TEACHERS' AWARENESS OF DIMENSIONS OF VARIATION: A MATHEMATICS INTERVENTION PROJECT

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This paper presents the findings of a 16-month longitudinal teaching intervention exploring how deliberate and systematic variation can be used to raise awareness in teaching and learning situations. The results indicate that intervention teachers attending to variation produce significant learning benefits for their students.

INTRODUCTION

This paper reports the findings of a 16-month longitudinal intervention called the Dimension of Variation Programme (DVP), investigating the deliberate and systematic handling of content and its consequences in the teaching and learning of mathematics. Informed by the findings of Runesson (1999), Watson and Mason (2004), and Marton and Tsui (2004), this study widens the application of a theoretical framework to a new area of mathematics.

The work of Marton (e.g. Marton and Booth, 1997) has drawn on research results spanning over 25 years culminating in an outlined theory on learning and awareness called variation theory. The theory asserts that, 'if one aspect of a phenomenon or event varies while another aspect or aspects remains unchanged, the varying aspect will be discerned'. The part of the content that varies is called the dimension of variation, hereafter referred to as DoV.

- For example, in the equation $x + 1 = 7$, there are several components that can vary. These include the variable: $y + 1 = 7$, the addend: $x + 2 = 7$, the sum $x + 1 = 8$, the operation: $x – 1 = 7$, the order: $1 + x = 7$, the position of the equals sign: $7 = x + 1$ and the variable coefficient: $2x + 1 = 7$. Each of these is a DoV.

Watson and Mason (2004) have defined the variation within a DoV as the range of change, hereafter referred to as RoC.

- Suppose in the equation $x + 1 = 7$ we choose to vary the dimension of the addend, then the values that the addend can take (i.e. natural numbers, negative numbers, rational numbers and so on…) would constitute the RoC of the DoV.
- If the DoV we are varying is the operator then the RoC would be +, –, ÷, x.

JUSTIFICATION FOR THIS APPROACH

All teachers naturally use variation in their teaching, either deliberately or accidentally. For the purposes of this study, it was assumed that no single 'standard approach' that used variation in a deliberate and systematic way to handle the algebra content existed. Instead it was recognised that all teaching would exhibit variation which could be distinguished by frequency and type. Hence, in order to determine whether there were noticeable differences between this continuum of 'other' approaches and one where there was an awareness of DoV, it was not a simple matter of identifying and observing two sets of classrooms where contrasting approaches were being employed. Instead, a specific intervention, the DVP, was designed.

INTERVENTION

The DVP was not of the form of giving tasks, material or instruction, but of providing information, discussion and support opportunities around variation theory. Both the comparison and intervention teachers retained overall control of their teaching style as well as the content of what was to be taught. The intervention teachers were committed to the use of variation as a means for handling the content of an elementary algebra lesson. The DVP was designed to encourage direct manipulation of expressions, making the process familiar and, as Lins (1994) would say, 'senseful'. This might lead to students appreciating generalities for themselves. The desire to then articulate these generalities creates the need for algebra (Brown and Coles, 1999). The field of algebra was chosen because the theoretical position that awareness is dynamic in structure had clear parallels to the following definition of algebra (Pimm, 1995):

Algebra is about transformation. Algebra, right back to its origins, seems to be fundamentally dynamic, operating on or transforming forms. It is also about equivalence; something is preserved despite apparent change.

By deliberately controlling the variation offered in progressively developed examples the teacher focuses the learners' attention on particular aspects of the algebra, hence increasing the chances of common experiences. Carefully selected and presented generic examples can provide algebraic experiences that develop both manipulative abilities (Bell, 1996) as well as generalisations of structure rather than surface features (Bills & Rowland, 1999).

- Example 1: the series of equations: $x + 1 = 6$, $x + 2 = 6$, $x + 3 = 6$, $x + 4 = 6$ might illustrate to the learner that the variable in this case is an unknown and the value of this unknown can vary from question to question.
- Example 2: the series of equations: $x + x = 2x$, $x + 2x = 3x$, $x + 3x = 4x$, $x + 4x = 5x$ might illustrate to the learner how like terms are added together.

DVP COMPONENTS

- Joint lesson and task design planning for each lesson
- In these sessions the mathematical content along with the principle points that the teacher feels need to be covered were discussed
- The teacher and I jointly decided on what concepts would be focused on
- Together we attempted to identify all the relevant DoVs in the situation
- Through negotiation it was decided which were the most appropriate DoVs for communicating these ideas to the teachers' particular class. Given that different

teachers were following different short-term plans with their classes, some teachers systematically highlighted different variation

- Classroom activities and homework tasks that would offer variation in these dimensions were then also co-planned

The DVP followed students through Year 7 and Year 8. It was a quasi-experimental design with 10 teachers and a total student cohort of 300 students. Stratified sampling was used to produce two groups of classes and to reduce the presence of confounding variables. While all teachers inevitably offered students dimensions of variation, the difference between the experimental groups was that the intervention teachers did so in a consistent, aware, deliberate and systematic manner while comparison teachers were not offered opportunities to develop this awareness.

DATA

Quantitative data in the form of pre-, post- and delayed post-tests using standardised national exams was collected for each student. The average time between pre- and post-tests was 16 months. After a subsequent 2 months a delayed post-test was administered. It would be over-simplistic to assume the intervention only happened between the pre- and post-tests as the experimental teachers' understanding of variation theory evolved and continued to develop up to and beyond the delayed post-test. This data was augmented by two forms of qualitative data. First, each teacher was observed for a total of 8 lessons, 4 consecutive lessons at the beginning of the study and 4 consecutive lessons towards the end of the study. Second, two target students from each of the 10 classes were clinically interviewed. Each student was interviewed twice using the same questions from the CSMS study (Kuchemann, 1981), near the times of the pre- and post-tests. While the quantitative data helps to establish whether or not the intervention has 'worked', the qualitative data can be used to conjecture the reasons why, and through which mechanisms it has worked.

RESULTS

Two independent t-tests were carried out on the adjusted mean post- and delayed post-test scores. This analysis produced a significant difference between experimental groups for the post-test [$t(225) = -1.95$; $p < 0.05$], however there was only a small effect size $r = 0.13$. There was also a significant difference between experimental groups for the delayed post-test [$t(223) = -4.97$; $p < 0.001$], with a medium effect size $r = 0.32$. Two multiple regression models were generated in order to investigate the temporal effects of the intervention. The first used the post-test marks as the outcome measure with the pre-test marks and the experimental group as predictors. While this model did suggest that the experimental group had a significant effect on the outcome measure, at $p < 0.05$, the related effect size, d, was only 0.13. The second model used the delayed post-test marks as the outcome measure with the experimental group, the pre- and post-test marks as predictors. The results of this model are below.

Outcome measure	Marks on delayed-post test		
	Coefficient of linear model, *B*	SE *B*	Beta
Marks on pre-test	0.28	0.08	0.25
Marks on post-test	0.65	0.08	0.55
Experimental group	12.84	2.05	0.24

In this model the experimental group again had a significant effect on the outcome measure, at $p < 0.001$, and represented a large effect size with $d = 0.51$. The respective outcome variables were chosen because they were considered those most linked to the developmental improvement of the students at those particular times during the intervention.

SYSTEMATICITY

A Kruskal-Wallis test was carried out on the mean variation offered by teachers. This analysis produced no significant difference between experimental groups [$H(1) = 0.046$; $p < 0.05$], which suggests that teachers across experimental groups do not use different amounts of variation and that there must be something else that the intervention teachers do to produce significantly better delayed post-test results. Any suspicion that this effect is due solely to the professional development atmosphere rather than their deliberate and systematic handling of the content and attendance to variation is dispelled when the qualitative data is considered. This data shows that in some cases the variation offered and generated seem to be independent of each other in nature. That is, students' sense of appropriate variation bears little relation to what teachers are trying to convey. In the intervention classes the variations offered and generated are more reciprocal in nature. Teacher and students collectively explore the mathematics of a situation by responding to the different Dimensions of Variation and related Ranges of Change that are opened up by the other party. This is perhaps because the intervention teachers are more aware that the variation generated by the students is indicative of what the students pertain to be critical in the learning situation. The desire to describe how the type, nature and degree of variation impacted the handling of the content necessitated the development of a new analytical tool. The systematicity of the variation can be used as a tool to analyse beyond the existence of dimensions of variations. Considering how the systematic nature of presentation of variation differed from teacher to teacher gives us the ability to describe sets of instances in more detail, hence facilitating discrimination on a higher level. Systematicity is different from RoC here in that it is sensitive to the number of examples presented. One good example can illustrate the RoC along a DoV (in the sense that it can encapsulate the potential along a particular DoV), but a sequence of examples is needed to discern the degree of systematicity. The degree is the number of embedded variations that are actively or deliberately being illustrated.

Intervention teachers are not systematic with the RoC of DoVs that are already well understood as this would be time consuming and inefficient. The systematicity of intervention teachers is illustrated in two different ways:

- The systematic handling of the RoC of a few selected DoVs pertinent to the teaching aim
- When a new DoV is generated or the students ask a question showing confusion relating to a particular DoV or its RoC then the teacher responds with a deliberate and systematic exposition of it.

GENERAL DISCUSSION

In this study an intervention designed to raise awareness through the use of deliberate and systematic variation was introduced in order to measure the effects, if any, on teaching and learning. The students' performance on three standardised national tests and the relationship between this performance and classroom observations leads to two main conclusions.

The first conclusion, as evidenced by amongst others the Kruskal-Wallis test, is that the intervention teachers handle the content differently. This was also the conclusion of an earlier paper by Runesson (1999). Further, an increased awareness of variation amongst intervention teachers was reflected by an increase in the systematicity of their teaching. Systematicity seems to explain the relevant regularities in the handling of the mathematical content by them.

The second conclusion is that students involved in the intervention programme performed significantly better than non-intervention students. The longitudinal data showed that the relationship between increased awareness of variation and understanding (test performance being just one of the criteria used to infer increased understanding) holds over an 18-month period. The smaller difference between experimental groups in the first 16-months of the study suggests that other factors may have exerted an influence while the intervention was still taking root. The subsequent finding that this connection was more pronounced over the last 2-month period indicates that the intervention had a more powerful influence in the long run.

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