TASK SELECTION AND TASK IMPLEMENTATION: SEVEN CONSTRAINTS AFFECTING THE TEACHER'S INSTRUCTION

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This paper examines an experienced teacher's task selection and task implementation teaching the function concept. There are seven constraints in the teacher's selection and implementation of the tasks and these factors potentially inhibit his students' engagement with the notion of function. The evidence suggest that task, by itself, does not grab the students' attention; it is the teacher's expertise in creating task conditions, such as establishing connections between the ideas and between the representations, which could promote students' understanding.

INTRODUCTION

The relationship between the types of mathematical tasks in which students engage and the mathematical notions they learn has been a subject of research over the last decades (Doyle, 1983; Stein & Lane, 1996). Prompting this interest is the idea that task provides context in which students learn to think about the subject matter. Different tasks pose differing cognitive demands on students (Doyle, 1983); thus tasks can potentially limit or broaden students' conceptions of the mathematical ideas (Henningsen & Stein, 1997). NCTM (2000) emphasised the importance of using worthwhile tasks in teaching mathematics and stated that "*Regardless of the context, worthwhile tasks should be intriguing; with a level of challenge that invites speculation and hard work*" (p.16).

Stein et al (1996) considered a mathematical task as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea or skill. They suggested that a mathematical task passes through three stages until it activates students' learning. The first is concerned with designing and presenting the tasks as they appear in the instructional materials. The second stage, *set up phase*, entails the teacher's introduction of the tasks in the classroom whilst the third stage, *implementation phase*, refers to process in which students perform the tasks they are given. Stein et al (1996) discuss two dimensions of a mathematical task: task features and cognitive demands. The former refer to aspects of a task which may require using more than one solution strategy, multiple representations, and various forms of communication styles; or they may request recalling and applying pre-presented rules and procedures. Cognitive demands refer to kind of thinking suggested by the teacher during the set up phase and the thinking process in which students engage during the implementation phase (ibid). Stein & Lane (1996) reported that when the teachers chose cognitively demanding tasks for use in the classroom and when they maintained the task demands during the implementation phase, there was an increase in the students' understanding. Henningsen & Stein (1997) indicated that teachers need to have several skills to maintain students' high-level engagement with the cognitively demanding task including providing scaffolding and asking students to give explanations for their answers.

This paper takes the interest further by examining a Turkish teacher's task selection and implementations teaching the function concept. In this study 'task' refers to function problems assigned to the students whilst 'task condition' refers to the instructional acts displayed by the teacher during his classroom teachings.

RESEARCH METHOD

As part of a larger project, a PhD study (Bayazit, 2005), the research employed a qualitative case study (Merriam, 1988). The participants were two experienced teachers and their 9th grade students (age 15). Data reported in this paper focuses on one of the teachers, Burak, who had 24 years teaching experience. Data reported in this paper was obtained through classroom observations and the document reviews (copies of student's notebooks). 14 lessons were observed. All the lessons were audio-taped and field notes were taken to record social and pedagogical aspects of the teacher's instruction and the visual attributes that the audio tape cannot detect.

CONCEPTUAL FRAMEWORK AND THE DATA ANALYSIS

The notions of 'task features' and the 'cognitive demand' (Stein et al, 1996) served the interpretation of the quality of the tasks assigned to the students and the teacher's task implementation. In particular, the notions of action-process-object conceptions of function (Breidenbach et al, 1992) provided a framework to identify key features of the teacher's task implementations. An action conception of function embraces completing a transformation of an element (an input) through step-by-step manipulations through an explicit algebraic formula (Breidenbach et al, 1992). A process conception is considered to be at a higher level in that the possessor is not only able to rationalise the actions associated with the previous step and talk about a function process in terms of inputs-outputs (ibid) but also capable of construing a function process in the light of concept definition. An object is attained through encapsulating a function process into a single unit and this level of understanding enables one to use a function in further process, for instance manipulating a function in the process of derivative (Cottrill et al, 1996).

The methods of discourse and content analysis (Philips & Hardy, 2002) were used to interpret the teacher's instructional acts displayed during the task implementations. The discourse analysis aimed not to interpret a specific teaching act in its own context but to construe it in the surrounding conditions. Lessons were fully transcribed and considered line by line whilst field notes and the copies of student's notebooks were used as supplementary sources. Initial codes were assigned to the units of meaning inferred from the texts. As this process went on the relationships between the units of meaning became clearer; thus, a system of pattern coding was employed to the data. Repetition of this second process led to creation of seven major categories which are discussed below.

RESULTS

There were seven constraints in Burak's selection and implementation of the tasks and these limitations were likely to confine his students' understanding to an action conception of function. The seven constraints are:

- Privileges procedural tasks — Burak greatly appreciated procedural tasks which could be, and were, solved through the application of rules and procedures, algebraic or otherwise.
- Prioritises procedure over the concept — concept was partially used during the task implementation; but it was not the driving force.
- Makes interference — Burak diverted his students' attention from the notion of function and engaged them with procedures or other mathematical ideas.
- Approaches to conceptual tasks in a procedural way — tasks were conceptual in that it was essential to use concept definition to solve the problems; yet during the task implementation the teacher ignored the concept definition and manipulated the tasks in a procedural way.
- Inconsistency in the presentation and implementation of the tasks — Burak did not care about the consistency in the task demands solved one after another. What he emphasised in one task was far from supporting his students' understanding of the ideas posed by the following task; and he did not utilise, when there was, the cognitive relations between the consecutive tasks.
- Does not establish connections between the representations — the representation systems, particularly algebraic and graphical forms, were not used in connection to each other though it was essential to establish such connections to support the students' understanding of the problem at hand.
- Oversimplifies the cognitively demanding tasks — tasks were cognitively demanding in that they allowed to encourage students' process or object conception of function; however the teacher totally ignored the task demands and manipulated them (functions) like an ordinary expression.

For reasons of space in this paper only two of these constraints will be discussed.

Makes interference

'Interference' refers to instructional acts that divert students' attention from the notion of function and engage them with the procedures or other mathematical ideas. It is internal to the teacher's approach to a task rather than to the epistemology of the task that is used. In Burak's teaching interference occurred at two levels: deliberate and un-deliberate interference. Apparently the former was caused by the external pressures, national and local exams, whilst the later was resulted from the teacher's failure in reorganising and presenting the students' previous knowledge to support their understanding of the function concept. The following presents a case which illustrates why and how the teacher made deliberate interference and how these acts

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might have affected his students' learning. Burak solved ten problems similar to:

> Given the function *f:R→R 2(fx)=(x+1) f(x+1)* and *f(1)=2*; what is the value of *f(2)+f(-20)*.

Burak's goal was to prepare his students for the external exams: **Episode 1:**

> … You would see such problems in the university exam leaflets… If you want to succeed in those exams you have to learn how to cope…

The key features of the teacher's instructions in similar situations can be seen as Burak explains:

> … Solving this type of problems we have to give attention to three things: the rule of function, here it is *2(fx)=(x+1)f(x+1)*; known value…*f(1)=2*; and the unknown value… *f(2)+f(-20)*. In this question we know *f* of 1… Yet, we need to find *f(2)* and *f(-20)*. To do this we have to make use of what have been given… … Let's start by putting 1 into the expression [inserts 1 into the expression, *2(fx)=(x+1)f(x+1)*, and gets the value of *f(2)* as 2]. … We are going to work out *f* of -20 in a similar way. … We are going to substitute integers from 0 till -20… [Constructs an equation system as follows]….
>
> $x=0 \Rightarrow 2\cdot f(0)=1f(1)$
>
> ……………………….
>
> $x=-20 \Rightarrow 2f(-20)=-19f(-19)$
>
> … Could you see the pattern going on in this equation system? … Do you have any idea? … [Silence in the classroom] … I will give you a hint; in such situations we usually conduct additions or multiplication among the terms on the both side of equations. … [Obtains the equation: $2f(0)\cdot 2\cdot f(-1)\cdot ...\ 2f(-20)= 1f(1)\cdot 0f(0)\cdot ...\cdot -19f(-19)$]… There is zero there; it makes the right side of the equation zero…so it has finished…

Clear enough that Burak makes great effort to ensure that his students understand the method of the solution that works in similar situations. In this respect he provides sound heuristics by decomposing the problem into three parts and explaining how to use the given expression and the values to get the unknown values. He encourages the students' inductive reasoning (seeing a pattern in the equation system), which is a valuable idea. Nevertheless, in every step of the task implementation Burak keeps the students detached from the notion of function. In the implementation of such problems the spoken language is the basic instrument to maintain the students' mental contact with the concept of function (Bayazit, 2005); yet Burak uses action-oriented language (which emphasises arithmetical procedures), "*...the value of f(-20) is 0...*", instead of using a process-oriented language (which points out the function process which does transformation), such as 'the function, *f*, transforms -20 to 0'.

Prioritises procedure over the concept

The essence of this approach was that the teacher started the task implementation with the concept — concept was introduced through one or two short sentences; yet as the instruction developed he left the concept there and emphasised rules, procedures, and factual knowledge; thus these routines became the subject of inquiry. For instance, consider the episode when the students were given the following task:

> What are the values of 'a' and 'n' for which $f(x)=(a-8)x+(2a-n+3)$ represents an identity function.

Burak explains:

> … An identity function matches every element to itself… We represented it in two forms, $f(x)=x$ and $I(x)=x$. … …it [the given expression] should be in the form of $f(x)=x$. … It means this expression should not involve a constant term and…the coefficient of the x must be 1. So, it has finished…we are going to equalise the coefficient of the x to 1 and get the value of 'a'. … We are going to work out [the value of 'n'] in a similar way. We told that the rule of identity function does not involve a constant term; so it means that the constant term must be zero. … [Sets up an equation $2a-n+3=0$ and solves it]…

The episode shows the extent to which Burak prioritises procedure over the concept. He focuses students' attention upon $f(x)=x$ and emphasises, through comparison, that the given expression must not involve a constant term and the coefficient of x must be one; yet he does not illustrate the underlying meaning of this factual knowledge in relation to the description of the identity function he gave in the first sentence.

DISCUSSION AND CONCLUSION

It is generally agreed that task shapes the students' way of thinking; thus they influence the students' learning (Doyle, 1983; Henningsen & Stein, 1997). Cognitively demanding tasks are particularly appreciated in teaching mathematics (NCTM, 2000). This might have psychological ground; cognitively demanding tasks, contrary to procedural ones, could provide a rich context to encourage students' critical thinking. Nevertheless, our evidence suggest that task does not engage the students. Any task, by itself, cannot be considered as a mediator between the mathematical ideas and the students' learning. It is the teacher's expertise in creating task conditions, such as using apposite language which is consistent with the epistemology of the function concept, establishing connections between the ideas and between the representations, caring continuity and consistency in the task demands performed in succession, and using the definition of function as a cognitive tool during the task implementation, which could foster the students' development of the function concept. As has been illustrated Burak's teaching includes seven constraints. Fundamental to all is that the teacher makes reduction in the task demands. In this case the teaching does not focus the students' attention on the key aspects of function necessary for them to make progress in their learning. Consider again the teaching episode on the task: *work out the values of 'a' and 'n' for which the* $f(x)=(a-8)x+(2a-n+3)$ *represents an identity function*. No matter whatever the cognitive demand this task poses; it can be solved in two different ways. It can be performed, as Burak did, through applying the procedure — make the coefficient of the x one and get rid of the constant term, without any connection to underlying meaning. Or it can be implemented in such a way that the definition of identity function is used as a

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cognitive tool to illustrate why the coefficient of the x must be 1 and why the constant term should be eliminated. These two different teaching approaches are likely to produce qualitatively different learning in the students.

In closing, it appears that the exam system has considerable effect on the teacher's task selection (see Episode 1). This pressure might be greater than that noted in this study and could be affecting the teacher's task implementation as well. This is an issue that needs to be explored with a large sample of teachers and the consequences should be considered with regard to curriculum design and the classroom practices.

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