

A SYSTEMATIC REVIEW OF THE ROLE OF ICTS IN LEARNING ALGEBRA

Maria Goulding and Chris Kyriacou

University of York

This paper focuses on a systematic review of studies on how different ICT tools can be used to develop understanding of functions. ICT tools included the use of different software (e.g. spreadsheets and graph plotting software) and different hardware (e.g. graphics calculators, computers used by pupils and computers used by teachers with the whole class). There was evidence of gains in understanding in particular aspects of functions, evidence of some difficulties in interpreting screen displays, and evidence of productive ways of working.

BACKGROUND

The third systematic review conducted by the Mathematics Education Review group coordinated by the authors at the University of York was commissioned by the government's Training and Development Agency for Schools (TDA) and managed through the Evidence Informed Policy and Practice Information and Co-ordinating Centre (EPPI) at the Institute of Education, University of London (see www.eppi.ioe.ac.uk for other reviews).

The TDA were interested under what conditions ICT could be used to develop mathematical understanding. Although the U.K has invested heavily in information and communications technology (ICT) in schools, simply providing ICT equipment and promoting its use is not enough to produce more than weak gains in attainment (Higgins, 2003). In mathematics, Ofsted (2004, pp. 4-5) reported that 'the use of ICT to promote progress in mathematics remains a relatively weak... aspect of provision...[and] is not as effective as in many other subjects...' On the other hand, Sutherland (2004) found that the mathematics teachers in the Interactive project had a legacy of ICT use which enabled them to incorporate it more smoothly into their practice and transform their teaching.

Algebra was chosen for the curriculum focus because of its centrality to the mathematics curriculum and its perceived difficulty. For many pupils the deeper meanings and purposes for algebra are hidden and they see it as a meaningless activity in which they have to memorise rules and methods for manipulating symbolic expressions (Kieran, 1994). We took algebra to be the use of symbols to express generality, with functionality as an overarching concept. We took algebraic understanding to mean making meaningful connections and relationships including being able to identify, describe and use functions represented in different ways.

We hoped to make comparisons between ways in which algebraic ideas are developed using different software and to make comparisons between similar algebraic ideas developed using different hardware e.g. when graph plotting is being

taught using devices with the whole class, stand alone computers or graphical calculators. We hoped to come to judgments about the relative merits of these different ICTs.

METHOD

Relevant studies (33 papers) were identified by electronic searches using keywords with bibliographic databases, hand-searching conference proceedings, citations and publications recommended by contacts, and matched against criteria for inclusion. They fell into three main categories (the development of symbolism, the relationship and interpretation of different ways of representing functions, operations on symbolic expressions). The second category (14 papers, see Appendix), was chosen for further in depth analysis using the review question:

How have different ICTs contributed to the development of understanding of functions for pupils up to the age of 16 with particular reference to the relationship between different representations and the interpretation of graphical representations?

FINDINGS

Gains in understanding: Godwin and Sutherland, Gray and Thomas, Hegedus and Kaput, each using pre-post test design, give evidence of general gains in interventions each using one type of ICT. Sivasubramaniam, using pre-post test design with a control group, found that pupils working in a computer medium performed better than those in a paper and pencil medium although both made gains in graphical interpretation. Isiksal and Askar, compared the performance of three classes, two taking computer based instruction (Autograph and Spreadsheet) and one without technological tools. The Autograph group outperformed the other two classes, but the non-technology class outperformed the spreadsheet class. This class had been taught how to use the spreadsheet but not in a mathematical context. This points to the importance of the design of the particular software and how it is introduced to the pupils.

Nature of understanding: Borba and Confrey, Doerr and Zangor, Friedlander and Stein, Godwin and Sutherland, Godwin and Beswetherick, Hegedus and Kaput, found some students successfully using visualisation with graphing software to fit graphs to datasets, interpret linear and non linear graphs, solve equations and transform functions. In interpreting rates of change, Sivasubramaniam found pupils working in a computer environment reached higher levels of thinking and could explain their thinking better than pupils working in a paper and pencil medium. On the other hand, Yerushalmy, using contextualized tasks, found lower attaining students preferring to work arithmetically with tables of values and only later integrating tables of values with computer generated graphs. Gray and Thomas found some pupils having difficulty moving between symbolic, tabular and graphical forms

when solving equations. Some of these differences may be accounted for by differences in the tasks and whether the tasks were context free or contextualised.

Difficulties of working with graphics calculators: Hershkowitz and Kieran, Mitchelmore and Cavanagh, found that students do not always know how to use the technology, have difficulty interpreting ambiguities in the output and exercising critical judgment when using advanced calculators. These studies show that the learner has to learn how to use the tool critically before it can be used effectively and also that difficulties in using the tool effectively may be exposing conceptual difficulties.

Ways of working : Doer and Zangor, Godwin and Beswetherick, Godwin and Sutherland found students working together in small groups and also interactively with their teachers in whole classes provided a learning environment in which the ICTs were harnessed productively. The individual/small group use of the technology gave pupils the opportunity for inquiry and experimentation (Gray and Thomas; Hershkowitz and Kieran; Yerushalmy) However, unless the teacher pulled this together and orchestrated whole class sharing, ICT use in particular could encourage each individual student to develop their own very idiosyncratic knowledge which may or may not accord with the common knowledge the teacher was intending to develop. In one study the connectivity of the computers allowed the teacher to demonstrate the work of individual pupils and build up collective knowledge in this way (Hegedus and Kaput).

Borba and Confrey found that when one student worked with a researcher the ability to listen carefully to the student was seen to be crucial. Ninness found that a behaviourist drill approach was successful with one dyslexic student. Freidlander and Stein found students could evaluate the respective advantages of several tools. Gray and Thomas; Godwin and Beswetherick; Mitchelmore and Cavanagh, found students who use ICT out of school are better able to use it effectively within school.

IMPLICATIONS FOR POLICY AND PRACTICE

This review offers some support for the use of ICTs in learning algebra as advocated in current government policy for England and Wales (DfEE, 1991; DfEE, 2001).

There is a strong emphasis in the associated documents which are helpful, including both pure and contextual examples. There is also a need for

- Making links between tables of values, symbolic representation and graphical representation in the process of modeling;
- critical use of graph plotters (scale, zoom, windows, interpreting pixel displays).

Flexible use of the three part lesson structure (DfEE, 2001) could support a mix of individual/group work and whole class interaction to allow the experimentation, direction and sharing which seems to maximise the potential of the ICTs. Time spent

on constructing meanings in this way would seem to be particularly important in algebra.

Graphics calculators and computers can be used by pupils in individual or group activity, interactive whiteboards or computers with projectors can be used for whole class work and *both have a place*. With increased use of the teacher controlled interactive whiteboard, pupils still need opportunities for autonomy and experimentation afforded by graphics calculators/computers with personal constructions shared with the whole class. The mathematics teacher has a pivotal role in structuring and supporting the learning using both digital and non-digital technology. Simply using ICT will not guarantee that students make more learning gains than using traditional paper and pencil methods.

Teachers need to be confident users of the technology themselves. They need to know how the scale, window and resolution may present misleading images. One way of overcoming these difficulties would be to smooth the path for students by setting the scale and window for them. Another way is to use cognitive conflict, to present students with a puzzling image (e.g. part of a parabola looking like a straight line because of the choice of scale) and encourage them to work through their misconceptions. Students need to be alerted to these possibilities.

Teachers need to help students make links between symbolic, tabular and graphical output by making these links explicit. A common approach to graphing functions is to start with a symbolic expression, make a table of values and plot these by hand. This can give students a point wise view of a function, a process to be done rather than an object in its own right. Graphical software has been hailed for taking plotting away from the learner and presenting the graph as an object which can be explored. This is very important when investigating families and transformations and checking whether functions are equivalent, but when solving equations a point wise view is also needed, as the coordinates of specific points on the graphs will give solutions. It is important then that these links are made explicit when working within any one of the representations. For instance, when solving equations, students could compare strategies involving tables of values (trial and improvement), symbolic manipulation methods, and reading off solutions from graphs. The review indicates that a full understanding of the links between different representations may take time and may be facilitated by regular access to the technology.

IMPLICATIONS FOR RESEARCH

Although most of the studies in the in-depth review were judged to be of high quality, there tended to be little justification given for the choice of sample and little attention to issues of reliability and validity at both the data collection and analysis stages. It would also be helpful to declare what counts as success in an intervention. Some interventions, although taking place with a whole class, select single students or pairs for report. Whilst this gives a valuable in depth picture of the potential of ICT we do not know how typical these responses were or why these pupils were selected for

report. More detail about the participants in the sample would enable the reader to gauge the limits on generalisability, and provide a useful starting point for large scale evaluation. One of the studies, whilst reporting statistical gains overall, was cautious in claiming too much for the intervention because only a minority of students attained multi-representational fluency. Other researchers, however, may have claimed this as a success.

In terms of substance, we need to know more about how teachers can provide carefully structured support in order to make full use of the ICT tools within the whole mathematical environment. In particular, we do not as yet have enough comparative evidence of individual/small group use, and whole class use with electronic whiteboards and/or computers with projectors. Here the focus would be on the mathematical understandings together with the ways of working.

APPENDIX

Studies included in the in-depth review

Borba MC, Confrey J (1996) A Student's Construction of Transformations of Functions in a Multiple Representational Environment. *Educational Studies in Mathematics* 31: 319-37.

Doerr H, Zangor R (2000) Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics* 41: 143-163.

Friedlander A, Stein H (2001) Students choice of tools in solving equations in a technological learning environment. In van den Heuvel-Panhuizen M (ed) *Proceedings of the 25th PME International Conference*, 2, 441-448.

Godwin S, Beswetherick R (2002) Reflections on the role of task structure in learning the properties of quadratic functions with ICT - a design initiative. In Pope S (ed) *Proceedings of the British Society for Research into Learning Mathematics conference*, pp 43-48. Nottingham University.

Godwin S, Sutherland R (2004) Whole-class technology for learning mathematics: the case of functions and graphs. *Education* 4: 131-152.

Gomes-Ferreira GV (1998) Conceptions as articulated in different microworlds exploring functions. In Olivier A & Newstead K (eds) *Proceedings of the 22nd PME International Conference*, 3, 9-16.

Gray R, Thomas M (2001) Quadratic equation representations and graphic calculators: procedural and conceptual interactions. In Bobis J (ed) *Numeracy and beyond: proceedings of the twenty- fourth annual conference of the Mathematics Education Research Group of Australasia Incorporated*. University of Sydney.

Hegedus SJ, Kaput J (2003) The effect of SimCalc connected classrooms on students' algebraic thinking. In Pateman NA, Dougherty BJ & Zilliox JT (eds) *Proceedings of the 27th PME International Conference*, 3, 47-54.

- Hershkowitz R, Kieran C (2001) Algorithmic and meaningful ways of joining together representatives within the same mathematical activity: An experience with graphing calculators. In van den Heuvel-Panhuizen M (ed) Proceedings of the 25th PME International Conference, 1, 96-107.
- Isiksal M, Askar P (2005) The effect of spreadsheet and dynamic geometry software on the achievement and self-efficacy of 7th-grade students. *Educational Research* 47: 333-350.
- Mitchelmore M, Cavanagh M (2000) Students' Difficulties in Operating a Graphics Calculator. *Mathematics Education Research Journal* 12: 254-68.
- Ninness C, Rumph R, McCuller G, Harrison C, Ford AM, Ninness SK (2005) A Functional Analytic Approach to Computer-Interactive Mathematics. *Journal of Applied Behavior Analysis* 38: 1.
- Sivasubramaniam P (2000) Distributed cognition, computers and the interpretation of graphs. In Rowland T & Morgan C (eds) *Research in Mathematics Education Volume 2*, pp 169-190. London: British Society for Research into Learning Mathematics.
- Yerushalmy M (2000) Problem Solving Strategies and Mathematical Resources: A Longitudinal View on Problem Solving in a Function Based Approach to Algebra. *Educational Studies in Mathematics* 43: 125-47.

REFERENCES

- Department for Education and Employment/Qualifications and Curriculum Authority (1999) *Mathematics: the National Curriculum for England*. London: DfEE/QCA.
- Department for Education and Employment (2001) *Key Stage 3 National Strategy: Framework for teaching mathematics: Years 7, 8 and 9*. London: DfEE.
- Higgins, S. (2003) *Does ICT improve learning and teaching in schools? A Professional User review of UK research undertaken for the British Educational Research Association*. Southwell, Notts.: BERA.
- Kieran, C. (1994) The learning and teaching of school algebra. In Grouws, D. A. (ed.) *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan. 390-419.
- Ofsted (2004) *Report: ICT in schools – the impact of government initiatives: Secondary Mathematics*. www.ofsted.gov.uk (accessed March 3rd 2006)
- Sutherland, R. (2004) Transforming teaching and learning: embedding ICT into everyday classroom practices. *Journal of Computer assisted Learning*. 20(6), 413-425.