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VARYING PEDAGOGICAL FILTERS: FORMING CONJECTURES THROUGH A SPREADSHEET LENS

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This paper is concerned with how both children and pre-service teaching students engage in mathematical investigation using spreadsheets. It examines how mathematical phenomena are shaped by the pedagogical medium through which they are encountered. It considers how the nature of pedagogical support influences different styles of social interaction, and how this interaction contextualises and hence conditions the mathematical ideas. A particular focus is on how alternative pedagogical media promote varying discourse networks that influence the approach to the forming and testing of informal conjectures. How these approaches might differ will also be examined.

INTRODUCTION

The study was part of an ongoing research programme exploring how spreadsheets might function as pedagogical media. When used as a tool for investigation, how might the social interaction they evoke colour the learning experience and hence influence the learner's perceptions and understandings? One aspect of this programme was to identify the ways participants approached the mathematical investigations as they negotiated the requirements of the tasks, and how they produced their conjectures and generalisations. An interpretive strategy was used to examine the participants' dialogue and action, evoked by engagement through the pedagogical media.

The prevalence of ICT media generally has begun to challenge the map of mathematical ideas encountered in schools. Access to many key elements of school mathematics has been altered, as different software offers new ways in which certain constructs are created and understood. Studies involving the dynamic geometry software, Cabri-geometre, (Laborde, 1999, Mariotti, 2002) assert that conceptualisation of mathematical phenomena will be different when engaged through the particular software lens. Meanwhile, spreadsheets have been found to offer an accessible medium for young children tackling numerical methods. With the potential to simultaneously link symbolic, numeric and visual forms, they can enhance the conceptualisation of some numerical processes (Baker and Beisel, 2001, Calder, 2004). Other characteristics, including their interactive nature and the capacity to give immediate feedback (Beare, 1993, Calder, 2004) appear to give the learner the opportunity to develop as a risk taker; to make conjectures and immediately test them in an informal, non-threatening, environment.

The capacity to provide instantaneous feedback also allows for conjectures to be immediately tested and perhaps refuted. Lin (2005) claims that refuting is an effective learning strategy for generating conjectures. Mathematical conjectures often have speculative beginnings, and by implication elements of logical guesswork (Dreyfus, 1999). Other researchers have sometimes considered them as generalised statements, containing essences distilled from a number of specific examples (Bergqvist, 2005). In their embryonic form they emerge as opinions, mathematical statements, generalisations, or positions. These can then be challenged or confirmed with explanation, leading to mathematical thinking. Suggesting counter-examples, or exposition of how two mathematical explanations are similar, indicate a more robust form of examination of the conjecture (Manouchechi, 2004).

APPROACH

The paper considers two settings where investigation takes place in a spreadsheet environment. The first situation located groups of three, first year, primary, pre-service students in a typical classroom setting while groups, from the same class, worked in an ICT laboratory, doing the same investigation using spreadsheets. Their discussions were audio recorded and transcribed, each group was interviewed after they had completed their investigation, and their written recordings were collected. This data, together with informal observation and discussions, formed the initial basis for the research. The second situation involved ten-year-old students, attending five primary schools, drawn from a wide range of socio-economic areas. There were four students from each school, eleven boys and nine girls. The data was produced in the same way as the first situation. The transcripts from both were then systematically analysed for patterns in the dialogue, within and between the settings.

RESULTS AND ANALYSIS

Two aspects were considered in the formation and testing of conjectures. Firstly, the data is examined for differences the pedagogical media may have evoked, with particular regard to the pre-service teaching students. An episode with the ten-year-old learners is then analysed with regard to the notion of subgoals.

Comparison of two pedagogical media

The dialogue in each situation demonstrated a contrast in the initial approach to engaging with the mathematics. In the classroom situation it began with a group member initiating the negotiation of the meaning, and requirements of the task, with a single discrete numerical example. For example, group one.

Karl: Let’s try each number one at a time. One times 101 is 101.

Group two likewise used this approach the initiate the process. For example:

Sarah: So if we had twenty three times a hundred you would have twenty three hundred…Let’s say we do twenty three times a hundred and one, we would get twenty three hundred plus twenty three ones

As they made further sense of what the problem was about, they began to predict, verify and reflect in a discrete numerical manner.

Rachel: We went through one at a time and solved them. We solved them on paper and we solved them with a calculator.

In contrast, those groups working in the spreadsheet setting used the spreadsheet to get a broad picture; they utilised the formulae and copy down functions to create a numerical table that could then be examined for any pattern. They used more algebraic language, while the pencil and paper groups had more numerical reference. For example:

Kyle: I haven't predicted. I was just going to put in A1 times 101 and drag it down.

Josie: So we're investigating the pattern of 1 to 16 times 101

This appears a more direct path to the patterning approach, and these groups quickly recognised a pattern, and explored further based on visual aspects. It also introduced a difference in terms of the technical language utilised. "Drag it down" is functioning language rather than mathematical, but the inference is clearly that there is a pattern, which might possibly lead to a generalisation; and that the spreadsheet by nature will enable users to quickly access that pattern. Most significantly, the social interactions appear to shape the analysis of the patterns in distinct ways. Given that the path to, and manifestation of, the patterns differs, the dialogue indicates a different approach once the patterns are viewed. Those using the spreadsheet used a more visual approach. They were observing and discussing visual aspects e.g. the situation of digits or zeros. For example:

With two digits you just double the number. You take the zero out.

Those using pencil and paper were more concerned with the operation aspects that generated the patterns. For example:

Basically, if you times your number by a hundred, and then by one, you would add them together, and get your answer.

To generalise a pattern in terms of the sequence of digits is significantly different to generalising in terms of an operation. In this aspect, the different settings have certainly filtered the dialogue and approach to forming conjectures, and by inference the understanding.

The influence on sub-goals

The characteristic of spreadsheets to produce immediate responses to inputted data permitted new sub-goals to be promptly set, assisting the emergence of a theory. The data produced relates to an investigation involving exploring terminating and recurring rational numbers, when one is divided by the counting numbers. In the first case they negotiated to gain some initial familiarisation of the task.

Sara: One divided by one is one - it should be lower than one.

They entered 1 to 5 in column A and =A1/1 in column B to get:

1 1

2 2 etc

This posed an immediate tension with their initial thoughts. After exploring various formulae and associated output, they settled on a way to easily produce a table of values to explore. The spreadsheet environment shaped the sense making of the task and the resetting of their sub-goal. Critically, it had enabled them to immediately generalise, produce output, then explore this visually. They generated further output:

1 1

2 0.5

3 0.333333..

4 0.25

5 0.2;

6 0.166666..etc.

Sara: So that's the pattern. When the number doubles, it's terminating. Like 1, 2, 4, 8 gives 1, 0.5, 0.25, 0.125.

Jay: So the answer is terminating and is in half lots. Lets try that =0.125/2; gives 0.0625-which is there. (Finds it on the generated output from above)

The structured, visual nature of the spreadsheet prompted the children to pose a new conjecture, reset their sub-goal and then allowed them to easily investigate the idea of doubling the numbers. The table gave them some other information however.

Jay: 1 divided by 5 goes 0.2, which is terminating too. (Long pause)

After further exploring, they reshaped their conjecture, incorporating their earlier idea.

Sara: If you take these numbers out they double and the answer halves.

Jay: That makes sense though, if you're doubling one, the other must be half. Like 125 0.008; 250 0.004.

Sarah: What's next. Let's check 500

Jay: Let's just go on forever!

They generated a huge list of output; down to over 4260.

Jay: 500 0.002; 1000 0.001. When you add zero to the number you get a zero after the point

Although this particular group didn't fully explore the relation of the base numbers to the multiples of ten, they have made sense of, explored, and generalised aspects of the investigation. The pedagogical medium through which they engaged in the task has influenced the contextualization and approaches they have taken. When asked: "When you saw the problem, how did you think you would start?" the children's responses corroborated this perspective.

Sara: Re-read to get into the math's thinking, then straight to a spreadsheet formula.

Greg: I type what I think and try it

As well, the spreadsheet groups progressed more quickly into exploring larger numbers and decimals. This appears to indicate a greater propensity for exploration and risk taking, engendered by the spreadsheet environment. It seems the spreadsheets have not only provided a unique lens to view the investigation, but have drawn a distinctive investigative response.

Fran: Using a spreadsheet made it more likely to have a go at something new because it does many things for you. You have unlimited room. You can delete, wipe stuff out.

Chris: Columns make it easier- they separated the numbers and stopped you getting muddled. It keeps it in order, helps with ordering and patterns.

CONCLUSIONS

This study demonstrated that the different pedagogical media provided a distinct lens to contextualise the mathematical ideas, frame the formation of informal mathematical conjecture, and condition the negotiation of the mathematical understanding. As Brown (1996) argued, the mathematical understanding is a function of the social frame within which it is immersed, and the social frame evolves uniquely in each environment. The data supported the supposition that the availability of the spreadsheet led the students to familiarise themselves with, then frame the problem through a visual, tabular lens. It is clear also that it evoked an immediate response of generalisation, either explicitly through deriving formulas to model the situation, or implicitly by looking to fill down, or develop simple iterative procedures. Tension, arising from differences between expected and actual output, and opportunities, arising from possibilities emerging from these distinctive processes, led to the setting and resetting of sub-goals. These, in turn, further shaped the understanding of the investigative situation, and the interpretation of mathematical conjectures.

The spreadsheet approach, perhaps due to the actual technical structure of the medium, led more directly to an algebraic process, with the language interactions containing both algebraic and technical terminology. It seemed, in fact, that the spreadsheet setting, by its very nature, evoked a more algebraic response. The participants in these groups were straight away looking to generalise a formula that they could enter and fill down. Their language reflected this, but the interactions also contained more language of generalisation, and it took them generally less interactions to develop an informal conjecture.

Those working in the classroom setting used a discrete numerical example to engage in the problem; to make sense of its requirements as well as initiating the process of solving. They tended to try, confirm with discussion and then move more gradually into the generalisation stage. Their conjectures were slower to emerge not only due to variation in computational time, but also due to the approach the spreadsheet evoked. The way they thought about the problem was different. Their initial dialogue seemed more cautious, and contained comments requiring a degree of affirmation amongst group members before moving into developing their conjecture. As a consequence the descriptions of the process undertaken and the mathematical thinking were more fulsome. This may be evidence of more fulsome understanding too.

The children also identified speed of response, the structured format, ease of editing and reviewing responses to generalisation, and the interactive nature as being conducive to the investigative process. While this particular medium has unfastened unique avenues of exploration, it has as a consequence fashioned the investigation in a way that for some learners may have constrained their understanding. The approaches and outcomes, as reflected in the dialogue, are different. If the dialogue between learners filtered the mathematical thinking and formation of conjectures in different ways, according to pedagogical media, then perhaps complementary approaches would give opportunity to enhance mathematical understanding.

REFERENCES

Baker, J. D., & Biesel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics* (Jan 2001) 23-31.

Beare, R. (1993). How spreadsheets can aid a variety of mathematical learning activities from primary to tertiary level. In B. Jaworski (Ed.), *Technology in Mathematics Teaching: A bridge between teaching and learning*. Birmingham.

Bergqvist, T. (2005). How students verify conjectures: teachers expectations. *Journal of Mathematics Teacher Education* 8, 171-191.

Brown,T. (1996) The phenomenology of the mathematics classroom. *Educational Studies in Mathematics* 31, 115-150.

Calder, N.S. (2004). Spreadsheets with 8 year olds: Easy to visualise? *Teachers and Curriculum.* 1, 125-143.

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics*.38, 85-109.

Laborde, C. (1999) Core geometrical knowledge for using the modelling power of geometry with Cabri-geometry. *Teaching Math and its Application.* 18(4).

Lin, F. (2005). Modelling students' learning in argumentation and mathematics proof. In Chick, H. & Vincent, J.L. (eds). *Proceedings of the 29th Conference of the International Group of the Psychology of Mathematics Education.* 1, 3-18.

Marriotti, M.A.(2002). Influences of technologies advances on students' math learning. In English, L. et al. (eds) *Handbook of International Research in Mathematics Education*, LEA, 695-723.

Manouchehri, A. (2004). Using algebra software to support a discourse community. *The Journal of Mathematical Behavior*. 23(1), 37-62.