PROFILES OF UNDERSTANDING AND PROFILES OF DEVELOPMENT IN EARLY ARITHMETIC

Camilla Gilmore

Learning Sciences Research Institute, University of Nottingham

Children's knowledge of arithmetic is made up of many components (e.g. conceptual understanding, procedural skill, factual knowledge) and is therefore difficult to capture with a single measure. Profiles of understanding map children's performance across tasks and can reveal differences among children in the relationship between components of arithmetical skill. A re-analysis of several studies examining children's conceptual and procedural knowledge of addition and subtraction revealed subgroups of children with different profiles of understanding. These subgroups may represent different routes to the development of arithmetic knowledge and so it may be important to consider profiles of development as well as profiles of understanding.

KNOWLEDGE COMPONENTS OF ARITHMETIC

Arithmetic knowledge consists of several components including procedural skill, conceptual understanding and factual knowledge. While there may often be a fuzzy distinction between these types of knowledge, conceptual knowledge generally involves "knowing why" and consists of a network of relationships among pieces of information while procedural knowledge involves "knowing how-to" and consists of sets of procedures executed in a specific sequence (Baroody & Tiilikainnen, 2003; Hiebert & LeFevre, 1986). These different knowledge components may be closely related and interact over development (e.g. Rittle-Johnson, Siegler, & Alibali, 2001). Given this multi-faceted view of arithmetic, it is difficult to capture children's understanding with a single measure. Tasks typically used to assess mathematics achievement tend to focus primarily on procedural skill.

PROFILES OF UNDERSTANDING

To obtain a complete picture of children's arithmetic understanding, multi-dimensional measures are required. These approaches can go beyond a single measure to examine children's understanding of different components of arithmetic, and their ability to apply this understanding to different tasks. Thus they provide a measure of children's 'profiles of understanding' across the arithmetic domain. Researchers have used different methods to examine children's knowledge of multiple arithmetic components.

Bisanz and LeFevre (1992) proposed a framework for gaining a richer picture of children's knowledge by examining understanding in multiple contexts. Different types of understanding can be revealed by using different activities (e.g. application, justification or evaluation of a procedure that reflects certain concepts). For each activity, tasks differ according to the generality of the situation. If children only demonstrate application of a procedure for one situation then this may reflect rote learning, however successful performance on a range of problems indicates greater generality and greater understanding. Within this framework, understanding is considered in terms of a profile of performance across a range of contexts, rather than performance on a single task. Development can be represented as a sequence of profiles or spread of understanding across contextual space.

Other researchers have examined the different types of arithmetic knowledge that children use. Canobi (2004) examined patterns of addition and subtraction knowledge in young children. She measured children's understanding of a range of arithmetic concepts using problem solving and judgement tasks and assessed procedural calculation skills. Several groups of children with different patterns of conceptual knowledge and procedural knowledge were identified, and children in higher conceptual groups tended to have more sophisticated procedural calculation skills.

INDIVIDUAL DIFFERENCES IN PROFILES OF UNDERSTANDING

One benefit of assessing children's mathematics performance using profiles of understanding is that they can reveal individual differences in the relationship between components of arithmetic skill. Rather than just differing in the absolute level of performance, children differ in the areas of arithmetic which are strengths and weaknesses. Profiles of understanding can reveal these different patterns of performance.

For example, Dowker (1998) found that, as a group, 5- to 9-year-olds' understanding of arithmetic concepts was related to calculation skills, but there was wide variation in the conceptual understanding shown by children with similar levels of arithmetic skill. Canobi (2004) grouped children aged 6- to 8-years old according to their understanding of arithmetic concepts, and also according to their level of arithmetic skill. In general, children in more advanced conceptual groups were also in more advanced procedural groups, however, there was wide variation with nearly 15% of children showing a marked discrepancy between conceptual and procedural grouping with either more advanced conceptual or procedural knowledge.

Gilmore & Bryant (2006) examined 8-year-old children's profiles of understanding of addition and subtraction. Children's understanding of the conceptual relationship between addition and subtraction was measured using problems that involved an inverse transformation (e.g. 10 + 5 – 5 = ?). Children's procedural skill at performing addition and subtraction transformations was also assessed using standard arithmetic problems (e.g. 10 + 8 – 3 = ?). To look at individual differences in profiles of understanding, statistical cluster analysis was used to group children according to their performance across these two measures. This revealed that there were three groups of children who showed different characteristic performance. One group of children had good conceptual understanding of arithmetic and also good calculation skills. A second group of children had poorer conceptual understanding and poorer calculation skills. The final group had good conceptual understanding but poor calculation skills. The final group had a surprising pattern of skills in that their understanding of addition and subtraction concepts were far advanced of their procedural skills. This study suggests that children may show different profiles of understanding of arithmetic, but cannot reveal why these differences come about, or how widespread they are.

CROSS-STUDY ANALYSIS

To discover whether the different profiles of understanding found in the study by Gilmore and Bryant (2006) were widespread and to understand why these groups exist, a much larger sample of children is required, including children of different ages. To do this, data was obtained from studies that had been carried out examining children's conceptual understanding and procedural skill with arithmetic (Gilmore, 2005; Canobi, 2005; Bryant, Christie & Rendu 1999; Rasmussen, Ho & Bisanz, 2003). In all of these studies, children were given a set of problems that involved an inverse transformation (e.g. 10 + 5 – 5 = ?) and a set of standard arithmetic problems (e.g. 10 + 8 – 3 = ?). In total, data was obtained from 13 different studies involving children aged from 3 years to 10 years old. The studies were carried out in the UK, Australia and Canada. The methods used to present the problems differed somewhat across the studies, with younger children being presented problems involving concrete materials, while older children tended to be presented problems using numerical digits. To compare across the studies, the proportion of correct responses on inverse and control problems was used as the measure of performance. Although the studies may differ in the absolute levels of performance, this is not critical since it is the profile of performance across different problem types that is of interest.

The data consisted of the scores of 655 children on inverse and control problems. To examine individual differences in performance on these measures, and to find out whether the findings of Gilmore and Bryant (2006) were replicable, a cluster analysis was performed on this data. This analysis grouped children according to their profile of understanding across the measures to reveal subgroups with similar profiles of performance. This analysis suggested that there were four different groups of children. Figure 1 displays the mean performance on the conceptual (inverse) problems and the control (calculation) problems for each group of children.

Groups 1 (n = 168), 2 (n = 99), and 3 (n = 300) show similar patterns of performance, although at different levels of success. These children score somewhat higher on the inverse problems than the control problems (indicating that they have some understanding of the inverse relationship between addition and subtraction) and their calculation skill, measured on the control problems, increases in line with their conceptual understanding. So, these three groups appear to consist of children whose understanding of arithmetic concepts is developing along with their procedural calculation skills. In contrast to these groups, the children in Group 4 (n = 187) show a very different profile of performance. Their conceptual understanding of arithmetic is very good and far more advanced than their calculation skills. This group appears to consist of children whose conceptual understanding of arithmetic is developing separately from their procedural skills, and who have particular difficulties with calculation.

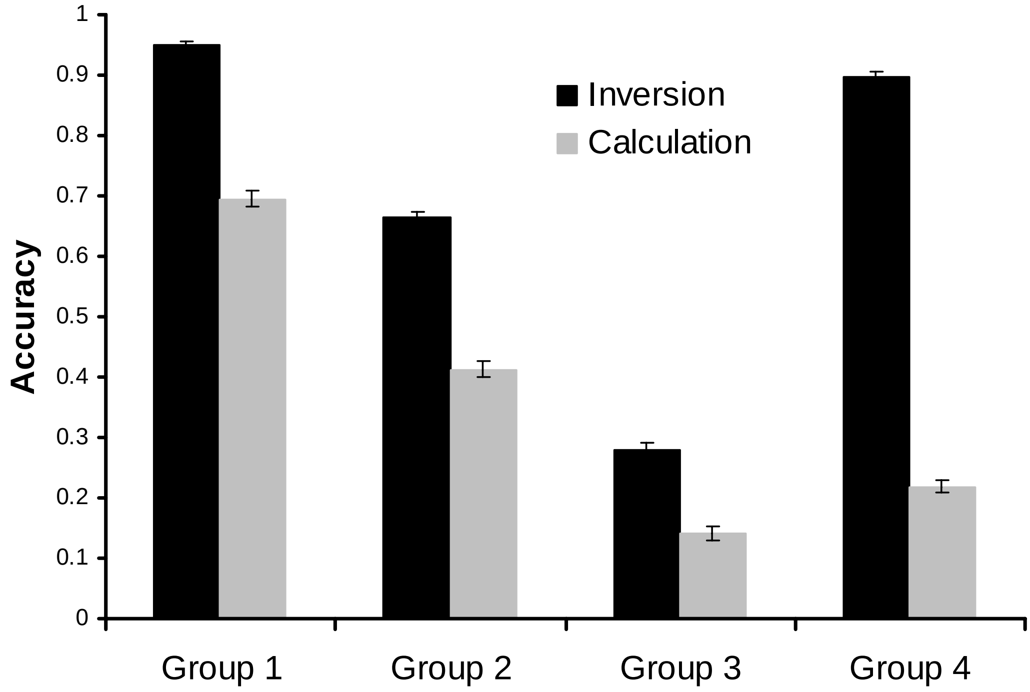

Figure 1: Mean performance on problems involving a conceptual relation (inversion) and standard arithmetic problems (calculation) by children in each subgroup.

The characteristics of the children in each group were compared. There was no difference in the proportion of girls and boys in each of the four groups ($\chi^2(3) = 6.97$, n.s.). However, the ages of children in each of the groups did differ. There was a significant difference in the ages of the children in groups 1, 2 & 3 ($F_{(3,645)} = 34.34$, $p < .001$). The children in group 1 were the eldest (mean 8 years 8 months), then group 2 (mean 7 years 10 months), with group 3 the youngest (mean 7 years 1 month). The children in group 4 (mean 8 years 4 months) did not differ in age from either groups 1 or 2.

This cross-study analysis found similar profiles of understanding to the smaller study by Gilmore & Bryant (2006). Most of the children showed conceptual understanding that was in line with their procedural skills. However, a substantial number of children had conceptual understanding that appeared to be developing in advance of their procedural skills.

PROFILES OF DEVELOPMENT

The different groups of children found in the cluster analysis outlined above suggest that children do not all follow the same route to learning arithmetic. While conceptual and procedural knowledge may develop together for most children, this is not always the case. Therefore we need to consider different routes to development of arithmetic skill.

Other studies have also found some evidence that there may be different routes to learning arithmetic concepts and procedures. For example, Canobi (2005, Exp. 1) found that some children understand arithmetic concepts first when problems involve concrete representations, while other children understand arithmetic concepts first when problems involve abstract numbers. A second study (Canobi, 2005 Exp. 2), demonstrated that some children learn about the inverse relationship between addition and subtraction first by understanding that adding and then subtracting the same quantity will result in no change (e.g. 5 + 3 – 3 = 5), while other children learn about this relationship first by linking a given addition sum with its complementary subtraction sum (e.g. 5 + 3 = 8 therefore 8 – 3 = 5).

The evidence that children may follow different routes to learning about arithmetic, suggests that we need to consider profiles of development as well as profiles of understanding (Figure 2). While profiles of understanding can be used to capture differences among children in their knowledge of different arithmetic components at a single point in time, profiles of development capture differences among children in how their knowledge of different arithmetic components changes over time. This allows us to capture different developmental routes. For example, some children's conceptual and procedural knowledge of arithmetic may develop together, while other children's conceptual knowledge develops in advance of procedural skill.

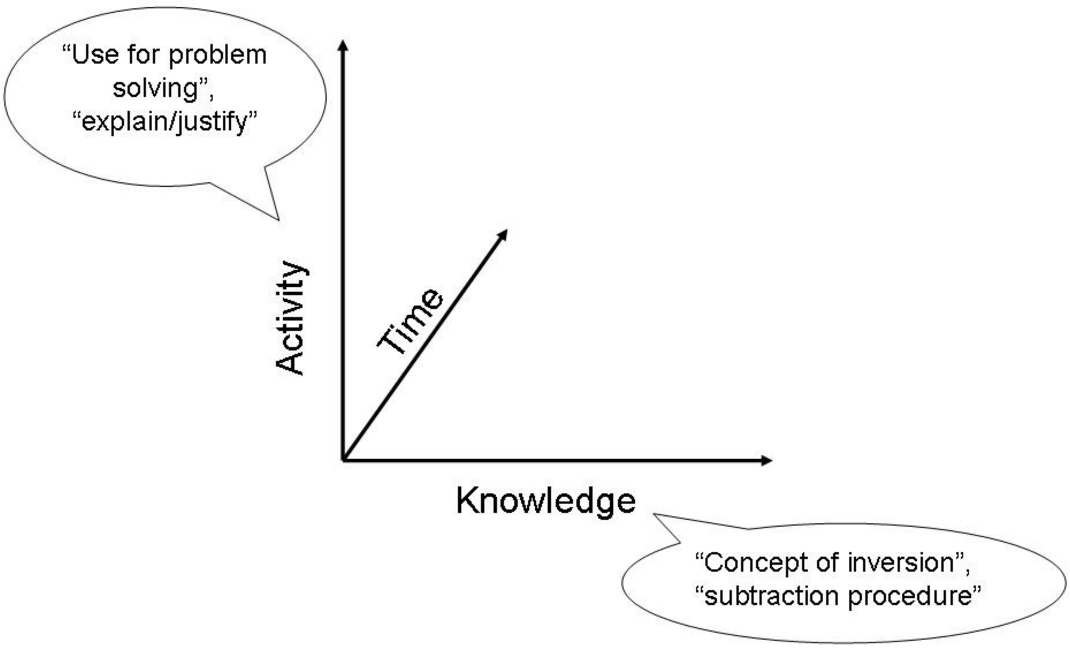

Figure 2: Children's understanding of arithmetic can be considered across different knowledge types (conceptual, procedural), different activities (explaining, using) and changing across time.

CONCLUSIONS

In order to capture a full picture of children's understanding of arithmetic, it is essential to consider both multiple components of arithmetic and individual differences among children. Multi-dimensional methods of assessment are needed to measure children's profile of understanding across different arithmetic components. Changes over time in the way that children's knowledge of different components develops must be considered to capture different profiles of development. To understand how different components of arithmetic knowledge interact over development and learning, longitudinal studies are needed. These studies can reveal whether children follow different paths in learning arithmetic.

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