MECHANICS SHOULD BE INTEGRAL TO SECONDARY SCHOOL MATHEMATICS

Stuart Rowlands

Centre for Teaching Mathematics, University of Plymouth

Mechanics has never been the most popular subject in A-level mathematics, either with the students, the teachers or educators. The 'innovative' attempts to popularise mechanics appear to have failed and it is conceivable that the subject will be dropped from the A-level syllabus within the next two decades. This article argues the importance of mechanics and why it should be integral to secondary school mathematics. Mechanics is the exemplar of mathematical modelling, is the logical point of entry for the enculturation into scientific thinking and provides the means to develop an understanding of the relationship between mathematics, the theoretical objects of science and the way science and mathematics speak of the world.

INTRODUCTION

Once considered boring, highly theoretical and hence irrelevant (Berry, 1990), mechanics still isn't a popular subject despite the attempts to make it relevant. Since the radical change in the A-level syllabi (now syllabus) during the late 80's and early 90's, when modelling was introduced in mechanics to make it relevant, there has been a decline in the proportion of A-level students doing mechanics (Kitchen et al. 1997). Although A-levels were changed to improve student motivation and success rate (Kitchen and Williams, 1993), with mechanics this has not been successful. There are many complex reasons for this, such as having to teach the way mechanics has been structured by the examination boards and their associated textbooks for assessment purposes. Yet despite its treatment and the straightjacket of having to teach it that way, mechanics as a subject in applied mathematics is so important that consideration ought to be given in treating it as a central topic in mathematics prior to the Sixth form. This article attempts to show the importance of mechanics both in terms of its logical character and the corresponding educational benefits.

It is perhaps safe to say that the unpopularity of mechanics extends to a large proportion of mathematics educators. Ironically, the same mathematics educators might agree on the virtue of modelling. The irony is that mechanics is the exemplar of mathematics modelling. In terms of its logical character this paper attempts to explain the importance of mechanics in modelling and how this offers the opportunity for even the most concrete thinkers to think not only in the abstract but to think how the abstract can model the world. There is a dual but related sense of the abstract here: the contemplation of (im)possible worlds (thought experiments) essential in explaining and modelling the physical world and how the theoretical objects of science and applied mathematics speak of that world. Such contemplation requires imagination (examples may include a world of no gravity, a frictionless surface, etc.) that is rule-governed by the logical character of mechanics. This paper offers a brief description of that logical character and attempts to explain how this character not only makes mechanics the central subject in science but also applied mathematics. What may initially seem contrary to the title, this article begins by situating mechanics in science so as to reveal its logical character, leading to the reasons why mechanics should be integral to the secondary school mathematics curriculum. Further reasons are then discussed, including mechanics as the exemplar of how research can inform practice.

THE LOGICAL CHARACTER OF MECHANICS AND ITS IMPORTANCE IN SCIENCE EDUCATION

There are many reasons for teaching mechanics as a central topic in science. For example:

- The history of the scientific revolution of the Seventeenth Century *essentially* began with mechanics. Essentially because the emergence of Seventeenth Century mechanics was paradigm changing: from the Aristotelian (cosmological and implied mechanical) world view to the modern one.
- Children can be introduced to mechanics without having prior knowledge of science. To contemplate the forces acting on a thrown ball, for example, requires no prior instruction.
- Mechanics is the logical point of entry for the enculturation into scientific thinking and is not merely a domain of physics that shares its place amongst many (Carson and Rowlands, 2005). This leads onto the next point:
- Mechanics is concept forming.

The last two points are the most important. Mechanics is not merely a topic amongst many but structures the very edifice of science. For example, without mechanics there would be no electromagnetism or kinetic theory of gases:

> Mechanic determines one form of description of the world by saying that all propositions used in the description of the world must be obtained in a way from a given set of propositions – the axioms of mechanics. It thus supplies the bricks for building the edifice of science (Wittgenstein, 1974, Proposition 6.341).

> Mechanics is an attempt to construct to a single plan all the *true* propositions that we need for the description of the world (Wittgenstein, 1974, proposition 6.343).

The basic interaction variable in mechanics is force (Hestenes, 1987) and so it follows that, as stated by Kitchen et al. (1997), Newton's laws should be a central component of student understanding of science. According to Galili (1995):

> A huge edifice, which today we call physics, consists of various domains. *The importance of mechanics is more than just being one of these domains. It determines the 'rules of the game', defines the main tools in physics, presents the most universal laws of nature. It actually describes the method of the discipline of physics which is then applied in all other domains in this discipline. This is why mechanics always opens any physics curriculum.* (Galili 1995, p.371, emphasis added)

Mechanics is concept forming because the axioms define force from which work can be defined and in turn the concepts of energy can be derived. The conservation laws rest upon the laws of motion. This is how the axioms supply the bricks for building the edifice of science (see Rowlands, 2003; Carson and Rowlands, 2005).

REASONS FOR TEACHING MECHANICS IN MATHEMATICS

Mechanics is an exemplar of mathematical modelling because of its concept forming character. A field may be modelled as a rectangle, the properties of the rectangle may be applied and a desired result calculated, but there is no scientific component that explains anything in the process. This is not to undermine mathematical modelling in general, but mechanics is a form of mathematical modelling that is able to explain phenomena as well as yielding the relevant data. That ability to explain is due to its logical character, giving rise to the relevant concepts. The following are reasons for why mechanics should be integral to the mathematics curriculum:

- Modelling in mechanics has primacy over other areas of applied mathematics (MA, 1965). It is the essential ingredient in understanding modern domains and is the only satisfactory vehicle for demonstrating applied mathematics as a discipline in its own right (Crighton, 1985). This may explain why “attempts to teach modelling in areas other than mechanics in A-level have, so far, proved inadequate” (Kitchen et al. 1997, p. 166).
- A mechanics course could (and should) provide the ‘linking framework’ to other areas of applied mathematics, such as differential equations which expresses the principle of many sciences and can begin with dynamics (MA, 1965).
- Unlike other forms of mathematics such as statistics and decision mathematics, mechanics provides the perfect opportunity to become fluent in algebra, trigonometry and calculus (Kitchen et al. 1997). Conversely, however, one of the few satisfactory ways in presenting the triangle law of vector addition in pure mathematics is by way of the parallelogram law of the addition of forces. This can be done using the Leeds mechanics kit. This is not to undermine the importance of other forms of mathematics, but mechanics provides the perfect opportunity to comprehend as well as to utilise pure mathematics.
- Modelling in mechanics has a ‘scientific component’ whereby the quantities modelled are well-defined, unlike other domains, such as economic theory (Crighton, 1985). Mechanics contrasts with the *pragmatic* models of finance management and population models which are “characterised by a purposeful avoidance of those fundamental concepts that the true mathematical model seek to describe. . . They seek to solve a problem and invariably omit any conceptualization procedure” (Hickman, 1986, p. 734).

This contrasts with what Clement (1982) described as the assumed modelling methodology, which is apparently structured by the ‘scientific method’ that begins with observation and is followed by hypothesis and validation. But modelling in mechanics is a process whereby an explanation is sought by formulating an appropriate model (Hestenes, 1992). As such mechanics not only provides an opportunity to understand the nature of science (NOS) but also theoretical modelling as compared with empirical or pragmatic models. NOS is not a clip board of recorded data from which relations are established, nor is theoretical modelling.

Compared with data collection, theoretical modelling engages different cognitive processes and research can inform us as to those cognitive processes and the teacher as to the likely cognitive state of her students. This is discussed next.

MECHANICS AS THE EXEMPLAR OF HOW RESEARCH CAN INFORM PRACTICE

Conceptual change is the largest area of research in science education and the biggest field of research in conceptual change is mechanics, particularly force and motion. This isn't surprising if we regard the importance of mechanics in structuring other domains in physics. Force and motion is not only fundamental with respect to the edifice of science, but also with regard to understanding learning processes, whether we frame those processes in terms of cognition or socially negotiated meaning. Mechanics can provide insights into the ways we think and learn. For example, from abstract considerations, such as thought experiments, to the way the abstract can enable learners to model real situations. Experience has shown that mechanics can also provide a means by which the concrete thinker can think in the abstract – in imaginative but rule-governed possible worlds. Research in these domains can inform the teacher the kind of intuitive responses she can expect from asking questions concerning force and motion and the strategies that can be employed in directing the mechanics class in understanding force and motion.

For example, in 'Widening Participation' sessions whereby year 9 'mixed ability' students are introduced to 'probably the most important question in science', students are asked to identify the forces acting on a vertically thrown ball going up. Research has shown that many students will identify a force pushing up along with gravity and air-resistance, and you can almost guarantee that many and if not most of the students in these sessions will do the same. The strategy adopted is to say that this answer is not the correct answer but it is a *good* answer because that was the answer given by Aristotle and accepted for two thousand years until challenged relatively recently by Galileo. The students are then engaged with the history of the subject which can put into perspective their own intuitive responses to force and motion. Mechanics provides the opportunity for research to inform practice.

DISCUSSION

In arguing for a broad mathematics curriculum at A-level and against the 'more specialist mathematical options at 16-19', Margaret Brown states

> The position of mechanics warrants special attention. Although it is a valid and important application of mathematics it should appear in the A-level mathematics course only as

> one of many, and should not have the favoured place that it currently occupies in England and Wales (and in no other country). Mechanics should properly be part of an award in engineering or physics. (Brown, 1999, p. 85).

Well before 1999 mechanics had lost its favoured place. Since the introduction of the modular scheme, many and if not most A-level mathematics students do not do M1 (the first module in mechanics) and very few do M2. Despite the recognition of the importance of mechanics in A-level mathematics, there seems to be the implicit desire to relegate mechanics to physics or engineering. Brown's argument is that 16 year old students should be given a wide choice of applicable mathematics at A-level because many of them are unaware of what career paths to take or what subject choice to make. Within the context of arguing against design options in accord with university course requirements, Brown states that 'university engineers may favour mechanics over statistics to ease their own teaching, but fail to recognise that statistics is as important a tool for engineers in employment as it is for other professionals in industry and commerce' (Brown, 1999, p. 85). Is the inclusion of mechanics really to do with 'ease of teaching'? Without undermining statistics, the converse could be argued but with a greater imperative: mechanics, because it is the exemplar of mathematical modelling, would benefit even those who go on to choose economics. Besides, to expose pre-16 year olds to mechanics as a subject in mathematics would go some way in influencing what subjects to study in mathematics at A-level. Such exposure would help develop the ability to model in mathematics as well as developing an understanding of the nature of science. Exposing pre-16 year olds to 'Data Handling' instead may well be superfluous as everything covered can quickly be taught in post-compulsory education.

That mechanics has a 'favoured place' in England and Wales is not a satisfactory argument in downplaying mechanics. The split between 'pure' and 'applied' is only just over a century old and many leading mathematicians of the past were also physicists and vice-versa. For example, Euler's mechanics for the rotation of a rigid body, Gauss's work on electro-magnetic forces, Galileo on probability. The point is, mechanics is very much a subject in mathematics as it is science and should be respected as such.

One module in mechanics is not enough to appreciate mechanics as a unified whole and there is an irony in all this: presumably M1 provides the basis for understanding mechanics (force and motion), yet after learning this 'difficult bit' students then take a module in statistics or decision mathematics. The irony is that once force and motion is understood then there is a basis for understanding all the various topics in mechanics, such as projectiles, circular motion, periodic motion etc. All these topics can be treated as the application of the laws of motion, but if M1 is the only mechanics module taught then the *significance* of M1 is lost. Mechanics needs to be treated with sufficient depth and that implies a linear course of half an A-level. This is unlikely given the status quo.

The whole argument presented above is not only a pedagogical issue but also a scholarly one. But as one commentator stated ‘deep philosophical arguments do not cut much ice with the increasingly pragmatic world of school mathematics’ (see Rowlands 2003). That world is dominated by the imposition of the state (QCA, formerly SCAA) and the examination boards ready to acquiesce in the pursuit of franchise. There has been scholarly criticism of the way the exam boards have tried to make mechanics practical, relevant and interesting, but unfortunately no response has been forthcoming. Arguments for a linear course in mechanics for all the reasons argued above may well fall on deaf ears, but change is possible. If there is a possibility for M1 to be redesigned by way of introducing thought-experiments, taking into account student misconceptions of force and motion, framing the topic historically and constructing a basis for understanding the subject as a whole – in short, a module that engages the learner with the logical structure of the subject and how that structure models the world - then there might be the demand for further modules in mechanics.

The attempts by the state and the exam boards to reform mechanics have failed and if the situation continues then there is every danger that mechanics will be dropped from A-level mathematics. If this does happen then arguably that would be a great loss to mathematics education. To prevent this from happening as *fait accompli* there must be scholarly discussion and debate and in a way that the issues cannot be ignored by those in positions of power regarding the curriculum. Practitioners, who are at the mercy of state diktat and exam board entrepreneurs, could well empower themselves by partaking in arguments concerning the curriculum and expressing the message: ‘contrary to the way we have been treated, we also have a say!’

REFERENCES

Because of space limitation, references can be found in Rowlands (2003) except:

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