THE ROLE OF AESTHETICS IN MATHEMATICS EDUCATION

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In this study, I examine appreciation of the relationship between aesthetics and mathematics and mathematics education. In this paper first I tried to give a definition of beauty followed by some natural phenomena. Then I made an effort to show the mathematics behind those phenomena. In the last a discussion on how appreciation of the confluence between mathematics and nature facilitate teaching-learning processes in mathematics education.

INTRODUCION

There is a lot of mathematics in nature, behind some of the things that we come across everyday. For instance, patterns like giraffes' spots, zebras' strips, branching process in the rivers and trees, and wave patterns in oceans and clouds, etc can be seen easily in nature. One of the main characteristics of all these patterns from art viewpoint is they are beautiful! That is the point which will be more clarified in the next section.

WHAT IS BEAUTY?

The relations between mathematics and the real world have always been central in mathematics education. The strong confluence between real world and mathematics is an issue which must not be neglected in mathematics education. Perhaps one of the main rings that can make a connection between mathematics and nature is the concept of beauty. Since many years ago philosophers and researchers in philosophy of art some mathematicians have tried to provide a satisfactory and convincing definition of the concept of beauty. Unfortunately we do not have a comprehensive and perfect definition for beauty in mathematics yet. When we decide to say a certain concept is beautiful? What properties of the concept make it to be beautiful? … There are plenty of these sorts of questions which are still debateable in philosophy and philosophy of mathematics (Adam, 2003). Properties like symmetry, simplicity, abstraction, concreteness, surprising products and findings, originality, connections, harmony, or a combination of all or some of those can be considered as standards of being beauty for a piece of art work or a mathematical concept, theorem, proof, or statement (Eisner, 1999; Betts, 2003). However, the paradoxical point is each of those aforementioned aspects of beauty can be taken into account as an aspect of ugly-ness. For instance, Hardy in his well known book, *A Mathematician's Apology* emphasizes that,

"The mathematician's patterns, like the painter's or the poet's must be beautiful, the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test; there is no permanent place in this world for ugly mathematics."

To my understanding Hardy's view of beauty is more tangible and concrete than ineffable and inaccessible impression (He used the term *beauty* against the term

ugly). To make the point more clear I would like to give another interpretation of beauty in the mind of a mathematician. Poincare asserts his opinion about beauty in mathematics in his book, *The Divine Proportion*, as follows:

"The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful"

For him, beauty is a matter of your conception and depends upon your perception. Something is beautiful because you *delight in it.* Another well known mathematician Paul Erdos expresses his view about the concept of beauty in mathematics in another way.

"Why are numbers beautiful? It's like asking why is Beethoven's Ninth Symphony beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is."

Apparently this view is more ineffable than the previous ones. Eaton (1989) explains that in the most of 19th and 20th centuries aesthetics appreciation of a work of art was focused on the properties of harmony, shape and colour. And he suggests that observer's feeling is neglected and one should take into account the influence of work of art on the observer. Subsequently, the meaning of beauty is not only a joyful and playful experiment in the mind. Escher (1989) asserts that "Anything that causes emotional disgust can also be art". They argue that the main problematic issue is that the concept of beauty is mainly and historically lies in the realm of philosophy and arts rather than mathematics. Consequently he works on the concept of beauty from a comparative approach between arts and mathematics. Therefore, one can say, an aesthetic experience in an emotional (joyful/painful) and insightful experience from an interaction with anything (Betts & McNaughton, 2004). Aristotle believed that humans by nature enjoy learning despite the pain that may be associated with learning and understanding.

EDUCATION AND MATHEMATICS

Education in mathematics is more based on either structural or functioning stances. And in both those viewpoints there is almost no place for explanation of beauty and joyful aspects of mathematics in the text books. In the structural point of view, the main focus is accumulating all the things which might be used somewhere in future in the mind of learners in a sequential and logical order. It is like for building a car we forced learners to study about gears, engines, and all other parts of it before giving them a chance to see the plans for the car and its functionality. In the other hand from functioning viewpoint, we are just giving them some engineering and technical problems, mostly, full of apparently complicated and lengthy differential or integral equations which are explaining a very narrow and tiny physical aspect of or world without giving a *wide picture* of applied mathematics, again lack of beauty and joy. The next section gives some examples of real world and nature which is somewhat provides a wide picture of mathematics.

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SOME EXAMPLES OF MATHEMATICS IN NATURE

As it has mentioned above there is a strong confluence between mathematics and nature. This great source of motivations and encouragements is almost overlooked by mathematics experts. Using these natural phenomena which can be easily seen and found around us has a fundamental role in mathematics education (See Geometry of Nature, Mandelbrot 1985). Few examples are shown below.

1- Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55 …

It is clear that each term of the sequence is addition of two successive previous terms.

In mathematical language it can defined recursively as follows.

Fibonacci sequence: $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ for all $n > 2$.

Apparently it is a mathematical sequence but there are plenty of examples of this sequence in nature, the number of rabbit's generations after n^{th} generation, the number of spirals in pinecones and cauliflowers …

2- Non-Euclidean geometry

   a. Fractals – these mathematical objects first introduced by Mandelbrot in 1975. Soon after become of interest of many researchers in a vide range of disciplines. Starting to work on fractal does not need any advance and complicated mathematical knowledge. So it can be used in the high schools and above. "Do not worry about your difficulties in mathematics. I can assure you that mine are still greater" Albert Einstein Due to a fantastic and amazing graphical images might be quite persuading to attract students and giving them the opportunity of doing a joyful mathematical experiment.

   b. Spherical geometry – among all non-Euclidean geometries, spherical geometry can describe to the learners. The result might be surprising because in this geometry there are no parallel lines! Each two lines meet each other at north and south poles. Also one can mention that the sum of angles in a given triangle in this geometry is not 180! Because it lies on the sphere.

3- Topology – perhaps, in topology non Orientable surfaces are good examples of attracting the learner interests in mathematics. Mobius band which is shown below is a surface with no edges! You can move on both sides of the band without crossing the edges of the band.

DISCUSSION

One of the main reasons of mathematics phobia in the society and among students is lack of awareness of beautiful aspect of mathematics in the textbooks and classrooms. While, there are plenty of everyday life applications of mathematics which can

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persuade students to learn mathematics easier in a more relax atmosphere. For example, using fractals and geometrical patterns around us in nature will help students to realize that mathematics is not only a box of hard, awkward formulas and equations. It shows that mathematics is not an ivory tower and it has many open doors welcoming to everyone who want and appreciate these beautiful aspects of it. As a teacher of mathematics we need to make students more familiar with these aesthetics views of mathematics. Recognition of beauty in mathematics may help to persuade learners and students to do mathematics joyfully.

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