

## **DIFFERENCES IN TEACHERS' SELECTION AND USE OF EXAMPLES IN CLASSROOMS: AN INSTITUTIONAL PERSPECTIVE ON TEACHER PRACTICE**

M. Kerem Karaağaç

University of Leeds

*The research presented here is an attempt to explore teachers' classroom practices in mathematics lessons in Turkish state schools and privately owned educational institutions. I will present data on the use of examples in mathematics lessons in two classrooms from different schools teaching the same content: inverse functions. The results indicate that the time given to student engagement on examples differs. In private college, the time allowed for students' engagement with problems is markedly less than in state schools. Further analysis on the use of examples shows that the examples teachers selected to use and the way they made use of them are also different. I will discuss the results in the light of socio cultural theories.*

### **INTRODUCTION**

Over the last 15 years there has been a shift in the assumptions and the direction of educational research. The general movement is from purely cognitive studies, which considers learning with in the 'head' of learner, towards recognition of social influences in the process of learning. The current trend in teacher research is examining teacher practice in its social context. However there appears to be a lack of research at institutional level. Although educators acknowledge that mathematical knowledge "strongly depends on the institutions where it has to live, to be learnt, to be taught. Mathematical objects do not exist per se but emerge from practices which are different from one institution to another one"(Artigue et al., 2001) there seems to be a "tendency to under-theorise differences between schools in terms of institutional effects on the social formation of mind."(Daniels, 2001) My research should be considered from this perspective.

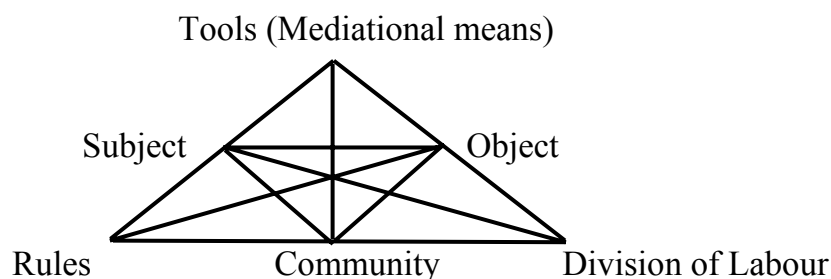
### **THE RESEARCH**

The research findings presented here are a part of an ongoing project to investigate teachers' beliefs about teaching and learning and their actual practices in two different educational institutions in the Turkish education system for 17-18 year olds. Many students of this age in Turkey are taught mathematics in two places. They attend state schools (SS), but at weekends or in the evenings many of them also attend courses in privately owned schools (PC). The main objective of private school courses is to prepare students for the university entrance examination (UEE), which is made up of multiple-choice questions. Unlike state schools, these private courses teach for test, the UEE.

I have used an exploratory case study methodology (Yin, 1994) in order to examine teaching in its natural context. Series of lessons of teachers from each kind of

institution (SS and PC) were video recorded. The recordings were made during teaching of the topic of functions to have comparable results. 24 teachers also gave semi structured interviews. The research reveals a difference between PC and SS teachers in their mathematical practices. In this paper I will draw on the data obtained from interviews and video recordings of teachers in both type of schools to interpret establish and explain the differences.

This research views teacher practice from socio-cultural perspective, in particular an activity theoretical framework. (Engeström, 1987). According to this approach an activity system (see Figure1) comprises: subject (agents); object (motive of the activity); tools (mediational means through which object is realised); rules (formal or informal, explicit or implicit rules and norms); community (within which activity takes place); and division of labour (how actions are divided horizontally and vertically among the participants).



**Figure1.** Engeström's (1987) model of the activity system

The reasoning behind this is the fact that it examines human practice as intertwined in the socio-cultural context and thus it offers “powerful sociocultural lens through which we can analyse most forms of human activity” (Jonassen & Rohrer-Murphy, 1999). The particular social context focused in this study is at the institutional level. This heuristic device (model of the activity system) is applied to teacher practice in order to make the link between teacher practice and institutional context more apparent. From an AT perspective examples are regarded as tools for teaching since they are used by subject to achieve the object. The ways examples are selected and used have significant implications as to how teachers practice teaching mathematics. This view is supported by the piloting as well as main data where teachers of both institutions primarily mention examples when they talk about their practices. This provides a strong link between theoretical framework and the data. I will now present the differences between teachers of two institutions.

## RESULTS

### Overall Privileging Patterns (R1)

Results presented elsewhere (Karaagac, 2004) indicate a strong distinction between PC and SS teachers use of examples. Teachers from two institutions allocate differing amounts of time for each phase (presentation, engagement, resolution) of examples presented. SS teachers spend most of their time on examples on supporting students' activities (48%, engagement phase). PC teachers, on the other hand, spend their time

explaining alternative and shortest ways of solving examples (59%, resolution phase). Moreover, PC teachers' practices involve example solving sessions where each example is given a short amount of time. SS teachers solve relatively fewer examples and allocate relatively more time for students to engage in the example. They also encourage students to come to the board and demonstrate a solution, which seems to result in more time for students to elaborate on the examples presented.

### Specific Differences in Video Data (R2)

Here is a sample of examples used in SS and PC classrooms when teachers were teaching inverse functions. These are typical examples used in SS and PC classrooms. On the left hand side is 5 examples from a SS teacher and on the right is 3 examples from a PC teacher in their actual presentation format.

SS teacher examples

$f(2)=3$	$\Rightarrow f^{-1}(3)=2$
$f(2x+3)=5$	$\Rightarrow f^{-1}(5)=2x+3$
$f^{-1}(3)=2$	$\Rightarrow f(2)=3$
$f^{-1}(2x-3)=x+2$	$\Rightarrow f(x+2)=2x-3$
$f(g(2x+1))=2$	$\Rightarrow f^{-1}(2)=g(2x+1)$

PC teacher examples

$f$ is defined as 1to1 and onto function.
If $f(x) = \frac{2f(x)-1}{x+3}$ then $f^{-1}(x)=?$
$f$ is defined as 1to1 and onto function.
If $x = \frac{3f(x)+2}{f(x)-5}$ then $f^{-1}(x)=?$
If $f(x) = \frac{3x-5}{4-x}$ then $f^{-1}(x)=?$

The SS teacher's examples suggest that the teacher is trying to establish an understanding of inverse function by providing a sequence of examples with slight modifications. The teacher is changing one aspect of the example in each step while keeping other aspects the same so that it will stimulate students understanding of the mathematical idea they wish to convey about inverse functions. This PC teacher's examples, however, are focused towards the solution of inverse function questions. Further elaboration of the examples used in the two institutional contexts suggests a distinction between PC and SS teachers' practices. To make the contrast more visible to the reader I make a distinction between 2 different types of examples based on an analysis of video data: active examples, passive examples.

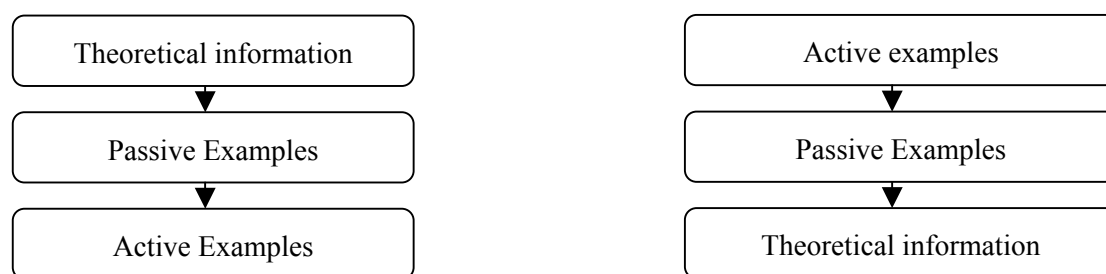
*Passive examples* (PE) are typical cases of broader categories, they do not ask for any physical action from students. They are intended to exemplify the concept or the procedure previously presented. Passive examples, in this respect, are part of teachers' explanations. For instance:

Conceptual passive example: teacher shows that  $f(x)=3$  is an example of the concept 'constant function'.

Procedural passive example: Teacher demonstrates finding inverse of ' $f(x)=x-1$ ' and then states that ' $x+1$ ' is the inverse of it.

*Active examples* (AE) are the ones that call for the audience to take action, to be precise, to solve it. Active examples require students or teacher to make use of variety of previously acquired mathematical knowledge. These examples can be regarded as exercises in the sense that their usage constitutes practice of prior knowledge.

The important difference between these two types of examples is that AE examples do not primarily provide understanding of mathematical ideas involved. However, PE helps students to understand the abstract mathematical information. My analysis of classroom practices suggest that SS and PC teachers use AE and PE in different ways, which seem to have implications for what is valued in each institutional context. PC teachers use large number of AEs and few PEs. They tend to use AEs first and then provide PEs afterwards, if necessary. SS teachers, on the other hand, consistently provided PEs after theoretical information is presented. A number of PEs are followed by AEs. I present this schematically in Figure2.



**Figure2.**

SS (on the left) and PC (on the right) teachers use of examples.

### Specific Differences in Interview Data (R3)

During the interviews teachers positioned themselves differently with regard to their use of examples. I will only be able to present two examples due to page restrictions. Here is a SS teacher.

<b>R</b>	<b>1</b>	Why do you put so much emphasis on examples?
<b>T</b>	<b>2</b>	As to the question ‘Why do I put emphasis on example?’ ...hmmm...I believe I
	<b>3</b>	teach better that way. That’s my opinion. You know that there are verbal
	<b>4</b>	(theoretical) parts in mathematics. When I give that, kids write it down but they
	<b>5</b>	cannot understand on their own when they read it. I don’t believe mathematics is
	<b>6</b>	such a subject that students read and understand or perform. That’s why I pay
	<b>7</b>	more attention to solving examples. I think the knowledge fits to its place, when
	<b>8</b>	they are solving examples. I mean, rather than knowledge written on the
	<b>9</b>	notebook with a certain template, (I prefer) solving examples...No matter how
	<b>10</b>	many times you give Menelaus’ theorem to students (they don’t understand). When they solve 10 examples it fits to its place. They don’t need to memorise it.

As can be observed SS teacher is mainly concerned with students’ comprehension of the topic. In explaining her practice, the teacher aims at ‘knowing that’. PC teachers positions themselves differently:

<b>R</b>	<b>1</b>	You gave two sets like A, B and then assign values in set A to set B. And also as pairs like (1,a) (2,b) etc. Also you present function such as $f = \frac{(x+1)}{(1+x^2)}$ What is the aim of this??
	<b>2</b>	
	<b>3</b>	
<b>T</b>	<b>4</b>	Well firstly it is because there are examples of past university entrance examination questions [related to this one]. If it wasn't in the past examination then I would not teach these and that's my first aim. Secondly, to make it more visual because you can switch one another. I mean if a student is asked one representation (of function) in the university entrance examination but given another representation, student will be able to switch. There isn't much other reasoning behind that. To be more precise we depend on university entrance examination..
	<b>5</b>	
	<b>6</b>	
	<b>7</b>	
	<b>8</b>	
	<b>9</b>	
	<b>10</b>	
	<b>11</b>	

In explaining his practice, this PC teacher clearly points out that he aims at students' high performance in the UEE and that he considers this as his primary objective. When prompted about solving examples by using a solution technique what PC teachers call 'numerical value technique', the teacher states that:

<b>T</b>	<b>12</b>	Yes, this is a part our system. In terms of university preparation, preparation for university entrance examinations, this is part of our system...Using numerical values is of interest to them [students] and they like it very much. 'Let's assign 1 to the value of 'a', and after that, lets give the options, lets put 1 for wherever you see 'a', what a simple thing, isn't it!' This is a part of our system, I mean, as a private course it is a part of us, we make use of it.
	<b>13</b>	
	<b>14</b>	
	<b>15</b>	
	<b>16</b>	
	<b>17</b>	

The teacher regarded the application of solution technique (knowing-how) that is aiming at UEE as part his teaching and he saw himself as part of the 'system'.

## DISCUSSION

The results show that there is a considerable difference between teachers' practices in the two institutions, specifically the selection (R2) and use of examples (R1, R2). SS teachers used a number of passive examples (R1) that help students to understand the mathematical concepts and/or procedures before proceeding to any application of the mathematics students learned. While SS teachers' stated aim is students' acquisition of the knowledge they present (R3, SS teacher line 5,6,8,11), PC teachers aim at doing mathematics (R3, PC teacher line 4,5,6,8,9,10,13,14,17). This difference is noteworthy since teachers from both institutions were examined while they were teaching the same topic.

Drawing on Chevallard's anthropological research (Artigue, 2002) makes a similar distinction regarding mathematical techniques that are "most often perceived and evaluated in terms of pragmatic value, that is to say, by focusing on their productive potential (efficiency, cost, field of validity). But they have also an epistemic value, as they contribute to the understanding of the objects they involve" (*italics in original*). Using these constructs with regard to my research, it can be argued that SS teachers prioritise the epistemic value of the tools because their primary object is teaching

‘knowing-that’. The situation is different for PC teachers. They prioritise the pragmatic value of examples since they have primary object of teaching ‘knowing-how’. From an activity theoretical perspective their objectives differ and this difference is intertwined with the way they use the tools. This situation is summarised in the table below.

	Privileged aspect	Object of practice
<b>SS</b>	epistemic value	Knowing-that
<b>PC</b>	pragmatic value	Knowing-how

**Table.** Difference between PC and SS teachers

Then the fundamental question to ask is: What it is that makes a difference in these teachers’ object of their practices? Although there may be some other possible explanations, the data from PC teachers strongly suggest that the teacher’s object is already decided for them before they start working for the PC. In a sense, it comes in a contextual package, where the object of the activity has little flexibility for personal preferences. Therefore, what makes a difference is largely the institutional context which fundamentally decides the object of practice and therefore what is to be prioritised and valued.

## REFERENCES

- Artigue, M. (2002). Learning mathematics in a cas environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7, 245-274.
- Artigue, M., Assude, T., Grugeon, G., & Lenfant, A. (2001). Teaching and learning algebra: approaching complexity trough complementary perspectives. In H. Chick, K. Stacey, J. Vincent & J. Vincent (Eds.), *he Future of the Teaching and Learning of Algebra*, Proceedings of 12 the ICMI Study Conference. The University of Melbourne, Australia, December 9-14, 2001.
- Daniels, H. (2001). *Vygotsky and Pedagogy*. London: Routledge.
- Engeström, Y (1987). *Learning By Expanding: An activitytheoretical approach to developmental research*. Helsinki: Orienta-Konsultit Oy
- Jonassen, D. H., & Rohrer-Murphy, L. (1999). Activity theory as a framework for designing constructivist learning environments. *Educational Technology Research and Development*, 47(1), 62-79.
- Karaagac, M. K. (2004). An Aspect of Teacher Practice in Turkish Classrooms: Differences in Time Given to Students to Do Mathematics in Two Different Types of Schools. *Proceedings of the British Society for Research into Learning Mathematics*, 24(2), 39-44.
- Yin, R. (1994). *Case study research: Design and methods* (2nd ed.). Beverly Hills, CA: Sage Publishing.

# **COMPUTER ALGEBRA RELATED CONCEPTIONS AND MOTIVATIONS OF UNIVERSITY MATHEMATICS LECTURERS AN INTERNATIONAL STUDY**

Zsolt Lavicza

University of Cambridge

*As a consequence of sizable investments in technology, Computer Algebra Systems are becoming more accessible and widely used in mathematics teaching and learning in universities. However, little is known about the factors influencing the integration of technology at the university-level. To better understand the affect of technology incorporation on mathematics education a number of school-level studies have focused on the relationships between teachers' conceptions of mathematics, mathematics teaching, and technology as well as on various social and cultural factors. My study investigates the relationships of these factors at the university-level paying particular attention to cultural elements by taking an international comparative approach.*

## **INTRODUCTION AND AIMS**

Mathematical understanding and computer proficiency are vital ingredients not only in the education of future scientists, engineers, and teachers but also for the economic well-being of any nation. However, universities in most developed countries report that their students are mathematically ill-prepared for their studies. A number of studies have attempted to tackle this problem by examining teaching, learning, and teacher preparation at the pre-university level. Nevertheless, university-level mathematics departments, the principal suppliers of mathematics teaching for other departments, are seldom studied. Mathematical software packages, which are increasingly being integrated into university classrooms, provide further didactic and research challenges. My research aims to investigate the integration of a significant example of a computational tool – Computer Algebra Systems (CAS) – into university mathematics teaching. It will focus on developing a model of university teachers' CAS-related didactic beliefs in relation to their manner of CAS integration into teaching. This model will assist in the better understanding of present CAS-enhanced teaching practices and will contribute to the development of a warranted pedagogy for CAS use. More specifically, I will examine

- the extent to and manner in which CAS are currently used in university mathematics departments;
- the pedagogic and mathematical beliefs and conceptions university mathematics lecturers hold with regard to CAS including factors influencing their professional use of CAS;
- the extent to which nationally situated teaching traditions, frequently based on unarticulated assumptions, influence lecturers' conceptions of and