SUBTRACTION OF FRACTIONS THROUGH THE EYES AND EARS OF FIFTH GRADE MODELLERS

Andreas O. Kyriakides

The Open University

The central question addressed in this paper concerns the ways in which modelling activities ground the conditions for a group of fifth graders to experience a progression in their awareness of subtraction of fractions. The teacher's narratives along with students' written work and transcripts of audio-taped class discussions constitute the primary data source for analysis. My attention is particularly drawn to a close examination of a teaching episode that appears to serve my research query. The study documents strong evidence that students could refine their fractional reasoning when exposed to a learning environment that sensitises them to detect problematicity through confronting impasses publicly, questioning themselves and peers, conjecturing and welcoming broken expectation.

INTRODUCTION

The Rational Number Project (Behr et al., 1992, 1993; Lesh et al., 1987) as well as Kieren (1988, 1993) marked a notable line of research in the area of teaching and learning fractions. Their major premise was rooted in splitting the concept of rational number into subconstructs and then binding up the complementary meanings arising from each system (e.g., part-whole, operator, etc) to achieve an integrative understanding of fractions. Though I acknowledge the importance of content in construing the meaning of such a complicated concept I remain dubious about the effectiveness of concentrating learners' focus on the particularities of each subconstruct; my uncertainty stems from Mason's (2001) caution that 'if the centre of gravity of students' attention is on the particularities of the example or model, then their awareness of the process is likely to be at best implicit and indirect' (p48). Against this background, the current paper describes an attempt to enter the world of a group of fifth graders (mostly by listening) and examine how in the midst of spoken and unspoken utterances students negotiate the meaning underlying subtraction of fractions.

THEORETICAL BACKGROUND

The study evolves around two spiralled frameworks: 'the constructivist teaching experiment' (Cobb & Steffe, 1983) and Mason's (2001) perspective of mathematical modelling. In the constructivist view, opportunities for learners to build their knowledge arise as they interact with both the teacher and their classmates; mathematical awareness is, thus, not arbitrarily generated but, instead, 'constrained by an obligation to develop interpretations that fit with those of other members of the classroom community' (p137). Sensitizing students' attention to the various stimuli conveyed either by peers or the teacher is an essential component of modelling, as well. For Mason (2001) the heart of modelling lies in an interactive shift between experiences. Once formulated, a model directs modeller's thinking 'through its particular stressings and ignorings' (p50). The key role of teachers in both the constructivist and the modelling perspective is to establish a classroom culture in which learners volunteer to struggle publicly, welcome the unexpected, feel free to conjecture and filter their world through a mathematical lens.

METHODOLOGY

The current work portrays part of a classroom teaching experiment implemented in an elementary school in Cyprus (September 2005 to June 2006). The participants in my study were a group of 22 11-year-olds (10 boys and 12 girls) taught by the author. Arising from my twofold profile as a teacher-researcher was the commitment to address all the objectives of fifth grade mathematics set by the national curriculum. Therefore, I coordinated only once a week modelling activities around fractions. In the end of each lesson I translated my experience from Greek to English and reported it in a journal; the teacher's narratives along with students' written work and transcripts of audio-taped class discussions constitute the data for analysis. As initially conceptualised, this paper aims to transmit a sense of how it is like to learn to subtract fractions in a public school classroom in which knowledge is not lectured but negotiated. The following research question is submitted as an entry point: How does the use of modelling activities in teaching subtraction of fractions influence students' learning processes?

FINDINGS

I will hereby display a teaching episode that emerged in March 2006. That day children struggled publicly to model the subtraction fact 1-2/3. It is important to note here that in the previous months my fifth graders have not been exposed to any formal instruction of fractional algorithmic procedures but have, instead, actively construed their own fractional knowledge 'in the midst of manipulating and expressing' (Mason, 1987, p210); rewording Mason's definitions to fit in the current context, I describe manipulating as the act of drawing either in one's head or on a palpable surface (paper or board) rectangular models of fractions. Expressing, on the other hand, points to learners' attempts to voice their inner experience either to themselves or others. Along with students' paper-and-pencil manipulations I occasionally employed technology-based representations as a window through which my students could glimpse a more accurate version of their own constructed models.

1 Teacher: Let's suppose that we have 1 whole minus 2/3

[Ria is coming on the board and developing Figure 1 (lines 1-7)]

2 Ria: I am drawing an area model and I leave it as a whole and I make another one and I divide it in two equal parts…. I divide it in two equal parts horizontally and in three equal parts vertically.

3 Teacher: Yes?

4 Ria: [Pause]

5 Teacher: Think aloud Ria, it doesn't matter if you make any mistake.

6 Ria: Then I will do the same with this one.

7 Teacher: Ria did the same drawing in the first area model as in the second one.

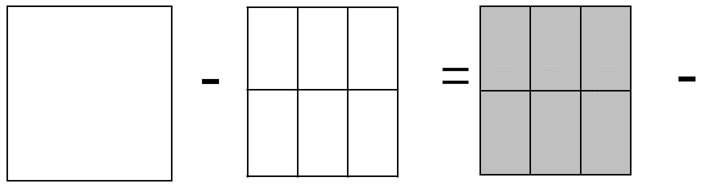

Figure 1: Ria's first attempt to model 1-2/3

8 Ria: No sir I realized my mistake; I should have divided it into 3 equal parts and take the two [Ria is developing Figure 2: lines 8-22].

9 Teacher: Okay, try this way then. So you divide the second one into 3 equal parts and you take the two. How about your first area model? What do you have there?

10 Ria: The denominator is 6 in both cases…

11 Teacher: Ria did you say that the denominator is 6 in the second one, as well? Could you explain a bit more?

12 Ria: No sir it is not 6.

13 Teacher: What is it then?

14 Ria: I divide the second area model into 6 small squares…. Hm, I divided it in half so that I could get six squares.

15 Teacher: And how much you have shaded now?

16 Ria: This one will be 6/6 minus 4/6.

17 Teacher: Why is it 4/6?

18 Ria: Because I divided this square in half and then I did…

19 Teacher: Correct! Could you tell us Ria how much did you shade now?

20 Ria: 4/6.

21 Teacher: Well done Ria and then what will you do?

22 Ria: 6/6 minus 4/6 is 2/6.

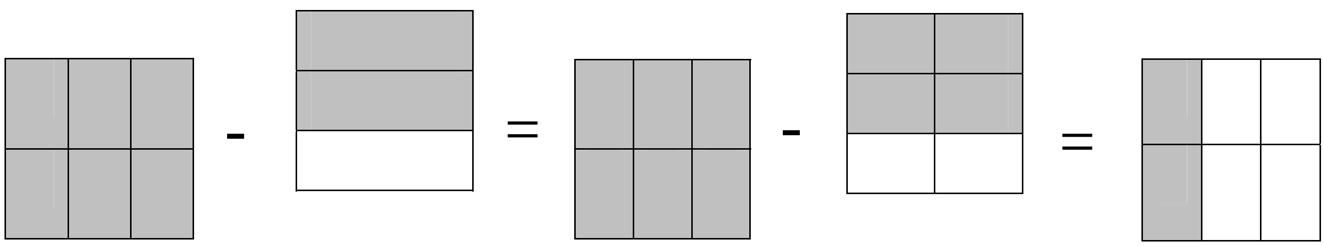

Figure 2: Ria's second attempt to model 1-2/3

23 Teacher: I saw some kids raising their hands. Who would like to say?

24 Jim: Sir I disagree with Ria's way.

25 Teacher: Would you like to say a bit more?

26 Jim: You said that the second area model is 2/3. We could divide the one whole into 3 parts and shade all of them and then the 2/3 there to transfer the columns…

27 Teacher: How about coming on the board Jim and show us? Jim will now show us another way of subtracting 1-2/3.

[Jim is coming on the board and developing Figure 3 (lines 28- 32)]

28 Jim: I draw an area model and I divide it into 3 columns and I shade 1…

29 Teacher: Why one?

30 Jim: I mean 2 because we have 2/3 and then I draw a whole. I will divide it into 3 horizontally and color all the rows. Then I will exchange the columns and we will have this one 9/9 and this one I will transfer the 3 horizontal. I will get here 6/9.

31 Teacher: So what would you have?

32 Jim: 9/9 minus 6/9 equals 3/9.

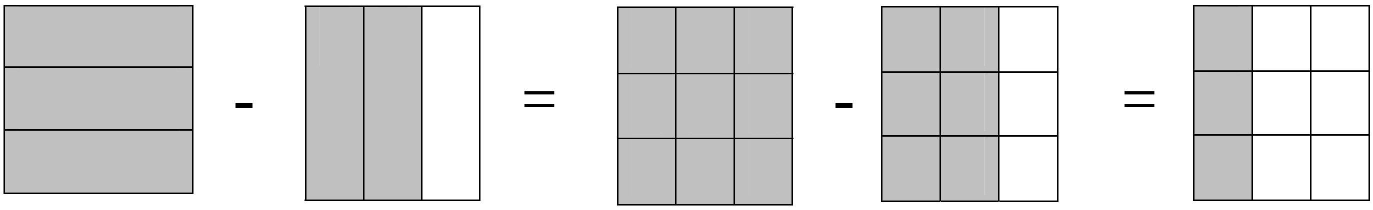

Figure 3: Jim's model of 1-2/3

33 Larkos: Is this correct sir? I think it's not correct…

34 Teacher: What would you like to say Larkos?

35 Larkos: I have another way. I will take one whole and leave it as a whole and then I will draw the 2/3…three rows and I will transfer the three rows into the one whole.

36 Teacher: Well done, and?

37 Larkos: Then because it is one whole I will color it all and then the 2/3 I will make it minus… 3/3 minus 2/3 and I will get 1/3.

38 Teacher: So we've seen three different ways of subtracting 1 – 2/3. All of the ways are correct. Let's now see how the computer works out this example.

39 Ria: It moved the one whole into the 2/3 and the 2/3 into the one whole?

40 Teacher: And why it did so?

41 Ria: To facilitate us to get the same denominators.

42 Teacher: What it did here?

43 Ria: It made them all thirds and now it will subtract… and we will get 1/3.

44 Teacher: What denominators did you use previously Ria?

45 Ria: Sixths.

46 Teacher: Sixths, it doesn't matter. And Jim?

47 Jim: Ninths.

48 Syria: But if we simplify 2/6 that Ria found won't it be 1/3?

49 Teacher:		Great Syria! All the three ways are correct. The important thing is to understand the way.
50 Xenios:		Can't we make the denominator 12?
51 Teacher:		Yes.
52 Xenios:		Basically the denominators should be multiples of 3.

DISCUSSION

There seems to be much "prima facie" evidence in the transcript signifying that students' fractional reasoning flowed as an extension of a growing awareness of an innate experience coupled with an externalisation of that experience (either in words or drawings). In the midst of Ria's public struggle to model 1-2/3 we are made witnesses of an impasse (lines 1-7); the rectangular models she developed at first do not seem convincing to her and this instantly brings to the fore a broken expectation. Instead of suffering in silence (line 4) the girl activates promptly an alternative way (line 8) whose initial vagueness (lines 10-12) motivates her to successively refine it. This finding reveals the great learning potential arisen from offering students the flexibility to discern problematicity within their own constructed models. Learning to subtract fractions is ill-defined if is interpreted in terms of input (change-into-common-denominators formula) and output (correct answer); this practice though efficient most of the times does not serve the very essence of teaching, that is, to broaden our students' horizons so that they could draw on their own conceptual repertoire when they become confused (like Ria for instance) or cannot recall a memorized rule.

Jim's (lines 23-32) and later on Larkos' (lines 33-37) publicly shared "opposition" to Ria's and Jim's approach, respectively, along with the implementation of technology-based representations sparked productive stimuli which eventually formed the ground for the transition Xenios exhibited from apparatus-situated cognition to abstract generality (line 52). In essence, Xenios re-invented the premise that to subtract a fraction from an integer the common denominator of the two numbers must be a multiple of the denominator of the subtrahend fraction. A question-mark reasonably generated is how this boy in the midst of a long silence formulated such an assertion. Instead of submitting my own interpretation I step back and consider the alternative; what if I assumed the role of the expert and displayed from the very beginning a series of routinized steps suitable for each distinct case of fractional subtraction? Would my students discern problematicity at all? Apparently not for, commonsensically thinking, there is no benefit earned from questioning the "authority". Hence, this line of research claims that true fractional reasoning emerges when students are channelled to sensitize their eyes and ears so that they could stress, as Syria did for instance (line 48), 'certain features of the what they see [and hear] as a result of their expertise' (Mason, 2001, p56).

I have hereby attempted to exemplify one perspective of what it is like to be a learner of subtraction of fractions. I offered my own situated experience as a public school teacher as evidence that incidents of conjecturing, surprise and questioning along with the use of modelling activities (technology-based or paper-and-pencil) give rise to multiple transitions in fifth graders' thinking. For the knowledgeable other, this study might not fit certain standards of methodological order, rigor and objectivity. To this end I resort to Mason's (2005) reflection; 'what has happened to the emergent, to the intuitive sensitivity that can develop between teacher and learner and that occasions the deep learning that we all value and seek?' (p472)

REFERENCES

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 296-333). New York: Macmillan.

Behr, M., Harel, G., Post, T., & Lesh, R. (1993). Rational numbers: Toward a semantic analysis - Emphasis on the operator construct. In T. P. Carpenter, E. Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 13-47). Hillsdale, NJ: Erlbaum.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83-94.

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 162-181). Reston, VA: National Council of Teachers of Mathematics.

Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. P. Carpenter, E. Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 49-84). Hillsdale, NJ: Erlbaum.

Lesh, R., Behr, M., & Post, T. (1987). Rational number relations and proportions. In C. Janvier (Ed.), *Problems of representation in teaching and learning of mathematics* (pp. 41-57). Hillsdale, NJ: Erlbaum.

Mason, J. (1987). Representing representing: Notes following the conference. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 207-214). Hillsdale, NJ: Erlbaum.

Mason, J. (2001). Modelling modelling: Where is the centre of gravity of-for-when teaching modelling? In J. Matos, W. Blum, S. Houston & S. Carreira (Eds.), *Modelling and mathematics education: ICTMA 9 applications in science and technology* (pp. 39-61). Chichester: Horwood publishing.

Mason, J. (2005). Coming of age in mathematics education: 17 characters in search of a direction? A review of classics in mathematics education research. *Journal for Research in Mathematics Education, 36*(5), 467-473.