TOWARDS CLASSIFYING QUALITIES OF QUESTIONS AND PROMPTS IN MATHEMATICS CLASSROOMS

Anne Watson

University of Oxford

My task is to analyse a collection of classroom videos in a way which draws attention to the range of mathematical possibilities available to teachers in the public tasks, questions and prompts of mathematics classrooms. To do this I have explored the use of several frameworks which identify different intended learning outcomes, different 'levels' of thought and different aspects of mathematical thinking. None of these pick up the subtle variations I see in the videos.

BACKGROUND

In the IAMP project (Watson, DeGeest & Prestage, 2003) we took a phenomenographic approach to charting the practices of teachers who deliberately worked to counteract disadvantage and underachievement. We found that, apart from a few features (such as giving learners space to learn, maintaining the complexity of mathematics) belief, persistence and courage seemed to be more important than specific teaching tactics (Watson & DeGeest, 2005).

In our current three-year project, Changes in Mathematics Teaching (CMTP) the target students are the same, those who enter secondary school below national target levels, but the unit of analysis has shifted to mathematics departments. We are chronicling the stories of three maths departments who deliberately set out, at the start of last year, to redeem a significant number of such students. We shall describe their practices, and identify factors which contribute to, or hinder, success. Currently we are analysing data from the first year. We have interviews with teachers and teaching assistants; interview and test data (where available) from some students; documents; observation notes of department meetings; copies of resources and so on.

First we do a content analysis, followed by analysis of content from a range of perspectives. Students' mathematical work is analysed from a mathematical perspective, using criteria derived mainly from a 100% coursework GCSE syllabus (SEG, 1988) to identify factual knowledge, technical knowledge, and adaptive and transformative thinking. Interview data from teachers is sorted using third generation activity theory (Engestrom, 1998), seeing interaction between departmental and classroom activity as a site for identifying parameters of change.

We are finding that, while third generation activity theory helps us 'lay out' the data in ways which manage the subsequent comparisons between teachers, between schools and for each teacher and department over time, this approach loses much of the detail which will eventually have to be reported. We are not looking for a theory of departmental change so much as pragmatic information for others. Activity theory allows us to construct the stories of the departments, but the range of practice which was so important to readers and policy makers who responded to our earlier research is not revealed in this approach. We also have a responsibility to talk about general classroom issues. For example, we fed back to heads of department a list of ‘comforts and conflicts’ which we identified in early teacher interviews – made anonymous and expressed in general terms – for department discussion; we also sent back to individual teachers analytical accounts of our first round of lesson videos. The analysis of these videos provides the focus for this paper.

LESSON VIDEOS

The purpose of videoing lessons was to collect a sample of classroom practice over the duration of the project to get some sense of its range and of any similarities and differences, or patterns, between and within schools. It is important to note that the departments appear to espouse similar overarching interests in the development of mathematical thinking. None of them chose ‘drill, skill and kill’ as an approach to rescuing learners. The videos are used only for research purposes, and individual teachers are given a copy which can be used for in-house professional development purposes if they wish. Teachers give informed consent to the video on this basis, and students are given the informed opportunity to opt out of the research.

The problem I face is how to analyse the videos so that I can produce a full description of the range of practices in classrooms, at a level of detail which is informative for the research schools and more widely. I do not want to merely reproduce knowledge from *Better Mathematics* (Ahmed, 1987), *Deep Progress in Mathematics* (Watson, DeGeest & Prestage, 2003) and elsewhere. For these two publications, similarity, condensation, and descriptions of ‘normal’ practice were important; for the current work, descriptions of dimensions of variation are important. In the TIMSS seven-nation video study, for example, descriptions of typical national lesson types were constructed (Hiebert et al., 2003) but while these enable cross-national comparison, we know that all types of lesson can be done well or badly from the point of view of students’ learning and that ‘typical’ lesson structures can be reduced to meaningless dogma, so we need detail.

The first stage of analysis was straightforward, which was to produce an account of what I could hear and see which related to the unfolding mathematical story of the lesson. In other words, what utterances and actions by the teacher and others were publicly available to structure the affordances and constraints of mathematical activity (Watson, 2004)? While making these accounts I had to work quickly and very openly so that I could soon send them back to the teachers to indicate the nature of the interest researchers were going to take in their teaching.

Just as third-generation activity lays out the parameters of the systems within which teachers were working, so the analytical accounts lay out the public discourse of each lesson. I now have to find a way to identify dimensions of variation within these discourses and the ranges of change within them (what prompts are used and in what circumstances). It might also be important to look at how sequences of such prompts are constructed, and the supportive utterances which surround them.

FRAMEWORKS

To situate my work in the literature I looked in a variety of places for suitable frameworks to inform the next stage of analysis. Teachers expect that what they say, and the tasks they set, will help learners achieve certain learning objectives it seemed sensible to start with Bloom's (1984) and Biggs & Collis'(1982) taxonomies. I may be able to see utterances, and sequences of utterances, as affording the construction of these kinds of knowledge. For example, if learners are only offered unistructural situations by the teacher they are less likely to develop multistructural understandings.

Bloom et al.'s taxonomy of learning objectives	Biggs & Collis' SOLO taxonomy
Knowledge Comprehension Application Analysis Synthesis Evaluation	Pre-structural Unistructural Multistructural Relational Extended abstract

Bloom's taxonomy does not provide for post-synthetic mathematical actions, such as abstraction and objectification, although it could be argued that the reflection involved in evaluation could include those. However, in classrooms it is more likely that 'evaluation' will be a separate, affective and/or target-accounting process. Blooms' taxonomy also underplays knowledge and comprehension in mathematics, both of which are multi-layered and require successive experiences in different mathematical contexts. The SOLO taxonomy is more promising, in that it allows us to count input and output variables, prioritises relationships, and allows for abstraction. Translated from a model of learning to a model of teaching, however, it does not allow for interplays between simple and complex examples, between symbols and images, and between examples and generalization, which characterise mathematical activity. It is also true of any such taxonomies that it matters whose view you are taking – what may seem multistructural to a teacher may be treated as unistructural by a student.

I turned to models which describe the human activity of working mathematically, such as the van Hiele model of geometric understanding (Usiskin,1982) which characterises visualising, analysis, informal deduction, deduction and rigour. If we treat the van Hiele model as a complex, rather than as a linear hierarchy, we begin to get some sense that different kinds of mathematical action could be triggered by different kinds of task, prompts and questions.

A description of learning activity is developed in a slightly different way is the Pirie-Kieren model of mathematical understanding (1994), which attempts to map kinds of mathematical engagement and allows for 'folding-back' to earlier levels rather than assuming a monotonic outward movement. When I tried to match these layers to teachers' utterances I lost some of the subtle differences I was seeing and hearing in videos, and had to make strong assumptions about the purpose of others.

I needed frameworks which describe teaching intentions, but take into account the mathematical complexity and kinds of learning activity detailed above. The METE project is promising (Andrews, Hatch & Sayers, 2005). This focuses on what teachers do, and it classifies features of mathematical meaning and structure without assuming that learners necessarily do what is intended. Thus it takes the viewpoint of what is afforded and constrained in the public mathematical discourse. Their framework looks at teachers emphasising and encouraging conceptual, derivational, structural, procedural knowledge and the use of efficient methods, problem-solving techniques and reasoning. A related resource is provided by Tanner, Jones, Kennewell & Beauchamp (2005) who, while focusing on the mode of interaction, imply the pedagogical focus: lecture; funnelling; probing; focus shifts; collective reflection.

These models synthesise categories of teaching so that comparisons can be made between teachers and lessons which might explain differences in engagement and learning. For our study, we aim to categorise teaching so that a wide range of possible pedagogical choices is revealed. If the categorisation is too compact, subtle differences which might lead to different kinds of learning are hidden.

At the BSRLM session I showed a video clip of a lesson and participants talked about the social and cultural context of the lesson, the emotional and affective dimensions of the lesson, the pace of the lesson, the way the teacher treated students' answers, and the way she modelled what she wanted them to do. These are all important dimensions of variation in teaching and need open discussion.

I have tried to probe within these dimensions and identify differences in the mathematical affordances of lessons, the mathematical orientation of tasks, the opportunities to be mathematical. I have attempted to combine the strengths of the taxonomies above with the intended focus of teachers in lessons, and present the following as a step towards this aim. The bold headings are dimensions of mathematical orientation, and represent the kinds of mathematical focus I identified in videos, organised according to some of the models described above. The words in normal text categorise the focus of the public tasks, questions and prompts of the classroom. The words in italics are experimental; I believe these describe task orientations which could provide bridges between the bold dimensions.

The list is intended to be complex, and does not as yet have the essential connecting and relating features of a model. Rather it is the contents for a future model. I have adhered to normal views of hierarchies of thought in its layout, but do not think that the teaching I am analysing is unidirectional within it, nor should it be. There is further work to be done and feedback is welcome.

DIMENSIONS OF VARIATION IN PUBLIC TASKS, QUESTIONS AND PROMPTS IN MATHEMATICS CLASSROOMS, AND RANGES OF CHANGE WITHIN THESE

Declarative/nominal/factual/technical • Information giving • Copy • Define terms • Know/ask facts • 'Research' facts, definitions, techniques *Remembering* **Actions** • Do/perform • Make *Report on actions* **Perceptual/directed attention** • Offer objects with single features • Offer objects with multiple features • Identification of characteristics/properties • Offer multiple objects • Classification • Comparison • Identification of variation *Directed Orientation* **Learners' response** • Visualization • Description • Informal deduction • Informal induction • Creating objects with multiple features • Creating objects with one feature • Exemplification • 'Own words' *Personal orientation*	**Recognising implications** • Explication/ Justification • Prediction • Induction • Deduction • Identifying relationships *Analysis* **Integrating and connecting** • Generalisation • Abstraction • Objectification • Formalisation • New definition *Synthesis* **Affirming** • Application to harder maths • Application to other contexts • Mathematical implications • Prove • Evaluation of process *Rigour*

REFERENCES

Ahmed, A. (LAMP):1987, *Better Mathematics,* HMSO, London.

Andrews, P, Hatch, G. and Sayers, J.:2005, 'What do teachers of mathematics teach? An initial episodic analysis of four European traditions', in D. Hewitt and A. Noyes:eds,, *Proceedings of the 6th British Congress of Mathematics Education held at the University of Warwick*, 9-16. Available from www.bsrlm.org.uk.

Biggs J and Collis K:1982, *Evaluating the Quality of Learning: the SOLO taxonomy,* Academic Press, New York.

Bloom, B.S.:1984, *Taxonomy of educational objectives*, Allyn and Bacon, Boston, MA.

Engestrom, Y.:1998, Reorganising the motivational sphere of classroom culture. An activity-theoretical analysis of planning in a teaching team. In F. Seeger, J. Voigt, U. Waschescio:eds, *The Culture of the Mathematics Classroom,* Cambridge University Press, Cambridge, 104-125.

Hiebert, J, Gallimore, R, Garnier, H, Givvin, K.B, Hollingsworth, H, Jacobs, J, Chiu, A.M.-Y, Wearne, D, Smith, M, Kersting, N, Manaster, A, Tseng, E, Etterbeek, W, Manaster, C, Gonzales, P, and Stigler, J.:2003,. *Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study:NCES 2003-013,. U.S. Department of Education,* National Center for Education Statistics, Washington, DC.

Pirie, S and Kieren, T.:1994, Growth in Mathematical Understanding: How can we characterise it and how can we represent it?, *Educational Studies in Mathematics,* 26:2-3,, 165–190.

SEG, Southern Examining Group:1988, *ATM/SEG Syllabus in Mathematics GCSE for the 1989 Examination.* SEG, Guildford.

Tanner, H, Jones, S, Kennewell, S, & Beauchamp, G.:2005, Interactive whole class teaching and interactive whiteboards, in P. Clarkson et al.:eds, *Proceedings of the conference of Mathematics Education Research Group of Australasia,* 28(2,, 720-727, Deaking University, Melbourne.

Usiskin, Z.:1982,, *Van Hiele levels and Achievement in Secondary School Geometry*, University of Chicago, Chicago.

Watson, A. & De Geest, E.:2005, Principled Teaching for Deep Progress: Improving Mathematical Learning Beyond Methods and Materials. *Educational Studies in Mathematics* 58, 209-234.

Watson, A, De Geest, E, & Prestage, S.:2003, *Deep Progress in Mathematics: Report of the Improving Attainment in Mathematics Project*. published by University of Oxford Department of Educational Studies.

Watson, A.:2004, Affordances, constraints and attunements in mathematical activity. In O. McNamara and R. Barwell (eds.) *Research in Mathematics Education, volume 6: papers of the British Society for Research into Learning Mathematics*, 23-34.