ANALYSING GEOMETRIC TASKS CONSIDERING HINTING SUPPORT AND INSCRIPTIONS

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The study was undertaken to determine the features of tasks within the secondary geometry curriculum that inhibit, or promote, access to the mathematics they contain. It was conducted as a part of a PhD research project considering the development of the skills needed to become a good geometer through interactions with resources. The resources considered were the textbook used to teach geometry, the examinations used to assess geometrical knowledge and understanding and dynamic geometry software packages used to underpin the understanding and learning that takes place within the regular lessons.

BACKGROUND

Geometry has traditionally been seen as one of the more difficult areas of the mathematics curriculum, both in terms of teaching and learning it. (Royal Society/JMC 2001) The major difference between tasks in geometry and those in other areas of the mathematics curriculum is one of language. Geometry is rich in specific, formal, mathematical language. For example, the following extract from a textbook gives details of one of the circle theorems:

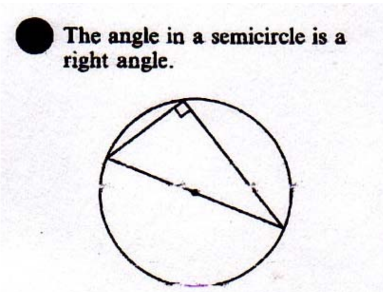

Figure 1: Thales' Theorem: Heylings (1987),

The diagram and accompanying statement give sufficient information for the theorem to be stated for those with an understanding of geometrical language and conventions. The square in the vertex of the triangle indicates that the angle is a right angle and the dot in the centre of the chord through the circle indicates that it is in fact a diameter. The statement, taken without the accompanying diagram, would not make clear the theorem as there are many possible angles shown in the semicircle; the diagram reinforces the message, making it clear to those who understand the conventions.

In the development of an analysis for considering such diagrams two definitions have been defined, *inscriptions* and *hinting support*. Pea (1993) describes the term 'inscriptions' to describe culturally created forms of representation that convey meaning whilst holding no intrinsic meaning themselves, such as the square and the dot in the diagram in Figure 1. Inscriptions are the compression of language into symbolic form, representing mathematical, or other technical, ideas or properties through easily constructed and replicated symbols that reveal their meaning to people accustomed to their use but withhold their meaning from those who are not inculcated into their meanings. Mathematics, and especially geometry, has many such inscriptions creating a community of practice amongst people literate in the inscriptional system used.

The accessibility of a problem in mathematics depends not only on the language and symbols used to frame the problem but also on the nature of the problem itself. We have appropriated the term 'hinting support' to indicate the level of support contained in the question, to mean the level of support inherent in a task or the presentation of a task that allows the students to access their geometrical knowledge more readily. 'Hinting Support' is a term originally coined by Chinnappan and Lawson (2000) in a discussion of the connectedness of geometrical schemas, or concept maps:

> A schema with components that are effectively organised is one for which minimal levels of cueing are required for activation. When a greater level of hinting support is needed for access, we argued that the knowledge schema is either less extensive or less well-connected. (p. 31)

These two aspects of problems, inscriptions and hinting support, offer a way to consider the accessibility of the problems in a range of geometry tasks from Durell to Cabri.

METHODOLOGY

The research undertaken was divided into two strands, a historical enquiry, considering the presentation and examination of proof and construction, and an empirical enquiry considering the impact of different presentation styles and resources on the accessibility of tasks. The historical enquiry considered textbooks and examination papers from 1939 to 2003, sources were chosen to reflect the prevailing trend in mathematics education at that time. This paper will look at the analysis for two examination questions from 1943 and 2003. The empirical study was conducted over two years with three groups of students and it is from this data that a third task will be considered, a task in the Cabri-Geometre II environment. Pupils were given a range of tasks and the data of their work collected as well as interviews of their responses. Guidance for this 'unobtrusive' data collection was taken from Webb et al (1966). Regular screen dumps of pupils' work provided a rich source of data.

ANALYSIS OF TASKS

Three tasks are considered, one question from Durell (1939), one from Edexcel (2001) and one Cabri task. To give context to the textbook task, each task is a part of an exercise intended to be completed by all students using the textbook. They are both questions involving the use of circle theorems, however, there the similarities end. Durell (ibid) has 121 pages of theorems and exercises. Theorems are introduced in groups of three or four with three exercises following each set, an exercise of 4 examples for oral discussion, three of which typically have accompanying diagrams,

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an exercise of 25 numerical questions, again three of which typically have accompanying diagrams and an exercise of 25 questions all requiring a proof with typically two diagrams. The Edxecel text (2001) introduces the theorems individually, taking 23 pages for all circle theorems. Each theorem is followed by at least one worked example showing how to use it and an exercise of 10 questions, all with accompanying diagrams and all requiring numerical answers. One exercise of 10 questions, after all theorems have been introduced, requires proofs. All ten questions are accompanied by diagrams. Analysing the books begins to reveal the chasm between the two texts. Analysing the questions using hinting support and inscriptions adds further to this distance.

Task 1: Durell (1939)

> Two chords **AB**, **CD** of a circle, centre **O**, intersect at a point **N** inside the circle. If $\angle$**ANC** is acute, prove that $\angle\mathbf{AOC} + \angle\mathbf{BOD} = 2\angle\mathbf{ANC}$. (p.312)

There is little hinting support offered in the wording of this question and the first demand placed on the examinee is to construct a diagram offering inscriptional support for their proof. The language used is deliberately formal but precise as without a diagram there needs to be no room for misunderstandings. The level of demand placed on the examinee is high in this question. Solutions cannot begin until the question has been appropriately interpreted to produce a diagram on which to place a proof. The relationships between the points and lines are not all given; important relationships have to be deduced and the proof does not follow immediately from the construction. Instead angles have to be defined and more relationships determined through a consideration of circle, triangle and angle properties. This is a demanding question, requiring a thorough and rich understanding of the topics being examined.

Task 2: Edexcel (2001)

18 *BCE* and *ADE* are straight lines. *AB* and *DC* are parallel lines.
Calculate the sizes of these angles:
(a) $A\hat{C}B$ **(b)** $D\hat{C}A$ **(c)** $C\hat{B}A$
Show that triangle *EBA* is isosceles.

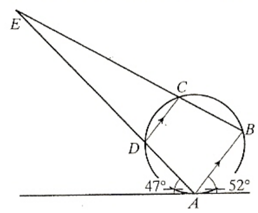

Figure 2: Edexcel question (p. 545)

This question is one of the few proof questions in the summary exercise at the end of the Chapter. The student is given a route towards a proof through the determination of the angles required prior to the proof. The diagram offers both hinting support and inscriptional support. It elucidates the information given in the question. The demands placed on the student by this question are very different from those made by Durell (1947). Finding the angles requires a repeated use of the alternate segment theorem and the angle properties of parallel lines. Proving that triangle *EBA* is

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isosceles requires only that angle $D\hat{A}B$ be stated as the third part of the question involves finding $C\hat{B}A$.

Task 3: Cabri Task

New curricula encourage the use of dynamic geometry software for teaching and learning so it is interesting to place one of the Cabri tasks alongside the two textbook tasks. One of the Cabri tasks was constructed as a 'black box task'. Laborde, (1995) defines such tasks as follows:

> A Cabri-drawing is given to the pupils; they do not know how it was constructed …The task for the pupils is to reconstruct the same Cabri drawing, i.e. a drawing on the screen behaving in the same way as the given Cabri-drawing when it is dragged. (p. 52)

These tasks can only exist in a computer environment. The facility for motion and the explicit nature of the relationships between the elements of the drawing mean that much geometrical knowledge needs to be accessed in order to complete the task. As Galindo (1998) explains

> This type of situation cannot take place in a paper-and-pencil environment because no drawing on paper can convey information about all the potential relationships among the drawing's components. (p. 79)

This task is equivalent to asking the students to produce a figure as defined by Laborde (1995)

> The "figure" which is the theoretical referent (attached to a given geometrical theory: Euclidean geometry, projective geometry …) …the "drawing" … is the material entity. (p. 37)

In other words a Cabri construction which remains invariant under dragging would be a figure as there is a theoretical basis for the construction used. A construction which appears correct but which does not survive the 'drag test' (Jones 2000, Healy and Hoyles 2001) would be classed as a drawing. This classification does not exist in a static, Euclidean environment as there is no equivalent of the drag test. The drawing cannot be shown to have less validity through its lack of robustness.

The first object to be constructed was an equilateral triangle. This was set up on a computer and projected onto a whiteboard. All the construction lines were hidden so that just the triangle remained. The students were shown the triangle and their attention was drawn to the way in which the triangle could move, by enlarging, rotating about two of its vertices and translating it across the screen. The facility for motion inherent in the dynamic geometry environment allows the students to develop strategies to monitor their progress, dragging the vertices to inspect the resultant figure. The task can be completed with a range of outcomes depending on the knowledge and understanding of the student.

The hinting support and inscriptions underpinning this task were analysed to assess what Cabri can bring to the analysis of being a good geometer. This task contained the hinting support inherent in Cabri allowing the students to monitor their own progress through the task. Such support was called *shadow hinting support* as it is inherent to the environment being used and does not depend on the task being performed. It lies in the menu options of the software. These can potentially give the students ideas about the methods to be used. As Straesser (2001) discusses in his analysis of the changes wrought by the introduction of dynamic geometry software on the teaching and learning of geometry, the menu options can provide prompts to the user.

> The DGS-use obviously favoured the tools immediately available in the menus – the rule of the tool seems to override the geometrical knowledge of the user (p.328)

The presentation of the task to the students was designed to eliminate geometrically trivial solutions that might be prompted by tools rather than geometrical knowledge. The shadow hinting support also extended to the use of measuring tools to check constructions as they were dragged. The final level of shadow hinting support came from the layout of the computer rooms at school. The computers were organised around the outside of the room and there is little space in between workstations. This gave the students an opportunity to monitor the progress of the students around them and so approaches that initially appeared to be successful spread rapidly through the class.

The task does not in itself contain much in the way of inscriptions. However a successful completion of any task involving computers requires a thorough initiation into the language and constraints of the programme being used and this can be considered to be a form of inscription. The menus of Cabri are very specific and use formal geometrical language that users need to be familiar with. An involvement with the programme also requires a familiarity with the programme's conventions.

DISCUSSION

Increasing the level of inscriptional use and decreasing the hinting support contained in the question causes an increase in the perceived difficulty of the task when it is rooted in static geometrical methods as can be seen from a comparison of the two textbook questions. Without doubt the Durrell task requires more understanding of geometry than the Edexcel task. Analysis of several such tasks exposes the paucity of geometrical challenge for our current students. However a pedagogic shift to using a DG environment presents new ways of working and new challenges rich in geometry. Working within a dynamic geometry environment changes the way in which the students interact with the task. They are not as inhibited by lack of hinting support as they were when faced with a static task. All the students were able to make some progress and continued to access the mathematics even whilst making little progress with their constructions. The level of learning from experience within the dynamic geometry environment benefited the students when faced with successive tasks. The curricula challenge to us all lies in the type of task we will offer our students for exploring their understanding of geometry.

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