USING VISUAL TOOLS TO PROMOTE MATHEMATICAL LEARNING

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This paper describes work done with low achieving 10-year olds. It explores the use and nature of successful visual tools. It discusses the importance of negotiating the labelling of the structure of tools to avoid the learning paradox and the use of tools as mediators between the children's ideas and the world of mathematical symbolism.

BACKGROUND

During my teaching career I found teaching mathematics to low achieving children difficult. Before beginning this research I successfully used mental imagery with a group of low achievers, which led to an investigation of mental imagery in a series of case studies with children of different achievement levels. During the analysis of the case study data, other factors emerged that I then investigated in a teaching project. This paper draws on data from the case studies and the teaching project.

VISUAL TOOLS

In this paper visual tools are defined as any mathematical visual representation of mathematical ideas or symbols. This includes 3D apparatus or 2D drawings, which are used to support the children's mathematical thinking. In the teaching project I used two main types of visual tools, models or drawings of the real life context we used for operations and mathematical structures unconnected to a real life context. This paper focuses on the second type and concentrates on visual tools used to help children understand the number system.

THE CASE STUDIES

Six case studies with children aged 5, 7 and 9 (a low and a high achiever from each age group). Each case study involved between 5 and 8 sessions of an hour. In each session I observed the child working in a whole class teaching session, worked with the child's group and worked with the child on tasks I had devised.

The low achievers needed step-by-step instruction to use visual tools. When counting with cubes Leon (aged 5) counted some more than once and missed others out. A teaching assistant provided step by step instruction by moving them to one side while he counted. Lucy (aged 7) was unable to use base-10 material to add 2-digit numbers. After her teacher told her how to move the pieces she was unable to remember the method required. Lee's (aged 9) teacher used a number square when teaching + and – 1 and 10. This was done as a whole class lesson using a large square. The children were shown how to move 1 square to the right to + 1, 1 down to + 10 and so on. Lee was unable to remember which way to go. His teacher provided clues by asking *'Do you go up or down to add 10?'* If he was right he was praised, if wrong corrected.

The low achievers used tools at their teacher's direction and were unable to use them when left alone to do a similar task.

The high achievers worked in a very different way, tending to use visual tools to help them solve problems. The following conversation with Harry (aged 9) illustrates this. We had talked about the infinite number of fractions between integers during which he had drawn a line labelled 0 and 1. The conversation continued.

SP *What about between negative numbers?*

Harry *No they wouldn't because after a while they get* (stopped speaking).

SP *Have a think about it.* (Silence) *What about between 0 and –1?*

Harry (Extended line from 0 to left to –1 and looked at it) *Yes they will be and between all the others.*

Harry used a visual tool to solve the problem and then extrapolated a general statement about the number system. His learning here is rapid and the only support I gave was to direct his attention to a specific example. Helen (aged 5) and Hope (aged 7) also tended to work with number lines, they also ignored tools used by their teachers. For example, Helen ignored the number square when doing the same task as Lee, simply increasing the first digit to add 10 to a 2-digit number and Hope used number lines rather than base-10 apparatus to add and subtract. All three high achievers appeared to prefer a linear visualisation of the number system.

Considering the data from the case studies, two ideas seemed important. Low achievers needed step-by-step instruction to use the tools. High achievers chose and adapted tools according to the task. This concurs with Bills (1998), who found that the representations teachers introduced in the classroom could be changed or abandoned. The difference between low and high achievers appeared to be that high achievers understood the mathematics inherent in the tool and low achievers did not.

THE LEARNING PARADOX AND MEDIATED ACTION

Cobb, Yackel and Wood's (1992) paper on representational view of mind included Bereiter's idea of the learning paradox. This paper provided a focus and for me an explanation not only of the difficulties I had seen in the low achievers in the case studies but also problems I had encountered in my own teaching. They described representations (or visual tools) as being devised by people who already understand the mathematical relationships being modelled. So to understand those relationships you need to be aware of them in the first place – which is a paradox. The high achievers in the case studies appeared to understand the relationships in the tools they chose, the low achievers did not appear to understand the relationships in the tools given them by their teachers. They suggested that the teacher's choice of representation would be based on his or her own mathematical views and experience. This was evident in the case studies in the use of the number square to add 10 and the idea of exchange with base-10 material. They said that it is not reasonable for mathematical educators to assume that expert interpretation of the tools we use with children matches the children's knowledge and understanding. They suggested that the learning paradox could be avoided if negotiation takes place between the learners' and teacher's position – in other words if time is given to work through the mathematics of the tool. This is a very different way of using visual tools than the way I saw during the case studies.

Nunes (1997) explored the idea of mediated action in which tools are used to help the learner make sense of mathematics. She saw mediated action as *'fundamental to the understanding of mathematical reasoning'* (p.32) but she saw this as a difficult process to achieve because the mathematics inherent in the tools is not obvious to learners. This linked to the idea of the learning paradox and explained the struggles of the low achievers in the case studies.

OUTLINE OF THE TEACHING PROJECT

The teaching project was an action research project over 55 consecutive mathematics lessons. I was responsible for all mathematics teaching during that time. Six 10 year-old low achievers, who were not expected to meet the government target (level 4) in the forthcoming national tests, took part. Each lesson was recorded on video and the data analysed on a day to day basis.

I used the literature and the case studies to inform my planning in the use of visual tools. When visual tools were used, I did not expect the children to understand the mathematics inherent in the tool but explored it with the children. The children would collectively construct the connections inherent in the tools and so increasingly participate in mathematics. The tools would mediate between what the children knew and the mathematics being learnt. My teaching method would emphasise talking together about mathematics rather than individual writing about mathematics.

This paper concentrates on two of the tools –number lines and a set of fraction shapes.

NUMBER LINES

The number line had 11 spikes, two of which would be labelled. The children were asked to label the remaining spikes and sometimes place numbers in a space between the spikes. To do the latter they learned a technique called 'magnification', where they identified and magnified the space by drawing a new magnified line for the space. They then worked out where on the magnified line the number was before finally transferring this information back to the original line. This built a familiarity with the visual tool and the number system.

Number lines were also used for work on decimals. There was a gap of a month between the work on magnification and beginning decimals. We began by magnifying spaces on lines labelled from 0 to 1000, 0 to 100 and 0 to 10. The children had no difficulty, two children driving the task forward. Sally then said *'If you magnify that bit there* (0 to 1) *you'll have decimals'*. When Fran asked what would happened if you magnify decimals, Lance replied, *'you get diddy numbers'*.

I appeared to be redundant as they completed the description of the magnification pattern and introduced decimals. Lance's answer showed a general awareness that each time a section was magnified the new sections were smaller than the original ones. The following day I introduced a 0 to 1 number line. The children had no difficulty labelling the spikes but in discussion they showed that they confused decimals and negative numbers. When I asked whether 0.1 was larger or smaller than zero Angela said, *'smaller like all the minus numbers'* and the others agreed. I used the number line to show the children that 0.1 is larger than 0 and they then used their fraction knowledge to decide how much larger.

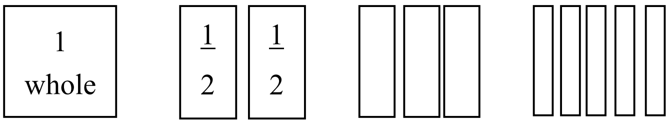
The children's contribution of ideas during this early work on decimals was a typical of how visual tools were negotiated. I knew from my experience of the children that they had formed ideas about mathematics from previous experience both in and out of the group. Ideas encountered in earlier years had caused problems in the past when the children had not understood in class and had tried to make to make sense of the concept for themselves using whatever knowledge they had. By allowing them to talk about their ideas I gained valuable information. Without this information I would not have mentioned negative numbers and they would have remained an unheard issue with the potential to confuse them.

After this the work progressed rapidly. Working with numbers to one decimal place, the children understood that the pattern established between 0 and 1 could be transferred to other integers. In a challenge game they were able to label any spike between any two integers. They had no difficulty magnifying the space between 0 and 0.1 or working with numbers to 2 decimal places. When ordering 0.7 and 0.51, I challenged them saying, *'Are you telling me that 7 is larger than 51?'*. They supplied the explanation, *'It's not 7 it's 70'*. They had no difficulty adding, subtracting or multiplying decimals. They did this a game situation without being taught, by applying the computation strategies they had used with whole numbers.

There were three unusual features to the children's work on decimals. First they transferred knowledge from work on whole numbers. Second they applied the concept to all integers. Third this work on decimals took far less time than I expected. The time spent on working with number lines early in the project appeared to have given the children an understanding of the connections and patterns in the number system and these connections were so embedded they were able to transfer their knowledge. The structure of the visual tool and the time given to negotiating the symbolic labels onto the structure appeared to play a part in that connection making.

FRACTION CARDS

It was an incident with fraction cards that alerted me to the importance of the negotiating the labels onto the structure. As so often happened during the project it was a problem that gave me insight. After a problem my analysis had a different flavour than when work was progressing smoothly. The presence of the question *'why did that not work?'* was a trigger to looking at new solutions new solutions. During earlier work on fractions the children had gained an understanding of the denominator but the numerator remained a problem. Fraction cards, see figure 1, were introduced to solve this difficulty.



Made of different coloured card

Also quarters, eighths, sixths, ninths, tenths

Figure 1: fraction cards

The first task involved using the cards to order 2 fraction symbols. This required three actions - choosing the correct type of fraction card by looking at the denominator; choosing the correct number of fraction cards by looking at the numerator; comparing to find the larger. The children selected the correct type of card but ignored the numerator and chose one of each type, for example, when comparing 2 thirds and 3 fifths they compared 1 third and 1 fifth.

In my analysis that evening I returned to the idea of the learning paradox. What was clear to me was not clear to the children because I had not allowed the children enough time to negotiate the meaning of the cards. In particular they did not understand how the labelling worked. I made a new set of cards without labels and the children labelled these cards and played a building fractions game. After that they were able to use them in the ordering and equivalence tasks without difficulty. Eventually they discarded the cards. This was a gradual process as they began to 'see in their mind's eye' whether a fraction built from fraction cards would be larger or smaller than ½. Eventually they began to use symbolic methods, working out the arithmetic relationship between the numerator and denominator. This way of working related to Nunes idea that tools can be used as mediators between the children's ideas and the mathematical world of symbols.

DISCUSSION

With the fraction cards both the learning paradox and the idea of mediated action were present. The first was a bar to learning, the second an aid to learning. With both the number lines and the fraction cards, the children had to negotiate how the symbols fitted onto the structure. The idea of negotiating the symbolic labelling of a structure to clarify the mathematics inherent in the tool seems an important point. Cobb, Yackel and Wood discussed negotiating representations to overcome the learning paradox. The idea of teachers providing a structure and the children negotiating the mathematical symbols onto the structure appears to be one practical way of avoiding the learning paradox.

It is important to ask what kind of structure is most suitable. The evidence showed that it should be a clearly defined structure that does not change. This enables the children to gain familiarity with the structure and to make connections. The children need to place symbols onto the structure. The structure needs to be adaptable for different numbers. Both number lines and fractions fitted the first two criteria but the fraction cards were less adaptable than number lines.

A supplement to the National Numeracy Strategy (DfES, 1992) suggested that pupils with dyslexia find number squares and pictures of base 10 materials difficult. It suggests that 100 bead frames should precede work on number squares and that children should handle base 10 material first. These suggestions do not appear to provide answers to the problems encountered in my research, which suggested that low achievers have difficulty in connecting symbolic labels to structures. The NNS supplement also suggested that these pupils have problems in interpreting the interval-based structure of a number line and that children would benefit from working on emptier lines. This appears to be a helpful suggestion. However, teachers need to be aware that the reason for working on emptier lines is to allow negotiation of labelling of the spikes and the nature of the intervals to take place. Policy makers need to give teachers more general guidelines as to what to look for in the tools they use with children so that they use the tools with more knowledge.

The teaching project was a small-scale project involving just 6 children – although it was over an intensive 3-month time scale and used a range of visual tools (far more than I have described here). The findings presented here need further research. Given the large-scale use of visual tools in primary classrooms and the way tools were being used in the case study classrooms, we need to know what are the best tools for low achievers and how they need to be presented and negotiated. This will help teachers avoid the learning paradox and chose tools that make good mediators.

Finally policy makers and teachers may be concerned that negotiating tools will disrupt the timetable given in the National Numeracy Strategy. However five of the children achieved the required level 4 in their tests disproving their low predicted targets, showing that the time spent negotiating mathematics is worthwhile.

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