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PATTERNS OF STUDENT INTERACTIONS: WHAT CAN THEY REVEAL ABOUT STUDENTS' LEARNING OF MATHEMATICS?

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This study is set within the context of a secondary school mathematics classroom in which a graph sketching software program is used. The interactions taking place as a small group of students works through the activities set by the teacher, captured on video, are analysed in terms of different 'modes of production' and mathematical processes and learning required to complete the task. The patterns of modes of productions leads to a more in-depth analysis of the mathematical learning of the students.

INTRODUCTION

The research presented here is part of a larger study concerning the use of computers in secondary school mathematical classrooms. One of the questions arising from this type of research is about the mathematical learning taking place: what did the students learn? Did they learn what they were intended to learn? What can we say about the nature and extent of their learning?

It is this last question that drives this paper. To begin, the study is briefly described, to provide a context for the discussion that follows. The theoretical framing for the study follows, laying out the basis on which the analysis is founded. A two-step analysis leads to concluding remarks, including some tentative findings and reflections on the findings and methods.

THE STUDY

The study here presents the development of analytical methods and preliminary results for my PhD study, which has the overarching aim to investigate the effective use of computer software in secondary mathematics classrooms, focusing on a number of areas including student learning of mathematics as they work at the computer. Here one lesson (of a series of six on the topic) has been chosen as a starting point for the analysis.

In the lesson, students are required to produce graphs representing quadratic functions using the software package *Autograph* and then to sketch the graphs onto a worksheet. The students arrive at the lesson having found the roots of the quadratic functions by factorisation. The teacher's intention is that, during this lesson, they will notice that the solutions are the same as the x-intercepts on the graph and that the constant in the function gives the y-intercept.

In *Autograph* a default scale ($-6 \le x \le 6, -4 \le y \le 4$) is used each time a new page is opened, but this can be changed at any stage, and in order to see the key points of the graph (intercepts and turning point) it needs to be changed for all the functions on the worksheet. If the graph is drawn before setting the scales, new scales can be entered and may be based on the output on the screen. More than one attempt may be required. However, knowing what the intercepts are may be a more efficient way to determine a suitable scale. The hope is that focusing on the three intercepts needed to set the scale will lead students to pay attention to the relationship between the solutions and the intercepts.

For example, consider the function $y= x^2 - 6x + 8$. To find the roots, factorise and solve for zero: $(x - 2)(x - 4) = 0$; the roots are 2 and 4. To find the y-intercept, set $x = 0$, which gives $y = 8$. This suggests that the axes should be set similar to those chosen here (figure 1, which shows the relevant part of the dialogue box). The graph produced by the software is shown in figure 2.

Figure 1 – changing scales

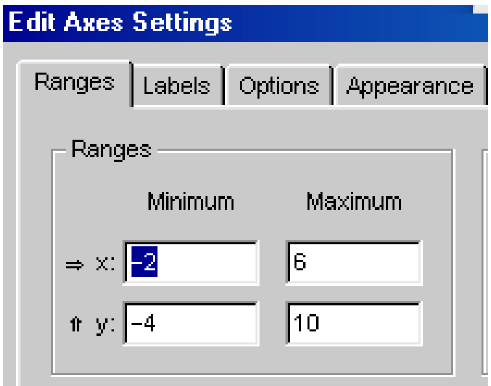
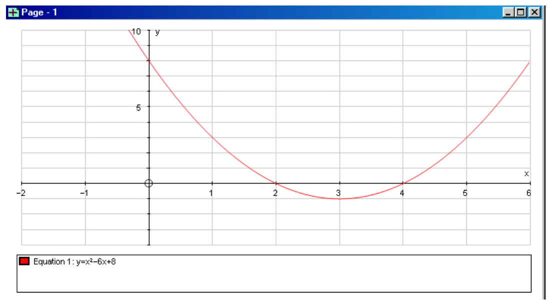


Figure 2 – the graph produced by the software



This paper explores the mathematical learning of the students as they engage in the activity, focusing, in particular, on the activity directed towards changing scale. These interactions are chosen because, as the discussion above points out, these interactions are tied so closely to an appreciation of the importance of the intercepts. Recall that it was 'noticing the intercepts' that the teacher had stated was her aim for the lesson.

What, though, do we mean by mathematical learning? What data is needed for this exploration, and through which theoretical lens shall we view the data? The next section addresses these questions.

THEORETICAL FRAMING AND INITIAL METHODOLOGICAL CHOICES

The perspective adopted for the study is a socio-cultural one; in particular, learning is seen as the active construction of knowledge (or 'knowings' (Sutherland and Balacheff, 1999)) by students (Vygotsky, 1978). Clearly much of this activity cannot be observed. What can be observed, however, is what the students say and do as they work and it is this activity (or behaviour) which may be able to provide clues about the nature and extent of (mathematical) learning (Balacheff, 2005). In particular, the way the students move through the task may be an indicator of learning.

This perspective suggests the need for a qualitative case study approach. In particular, the need is for rich data on interactions; video was used to capture these. Within the classroom, a small group of students was chosen by the teacher as a 'focus' group, to be studied in depth. The activity of this small group of students, as they interact with one another, with the computer and with the teacher, is the focus of the analysis presented in this paper.

Brousseau's (1997) notion of 'modes of production' is used as a basis for the development of coding categories; he suggests that students engage in three types of mathematical activity in mathematics lessons: action, formulation and validation.

The modes of production are explained below, drawing examples from the context of the study.

Brousseau's modes of production

Situations of action are described as:

> The process by which the student forms strategies, that is to say, “teaches herself” a method of solving her problem. (p9)

Further, in situations of action students use 'implicit models' in which they make decisions based on rules and relationships of which they may not yet be conscious. Communications in this category 'appear in a code so easy that … it will play no role in the game.' (p 61) Actions could be, for example, typing the equation of the function into the software to produce a graph, or copying a graph from the computer screen to the worksheet.

Formulation occurs when a student becomes conscious of her strategies and begins to make suggestions. Brousseau includes in this category 'classifying orders, questions etc….'(p 61) He goes on to say that in these communications students do not 'expect to be contradicted or called upon to verify … information'. (p61) A formulation could be, for example, a 'what if ….?' question, a suggestion, or a guess. An example would be a suggestion about where the graph will cut the x-axis.

It is when an interaction intentionally includes an element of proof, theorem or explanation and is treated thus by the interaction partner (or interlocutor), that it becomes a validation: 'this means that the interlocutor must be able to provide feedback…' (p16). Examples of interactions falling into this class would include, for example, an explanation about why the choice of scale was correct.

Finally consider the relationship between the three modes of production; we can see them as embedded, as Brousseau suggests in the concluding remarks on validation:

> A dialectic of validation is itself a dialectic of formulation and therefore a dialectic of action.(p17)

Expanding and adapting the theoretical framework:

Brousseau uses a game to contextualise his theory; it provides an excellent exemplar but to some extent could be seen as designed precisely to draw out the three modes of production. The theory is used to inform the engineering of the didactical situation. However, if the categories are to be useful in the analysis of authentic classroom interactions, where the focus is on the students' behaviours rather than on the design of the task, some expansion and adaptation of these categories is needed.

In particular, if they are to be used to investigate how the students progress through the task, a problem arises with formulations.

Consider two types of formulations; formulation A and formulation B. Formulation A is a suggestion, a question, a guess about the mathematics; perhaps a suggestion about where the graph cuts the x-axis. The student, mid-action, is suggesting a strategy which will allow the students to complete the task. Formulation B, however, is a response to feedback, perhaps from the computer, which acknowledges that the strategy employed was correct or not. For example, the graph produced on the screen has the wrong scale; the formulation now is a suggestion that a new strategy should be used, but it is not a suggestion of what the strategy might be. The categories used need to address this distinction: formulations of type A will be called formulations, those of type B will be called feedback.

Further categories

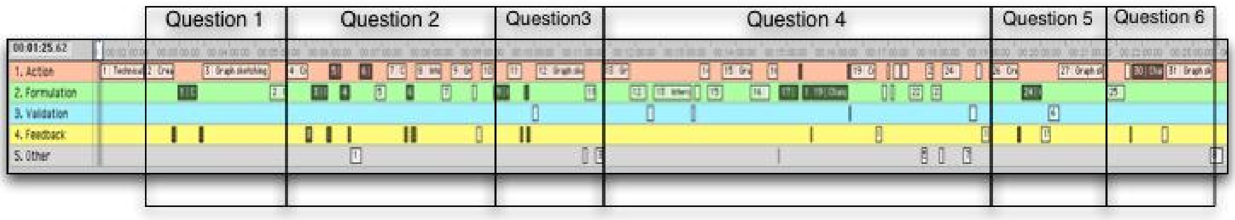
Some interactions between the students are not part of the *mathematics* of the situation. These interactions fall into three principal types: chatting (such as talking about what happened at lunch time), technical (such as trying to get the printer to work) and outside interactions (such as non mathematical intervention by the other students or the teacher) These interactions, while not necessarily providing any explicit information about the mathematical learning taking place, do provide clues about what may or may not be instrumental in preventing or advancing further progress. They may also give an indication of the level to which the students were 'on task'.

For the purposes of coding, the four categories are treated as mutually exclusive.

RESULTS AND DISCUSSION

The data was analysed using software called StudioCode, which produces a timeline showing how the interactions are distributed, within their categories, over time. (See figure 3 below). The top line shows the 'action' interactions, the second shows the formulations, the third the validations and so on. The interactions focusing directly on changing scale have been highlighted (they are darker). The six questions from the worksheet have been superimposed to help understand how the lesson was structured.

Figure 3 – the timeline showing the interactions with worksheet questions superimposed.



What can the distribution, length and patterning of the changing scale (cs) interactions reveal? Can they tell us anything about the students' learning of mathematics? The discussion below focuses on the changing scale interactions, but with reference, as necessary, to other interactions.

Notice, first, that in the first part of the lesson, there are many more cs interactions than in the second part. Further the interactions are generally shorter. The pattern has changed, and points to the need for more in depth investigation; questions 2 and 5 provide a suitable comparison; being neither the first nor the last, appearing to have significantly different patterns, and far enough separated in time to allow for progress.

Both questions begin with the students entering the equation for the graph, followed by feedback which indicates that the current scale settings are not appropriate, which is in turn followed by suggestions for better scales.

In question 2 three attempts at changing scale are needed. (These are the cs formulations). To begin, the suggested scale is entered (change y min to -8)

> 'Go down to -8 and um… um….[*points to the x inputs*] minus…. minus …. ' (Nafessa, question 2)

but the computer feedback indicates that the scale is not suitable. The teacher then arrives and asks if they have a problem. Katie says to her:

> 'Yeah the second one, we can't work out the axes…' (Katie, question 2)

At the same time, Natalie suggests (to Nafessa)

> 'Go down to, like, minus 10' (Natalie, question 2)

In response to the feedback, Katie says

> 'Nearly' (Katie, question 2)

A final attempt (change y min to -12) is needed before enough of the graph can be seen.

The discussion in question 5 provides a marked contrast, both in content and composition. There is only one cs formulation;

> We could have fiddled the axes because we know where…you need to go down to minus 8… just leave the maximum….[*Natalie points the mouse to the x minimum, asking what it should be ....*] Minus 2 and 7 (Katie, Q 5)

The graph now shows all the key points. Note that, not only does Katie suggest suitable scales at the first attempt, but also that she also begins to say that they know how to work it out.

CONCLUSION

This section begins by drawing tentative conclusions based on the data, and goes on to consider further questions raised by the methods and the findings.

What can the evidence above tell us? First, as indicated by both the number of interactions and the content of these interactions, we may be able to say that in the latter part of the lesson the students are choosing their scale with more efficiency. Second, it appears that their confidence about choosing scale has increased. Third, we know from both the video data and the completed worksheets that the students successfully completed the work set. The extent to which this progression corresponds to mathematical learning is left to the reader to consider.

Thinking about the evidence prompts reflection on the usefulness of the framework and the software used. The framework initially considered, with Brousseau's three modes of production, did not seem to have a place for the special role of feedback; given the way feedback drives the classroom activity (to some extent), it became clear that a separate category was required. The output from the software is the timeline, where the interactions, within the analytical framework are shown over time. How, then, do the timeline and categories help in the analytical process? What further analysis is needed? In this study, the timeline first allowed a clear overview of the pattern of the lesson. It showed that the types and patterning of interactions changed over time, but it also demonstrated the need for a deeper exploration of what the students were doing and saying. Here its function was to point out which areas of the lesson may provide the evidence required.

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