text]. (S2)

24 S22 [to the R]: Andri has 99… And I have found the other two so it is correct. (S4)

The analysis followed several general principles in order to achieve high levels of consistency. For example, there is a difficulty in deciding whether a student was talking to herself or she was explaining ideas to the researcher. To deal with it, I decided to focus on whether the student was looking at the researcher or at the text of the problem. For instance, in excerpts 4 and 5 the decision of the type of strategy was made according to the direction of the eyes of the student[2]. When the student was looking at the researcher while she was deliberately trying to explain her ideas then this was labelled as an ‘explaining ideas’ strategy (excerpts 5, 15) whereas if the direction of her eyes was steady at the text or at any directions besides the researcher then this was assigned as a ‘talking to herself’ strategy (e.g. excerpts 4 and 9). Another example sets out from the difficulty to determine the boundaries between the three stages and decide at which stage of the scheme a student is at a certain time of her work. What I considered to be crucial was the behaviour of the student around the answer to the problem. For instance, when the student was trying to understand the problem but without directly pursuing to give an answer then her behaviour was being in the forethought stage (e.g. excerpts 1 to 3). When the student was struggling to give an answer to the problem by manipulating the numbers, this entailed that she was in the performance stage (e.g. excerpts 4 to 9).

Table 1: The coding scheme for self-regulated mathematical problem solving

Strategy [*type of strategy*]
Forethought stage
F1: Distinguishing relevant from irrelevant data: underlining or verbalizing key words or phrases, passive note-taking of the relevant data. [*Cognitive-rehearsal*]
F2: Paraphrasing the material to be learnt in her own words. [*Cognitive-elaboration*]
F3: Explaining ideas or plans. [*Cognitive-elaboration or Metacognitive-planning*]
F4: Using representations to organize thought: a table, a diagram, a drawing or a list. [*Cognitive-organisation*]
F5: Going back and rereading a portion of text. [*Metacognitive-regulation*]
F6: Thinking aloud: talking to herself, self-questioning. [*Metacognitive-monitoring*]
F7: Seeking for assistance. [*Resource management*]
F8: Saying the words aloud as reading. [*Cognitive-rehearsal*]
Performance stage
P1: Adding data to the existed representation. [*Cognitive-organisation*]
P2: Using a mathematics strategy: selecting the right calculation, writing the mathematical equation that fits the problem, using guess and check method, looking for a pattern, working backwards, using logical thinking. [*Mathematics-task*]
P3: Thinking aloud: talking to her self, self-questioning. [*Metacognitive-monitoring*]
P4: Explaining ideas. [*Cognitive-elaboration*]
P5: Checking and correcting her last steps. [*Metacognitive-regulation*]
P6: Seeking for assistance. [*Resource management*]
Self-reflection stage
S1: Checking if the outcome is reasonable and meets the conditions of the problem. [*Metacognitive-monitoring*]
S2: Reviewing her notes and the answer obtained. [*Metacognitive-regulation*]
S3: Checking if there is another solution. [*Metacognitive-monitoring*]
S4: Explaining ideas. [*Cognitive-elaboration*]
S5: Seeking for assistance. [*Resource management*]
S6: Thinking aloud: talking to herself, self-questioning. [*Metacognitive-monitoring*]
S7: Saying the words aloud as reading. [*Cognitive-rehearsal*]
S8: Trying different ways of solving in order to check if the same outcome is obtained. [*Metacognitive-monitoring*]

CONCLUSION

The analysis of the transcript presented above indicate that the employment of various cognitive, metacognitive, resource management and mathematics strategies at every stage of word mathematical problem solving is crucial to achieve self-regulated mathematical problem solving behaviour. The paper presents a coding scheme that enables the analysis of such behaviour at every stage of MPS. However, the scheme needs to be tested in terms of inter-rater reliability in order to be suitable to be used as a tool for analysing primary students' mathematical problem-solving behaviour as this is being delineated through the process of clinical interviews.

NOTES

1. Strategies that were not frequently observed were excluded from the scheme. To see in detailed how the coding scheme was developed see Marcou and Lerman (2006).

2. There are cultural biases about looking at a person. However, since all the interviewees were Cypriot students, such biases seem to do not cause any problems to the coding procedure.

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